Name: ______________________ March 27, 2008

16.513 Midterm Exam (Spring 2008)

There are 5 problems (30 pts) and one bonus problem (6 pts). Please show the detailed computation.

1. (5) Find the rank for the following matrices

$$a)\ A_1=\begin{bmatrix}0&1&-1\\1&2&0\\2&3&1\\3&4&2\end{bmatrix},\qquad b)\ A_2=\begin{bmatrix}1&1&0\\-1&1&1\\\cos\theta&\sin\theta&0\end{bmatrix},\ \theta\in[0,2\pi],\qquad c)\ A_3=\begin{bmatrix}1&a&1\\1&-1&-a\end{bmatrix},\ a\in R$$

2. (5) Given the old basis $\{e_1,e_2,e_3\}$. Let the new basis be $\hat{e}_1=e_2+e_3$, $\hat{e}_2=e_1+e_3$, $\hat{e}_3=e_1+e_2$.
   a. For $x=e_1-e_2$, find a,b,c such that $x=a\hat{e}_1+b\hat{e}_2+c\hat{e}_3$
   b. For $y=2\hat{e}_1+4\hat{e}_2$, find α,β,γ such that $y=\alpha e_1+\beta e_2+\gamma e_3$

3. (7) Find the general solution to the following equations:

$$(1)\ \begin{bmatrix}0&1&-1&-1\\1&1&0&1\end{bmatrix}x=\begin{bmatrix}2\\3\end{bmatrix};\qquad (2)\ \begin{bmatrix}-3&1\\6&-2\end{bmatrix}x=\begin{bmatrix}1\\-1\end{bmatrix},\qquad (3)\ \begin{bmatrix}1&0&-1&2\end{bmatrix}x=-2$$

4. (7) For $A_1=\begin{bmatrix}5&6\\-3&-4\end{bmatrix}$, $A_2=\begin{bmatrix}2&0&0\\3&-2&-1\\-3&1&0\end{bmatrix}$, find the diagonal form (or Jordan form) $\overline{A}_i$ and Q_i such that $\overline{A}_i=Q_i^{-1}A_iQ_i$.

5. (6) For $A_1=\begin{bmatrix}3&2\\-1&0\end{bmatrix}$, find β_0,β_1 such that $A_1^{k+2}=\beta_0I+\beta_1A_1$; For $A_2=\begin{bmatrix}2&0&0\\1&0&-1\\-1&1&2\end{bmatrix}$, find β_0,β_1,β_2 such that $A_2^k=\beta_0I+\beta_1A_2+\beta_2A_2^2$. Assume k>3.

Bonus problems (6) : For the following circuit, let $x=\begin{bmatrix}v_1\\v_2\\i\end{bmatrix}$ and $u=\begin{bmatrix}u_1\\u_2\end{bmatrix}$. Find matrices A, B such that

$\dot{x}=Ax+Bu$