PRODUCTION AND OPERATIONS MANAGEMENT
Vol. 9, No. 4, Winter 2000
Printed in U.S.A.

COMBINED PRODUCTION AND MAINTENANCE SCHEDULING FOR A MULTIPLE-PRODUCT, SINGLE-MACHINE PRODUCTION SYSTEM*

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Traditionally, the problems of equipment maintenance scheduling and production scheduling in a multi-product environment have been treated independently. In this paper, we develop a Markov decision process model that simultaneously determines maintenance and production schedules for a multiple-product, single-machine production system, accounting for the fact that equipment condition can affect the yield of different product types differently. The problem was motivated by an application in semiconductor manufacturing. After examining structural properties of the optimal policy, we compare the combined method to an approach often used in practice. In the nearly 6,000 test problems studied, the reward from the combined method was an average of more than 25 percent greater than the reward from the traditional method.
(PRODUCTION SCHEDULING; EQUIPMENT MAINTENANCE; MARKOV DECISION PROCESSES; SEMICONDUCTOR MANUFACTURING)

1. Introduction

In the semiconductor industry, it is common for a production facility (called a *fab*) to produce many products of varying types and technological complexities. Using the same equipment set, one fab can produce memory devices, logic circuits, and microprocessors, each with different processing requirements. One way to differentiate products is by their *minimum feature size*, the smallest component in the circuit. Minimum feature sizes range from 10 to 0.25 microns (1 micron = 10^{-6} m; a human hair is roughly 100 microns wide). *Yield*, the percentage of working devices that emerge from the fabrication process, is undoubtedly the most important performance metric for most semiconductor fabs, and particulate contamination is one of the primary sources of yield loss (Borden and Larson 1989). As one might expect, devices with smaller minimum feature sizes are more susceptible to this kind of defect, so minimum feature size can be considered an indicator of product sensitivity.

Products with different minimum feature sizes have significantly different processing requirements in terms of the precision and cleanliness of the production equipment. Consider, for example, the etch operation in which a reactive ion plasma is used to strip certain chemicals from the surface of the semiconductor device. Each device undergoes the etch

* Received October 1996; revisions received September 1999 and October 2000; accepted October 2000.

1059-1478/00/0904/379$1.25

operation up to 15 times during its production cycle, and the success of this operation depends heavily on the cleanliness of the machine chambers. As a machine is used more and more between cleaning operations, particles accumulate in the chambers, and the risk of product contamination increases. In other words, the "state" of the machine deteriorates over time until production is stopped and the machine is cleaned. Several operations in the semiconductor manufacturing process follow this pattern.

In the past, the machine condition was gauged by processing a test unit and then examining the unit for particle contamination. The development of *in situ* particle monitors has made it possible to accurately assess the level of machine contamination in real-time, without processing test units (Peters 1992; May and Spanos 1993). Using such information, it would seem beneficial to produce more-sensitive products immediately after the equipment is cleaned and to produce progressively less-sensitive products as the machine state deteriorates. Surprisingly, although *in situ* monitors are increasingly being used to improve cleaning procedures, little effort has been made to use information about the machine condition for lot dispatching decisions.

A review of the literature related to this problem reveals that the issues of dispatching and equipment maintenance have been treated independently by researchers as well. Maintenance scheduling models do not account for the effect of equipment condition on yield. Production scheduling models in which equipment condition affects yield assume that either a single product is produced or that the effect on all products is the same. The purpose of this paper is to demonstrate the advantages of combining elements of both areas into a single decision process.

We develop a Markov decision process model of a single-machine production system with multiple products and with multiple machine states. The problem is to determine a production and maintenance schedule that maximizes long-run expected average profit. Possible actions include producing one of the product types and cleaning the machine. After examining structural properties of the optimal policy, we compare the combined method to the approach often used in practice: a simple threshold maintenance policy and a first-come, first-serve dispatching policy. The results of nearly 6,000 test problems covering a wide range of parameter values suggest that the improvement provided by the combined method increases as the diversity of the products increases and that most of the improvement is due to changing the production schedule rather than the maintenance schedule. In the problems studied, the reward from the combined method was an average of more than 25 percent greater than the reward from the traditional method.

2. Literature Review

The literature related to the problem described above can be divided into three categories: maintenance models, production models, and combined production and maintenance models. Each category is discussed below.

2.1. *Maintenance Models*

Maintenance models address the problem of when to inspect, repair, and/or replace equipment that deteriorates over time and eventually fails. McCall (1965), Pierskalla and Voelker (1976), Sherif and Smith (1981), and Valdez-Flores and Feldman (1989) provide detailed reviews of this literature. Cho and Parlar (1991) and Dekker, Wildeman, and Van der Duyn Schouten (1997) review the literature on maintenance of multi-component systems.

Many maintenance models consider only two possible actions: replace and do nothing. The aim of such models is to determine the machine condition—sometimes called the *control limit*—at which it is better to replace the machine than continue operating it. Derman (1963), Kolesar (1966), and Kalymon (1972) all develop models of this type, examining the effects of different deterioration patterns and cost structures. Other maintenance models allow the possibility of intermediate repair actions or preventive maintenance. Kamien and Schwartz

(1971), Bobos and Protonotarios (1978), Tijms and Van der Duyn Schouten (1985), and Hopp and Wu (1990) propose models of this type, studying the effects of different constraints on inspection and effectiveness of the repair actions.

The models listed above do not address or incorporate the interaction between equipment condition and production. The models developed by Van der Duyn Schouten and Vanneste (1995) and Meller and Kim (1996) are two exceptions. Both papers model two-machine production systems in which the two machines are separated by an inventory buffer, and the first machine is subject to failure. The maintenance policy for the first machine clearly has an effect on the inventory buffer and, hence, the operation of the second machine. However, in both models the implicit assumption is that a single product is produced or that the equipment condition does not affect the products differently.

In summary, these models address questions of when equipment maintenance should be performed, and what type of maintenance should be performed. They do not address questions related to how much to produce and which product type to produce.

2.2. *Production Models*

Production models address the questions of how much and what to produce. Here, we consider only production scheduling models that incorporate equipment condition in some fashion. The most common way that production scheduling models incorporate equipment condition is simply to allow for machine failure. Machine failure has been studied in the context of continuous flow control models (Akella and Kumar 1986; Lou, Sethi, and Sorger 1992), discrete part production systems (Hong, Glassy, and Seong 1992; Taylor, Clayton, and Grasso 1982), and lot-sizing models (Groenevelt, Pintelon, and Seidmann 1992a, 1992b). The assumption in these models is that the equipment has only two states: operational and non-operational. The equipment condition does not affect the quality or yield of the production process.

Other production models related to the problem that we study are those that incorporate variable yield. Yano and Lee (1995) present a comprehensive review of this literature. Most of the work in this area has focused on single-product systems. Notable exceptions are the works by Akella, Rajagopalan, and Singh (1992) and Gong and Matsuo (1997), which model multi-product, multi-stage systems. But these models, as well as most of the others reviewed by Yano and Lee (1995), do not model yield as a function of the process or equipment condition. Several papers, however, have addressed the interaction between equipment condition and yield, and these are discussed below.

2.3. *Combined Production and Maintenance Models*

Relatively few models combine production and maintenance scheduling issues. Boukas and Haurie (1990) and Boukas, Yang, Zhang, and Yin (1996) examine combined production and maintenance decision problems for a single-product system using continuous-time stochastic flow control models. Boukas and Haurie (1990) assume a fixed corrective maintenance policy and determine the production and *preventive* maintenance policy. Boukas, Yang, Zhang, and Yin (1996) assume a fixed preventive maintenance policy and determine the production and *corrective* maintenance policy. It is important to note that in these models, equipment condition affects costs but does not affect the quality or yield of the output. This is the key difference between these models and the decision problem that we address.

Dedopoulos and Shah (1995, 1996) study maintenance and production scheduling for multi-product chemical processing plants. In the first paper, they formulate a mixed integer linear program to determine how to allocate production equipment, assign maintenance tasks, and plan the flow of products through the system in a way that maximizes value-added. The optimization problem requires information about the product recipes and output targets; the equipment capacity, functionality, and degree of degradation; and the availability of maintenance personnel. In the second paper, they formulate a mixed integer optimization problem that determines an aggregate maintenance plan (when to perform maintenance and what type of maintenance to perform) that takes long-term production needs into account. The primary difference between these models and ours is that while the equipment condition may affect costs and throughput, it does not affect the quality or yield of the output.

Rosenblatt and Lee (1986) propose one of the first models that explicitly links equipment (or process) condition, product quality, and production scheduling. They extend the framework of the classical economic manufacturing quantity (EMQ) model to examine the effects of imperfect quality on the determination of an optimal production run length. Porteus (1986) uses a similar approach. Rosenblatt and Lee (1986) and Porteus (1986) both show that the possibility of producing defective items results in a batch size smaller than the classical EMQ.

Several researchers have extended these models in various ways. Lee and Rosenblatt (1987) allow inspections during the production cycle. Porteus (1990) also allows inspections, but there is a delay between inspection and detection of defective units. Lee and Rosenblatt (1989) incorporate variable restoration costs and shortage costs. Lee and Park (1991) consider the case when the cost of defective items depends on whether the defect is discovered before or after the sale of the item. Makis and Fung (1998) allow inspections during the production cycle and incorporate machine failures. The key distinction between these models and ours is that our model treats the case of a multiple-product environment.

In summary, much work has been done in the areas of maintenance scheduling and production scheduling, and some work has been done to combine these two areas. However, none of this work examines the situation in which the equipment condition affects different products in different ways.

3. Combined Production and Maintenance Scheduling Model

3.1. *Model Formulation*

We consider the problem of scheduling production and maintenance for a single-machine, multiple-product system. The condition, or *state*, of the machine deteriorates over time, and this condition affects the probability of successfully producing the various products. Let X_n denote the state of the machine in period n, where $n = 1, 2, \ldots$. The machine can be in any one of $M + 1$ states, indexed by $i = 0, 1, \ldots, M$, where state 0 denotes the best possible condition and state M denotes the worst possible condition. Let a_n denote the action taken in period n. Possible actions include producing one of the product types (denoted as actions $1, 2, \ldots, K$, respectively) or cleaning the machine (denoted as action $K + 1$). We use the index a when referring to an action in the set of all actions, $\{1, 2, \ldots, K + 1\}$, and we use the index k when referring to a product in the set of all products, $\{1, 2, \ldots, K\}$. The probability of successfully completing action a while in state i is denoted as β_{ia}. We also refer to β_{ik} as the *yield* of product k when the machine is in state i. When action a is taken in state i, a reward R_a is earned with probability β_{ia}.

We assume that changes in the machine state depend only on the current state and the action taken. This is expressed as

$$\textit{Assumption 1: } p_{ij}^{a} = \Pr\{X_{n+1} = j | X_n = i, a_n = a; X_{n-1}, a_{n-1}; \ldots ; X_1, a_1\}$$

$$= \Pr\{X_{n+1} = j | X_n = i, a_n = a\}.$$

Then $\{X_n, n \geq 1\}$ forms a Markov chain, and the problem can be treated as a Markov decision process.

A *policy* is a decision rule that prescribes an action for each state, and our objective is to determine a policy that maximizes the long-run expected average profit. Define $\Phi_\pi(i)$ as the long-run expected average profit per unit time when policy π is employed and the initial state is i. More precisely, let

$$\Phi_\pi(i) = \lim_{N\to\infty} \frac{E_\pi[\sum_{n=1}^{N} r(X_n, a_n)|X_1 = i]}{N},$$

where $r(X_n, a_n)$ is the single-period reward earned for taking action a_n in state X_n.

It should be noted that we have implicitly assumed that period lengths, whether producing or repairing, are independent and identically distributed.

Several assumptions are needed to ensure the existence of an optimal policy. We assume that rewards are bounded and that the cost to clean the machine, denoted as C, does not depend on the machine condition. The primary cost associated with cleaning the etch equipment is related to disrupting production and opening the machine chambers, so the level of contamination will not significantly change the cost to clean the equipment. Therefore a constant cleaning cost is appropriate in this context, but a variable cleaning cost could easily be incorporated. We assume that as the machine condition deteriorates, the success probability (or yield) decreases, and the yield for all products is zero when the machine is in the worst state. Cleaning can be initiated from any state, and this action is successful with probability one. These assumptions are summarized as follows:

Assumption 2: $|R_a| < \infty$ for all a.

Assumption 3: β_{ia} is nonincreasing in i for all a.

Assumption 4: $\beta_{Ma} = 0$ for $a \neq K + 1$.

Assumption 5: $\beta_{i,K+1} = 1$ for all i.

We assume that once in state M, the machine cannot leave this state unless it is cleaned. If production is chosen, the state transition is not affected by the choice of which product to produce. This reflects the etch operation in semiconductor manufacturing where we do not expect the deterioration to be influenced by product type because the processing of one product type does not generate more particles than any other. In other contexts, this may not be true and the notation and computation would have to be modified accordingly. We also assume that cleaning the machine returns it to state 0 with probability one. These conditions are expressed as

Assumption 6: $p_{ij}^a = \begin{cases} p_{ij}, & \text{for } a = 1, 2, \ldots, K, \\ 1, & \text{for } a = K + 1, j = 0, \\ 0, & \text{for } a = K + 1, j \neq 0. \end{cases}$

We assume that as the machine condition deteriorates, it is more likely that the machine will go to a worse state than a better state. We can express this as

Assumption 7: For each l, $\sum_{j=l}^{M} p_{ij}$ is nondecreasing in i.

A *stationary* policy is a decision rule that is non-randomized and does not depend on time, i.e., a policy that depends only on the current machine state. The following proposition states that a stationary average-reward optimal policy exists if Assumptions 1 through 7 hold. Furthermore, the optimal policy will be of the control limit type.

PROPOSITION 3.1. *If Assumptions* 1 *through* 7 *hold, then*
(*a*) *there exists a stationary policy* π^* *such that*

$$\Phi_{\pi^*}(i) = \max_\pi \{\Phi_\pi(i)\}, \qquad \textit{for all } i \geq 0; \qquad \textit{and}$$

(*b*) *there exists a threshold state, $\hat{\imath}$, such that the optimal policy cleans the machine when the state is i if $i \geq \hat{\imath}$ and does not clean if $i < \hat{\imath}$.*

The proof can be found in the Appendix.

3.2. *Monotone Optimal Policy*

Thus far we have only made assumptions about the ordering of values with respect to the machine state. Now we consider the implications of imposing an order on values with respect to the actions. Specifically, we consider a case commonly found in the production of semiconductors in which leading-edge products have very small minimum feature sizes. A device with a smaller minimum feature size has more circuitry per unit area than a device with a larger minimum feature size, and, other factors being equal, is worth more. However, devices with smaller minimum feature sizes are also more sensitive to the equipment condition, and, therefore, have lower yields than devices with larger minimum feature sizes. To model this phenomenon, we rank the products in descending order of sensitivity to the machine state; that is, a product with a lower index is *more* sensitive than a product with a higher index. We express these assumptions as

Assumption 8: R_k is nonincreasing in k.

Assumption 9: β_{ik} is nondecreasing in k for all i.

We can go one step further by assuming that as the condition of the machine deteriorates, the expected profit decreases less for less-sensitive products than for more-sensitive products. This can be expressed as

Assumption 10: $R_{k-1}\beta_{i-1,k-1} - R_{k-1}\beta_{i,k-1} \geq R_k\beta_{i-1,k} - R_k\beta_{ik} \geq 0$,

for $k = 2, 3, \ldots, K$ and $i = 1, 2, \ldots, M - 1$. We can now state the following proposition, based on a theorem from Puterman (1994).

PROPOSITION 3.2. *If Assumptions* 1 *through* 10 *hold, then a stationary average-reward optimal policy exists. Furthermore, the optimal action, $a(i)$, is nondecreasing in i.*

The proof can be found in the Appendix. The fact that the optimal action is nondecreasing simply means that we will produce more-sensitive products when the machine condition is good, and as the machine condition deteriorates, we will produce less-sensitive products. Eventually, the machine condition will reach the cleaning threshold, and we will stop and clean the machine. Such monotone policies are desirable because they are intuitive and easy to implement.

3.3. *Solution Method*

We can find the optimal stationary policy using a linear program, first shown for problems of this type by Manne (1960). To solve for the optimal policy, we consider policies that are *randomized*, that is, in which actions are picked according to some probability distribution. Let $\pi(i, a)$ denote a decision rule that specifies action a given that the machine is in state i. Define x_{ia} as the probability that the machine is in state i and action a is taken. The optimal policy can be found by solving the following linear program, which we refer to as LP:

$$\text{maximize} \qquad \sum_i \sum_a x_{ia} r(i, a) \tag{1}$$

$$\text{subject to} \qquad \sum_a x_{ja} = \sum_i \sum_a x_{ia} p_{ij}^a \qquad \text{for all } j,$$

$$\sum_i \sum_a x_{ia} = 1, \qquad \text{and}$$

$$x_{ia} \geq 0 \qquad \text{for all } i,\ a,$$

where $r(i, a)$ is the reward earned for taking action a when the machine is in state i. We denote the optimal solution to this linear program as $\mathbf{x}^* = [x^*_{ia}]$. As shown by Ross (1983), the optimal stationary policy is $\pi^*(i, a) = x^*_{ia}/\Sigma_a x^*_{ia}$. Furthermore, $\mathbf{x}^*$ has the property that for each i, $x^*_{ia} = 0$ for all but one a. In other words, the optimal policy is *not* randomized.

3.4. *Meeting Production Requirements*

In many production environments, it is necessary to meet some pre-specified production requirements. Suppose that we must meet some long-term production schedule in the sense that a certain proportion of total production must consist of a particular product type. Specifically, let γ_k be the long-run proportion of product k required, where $k = 1, 2, \ldots, K$, and $\Sigma_k \gamma_k = 1$.

We can interpret x_{ia} as the long-run proportion of time that the process is in state i and action a is taken. We can then modify LP to include additional constraints to ensure that the long-run average production requirements are met; that is,

$$\frac{\Sigma_i x_{ik}\beta_{ik}}{\Sigma_k \Sigma_i x_{ik}\beta_{ik}} = \gamma_k \qquad \text{for } k = 1, 2, \ldots, K. \tag{2}$$

This can be rewritten as a linear equation as follows:

$$\sum_i x_{ik}\beta_{ik} - \gamma_k \sum_k \sum_i x_{ik}\beta_{ik} = 0 \qquad \text{for } k = 1, 2, \ldots, K.$$

We refer to the linear program with these additional constraints as LP$'$. The optimal policy is determined as follows:

1. Find an optimal basic solution to LP$'$, $\mathbf{x}^*$. Define the set $S^* = \{i\colon x^*_{ia} > 0, \text{ for some } a\}$.
2. For $i \in S^*$, $\pi^*(i, a) = x^*_{ia}/\Sigma_a x^*_{ia}$; for $i \notin S^*$, $\pi^*(i, a) = 1$ for some arbitrary a.

As shown by Ross (1989), a problem of this form will be randomized in at most K states.

3.5. *Sequential Approach*

The combined production and maintenance scheduling model above solves two problems simultaneously. Traditionally, the problems of maintenance scheduling and production scheduling have been solved sequentially. First, one determines the maintenance schedule, ignoring how the machine condition may affect different products differently. Then, given a maintenance schedule, one determines a production schedule. This is often the approach used in industry.

Since the traditional approach to maintenance treats all products the same way, the problem is reduced to choosing between "produce" and "clean" for each state. We find the point at which it is more profitable to stop and clean the machine than to continue producing. When the produce decision is made, we choose which product to produce on a first-come, first-serve (FCFS) basis. On average, we want γ_k of our production to consist of product k. So in the long run, FCFS scheduling would be equivalent to choosing the products based on the production proportions. Put differently, we would choose product k with probability γ_k. Thus, when the machine is in state i, the expected reward for the produce decision is simply $\Sigma_k \gamma_k R_k \beta_{ik}$. The cost of the clean decision is C, regardless of the machine state.

We wish to determine how much of a difference it makes to solve the problems simultaneously rather than sequentially and also how much of a difference the dispatching policy makes. We examine three different approaches, each of which solves the two problems in a slightly different way. The methods are discussed below.

METHOD 1 (SEQUENTIAL, FCFS DISPATCH). This is the traditional approach. A cleaning threshold is determined by solving LP, and an FCFS dispatching policy is used. Thus, the production schedule is determined by solving LP′ with $r(i, k) = \sum_k \gamma_k R_k \beta_{ik}$ and $r(i, K + 1) = -C$ and with the additional constraint that the cleaning threshold be enforced.

METHOD 2 (SEQUENTIAL, YIELD-BASED DISPATCH). As with Method 1, a cleaning threshold is determined by solving LP. Next, a production schedule is determined by solving LP′ with $r(i, a) = R_a \beta_{ia}$ and with the additional constraint that the cleaning threshold be enforced.

METHOD 3 (SIMULTANEOUS, YIELD-BASED DISPATCH). The cleaning and production schedules are determined simultaneously by solving LP′ with $r(i, a) = R_a \beta_{ia}$.

In the next section, we present the results of test problems that illustrate the benefits of solving the problems simultaneously.

4. Numerical Results

In this section, we report the results of numerical experiments designed to explore the effects of making production and maintenance scheduling decisions simultaneously rather than sequentially. First, we describe how the experiments were designed. Next, we summarize the results, show the value of incorporating information about the equipment condition, and examine the effects of changes in each of the model parameters.

4.1. *Experimental Design*

The primary purpose of the experiments is to understand the value of using more information about the equipment condition when making production and maintenance decisions in a multiple-product environment. Specifically, we wish to know the following.

- How much improvement can be made by simultaneously determining the maintenance and production schedules?
- Under what conditions does the simultaneous scheduling have the biggest impact?
- How much improvement comes from improving the production schedule versus the maintenance schedule?
- How important are Assumptions 8, 9, and 10? That is, does the ordering of the products have an effect?

We propose two experiments to answer these questions, one for each set of assumptions. Experiment 1 incorporates Assumptions 1 through 7, and Experiment 2 incorporates Assumptions 1 through 10.

We examine a single-machine system that produces four products. The machine has five states. The model developed in the last section has five parameters, and we treat each of these as a factor in our experiments. Table 1 reports the number of factors and the levels for each factor in the experiments. We use a full factorial experimental design, so we have a total of

TABLE 1

Data for Test Problems

Factor	Description	No. Levels	Factor Values
γ_k	Product mix	11	See Table 2
p_{ij}	Equipment deterioration	3	slow, med., fast
C	Cleaning cost (\$)	3	100, 200, 400
R_k	Reward (% difference)	3	50, 100, 200
β_{ik}	Yield matrix	10	See Appendix

TABLE 2

Product Mix (γ_k) Levels

Level	γ_1	γ_2	γ_3	γ_4
0	0.900	0.034	0.033	0.033
1	0.034	0.900	0.033	0.033
2	0.034	0.033	0.900	0.033
3	0.034	0.033	0.033	0.900
4	0.400	0.100	0.100	0.400
5	0.400	0.100	0.400	0.100
6	0.400	0.400	0.100	0.100
7	0.100	0.400	0.100	0.400
8	0.100	0.400	0.400	0.100
9	0.100	0.100	0.400	0.400
10	0.250	0.250	0.250	0.250

2,970 test problems in each experiment. Below we describe each factor and explain the levels to be tested.

PRODUCT MIX (γ_k). Product Mix describes how much of each product must be made, expressed as a fraction of total production [see Equation (2)]. We test 11 factor levels, where each level is a vector of fractions that sum to one. The levels range from uniform (i.e., each of the four products accounts for 25 percent) to extreme (i.e., one product accounts for 90 percent). The actual values used are reported in Table 2.

EQUIPMENT DETERIORATION (p_{ij}). This factor describes the "rate" at which the machine deteriorates. Each level is a matrix of probabilities, and we test three levels: slow, medium, and fast. Slow deterioration means that the probability of moving to a higher (i.e., worse) state is low. Fast deterioration means that the probability of moving to a higher state is high. Recall, that $p_{ij}^a = p_{ij}$ for $a = 1, 2, \ldots, K$, $p_{i0}^{K+1} = 1$ for all i, and $p_{ij}^{K+1} = 0$ for all $j \neq 0$ by Assumption 6. See Table 3 for the actual p_{ij} values used.

CLEANING COST (C). Cleaning Cost is the cost to clean the equipment and return it to like-new condition (state 0). We test three levels: \$100, \$200, and \$400.

REWARD (R_k). Reward is a vector of rewards, i.e., the selling prices (in dollars), of the finished products. We would expect the relative rewards rather than the absolute rewards to make a difference in the decision problem. Thus, a level describes the percent difference between the rewards of adjacent products. For example, a level of 100 percent means that $R_k = 2R_{k+1}$ for $k = 1, 2, 3$. We set $R_4 = \$100$ throughout the experiments, and we test three levels: 50, 100, and 200 percent. The reward values are reported in Table 4.

YIELD MATRIX (β_{ik}). This is the matrix that indicates the expected yield when producing each product in each state. Probabilities were generated from a beta distribution and sorted

TABLE 3

Equipment Deterioration Probabilities (p_{ij})

Level	Matrix	Level	Matrix	Level	Matrix
slow	$\begin{bmatrix} .9 & .1 & 0 & 0 & 0 \\ 0 & .9 & .1 & 0 & 0 \\ 0 & 0 & .9 & .1 & 0 \\ 0 & 0 & 0 & .9 & .1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$	med.	$\begin{bmatrix} .5 & .5 & 0 & 0 & 0 \\ 0 & .5 & .5 & 0 & 0 \\ 0 & 0 & .5 & .5 & 0 \\ 0 & 0 & 0 & .5 & .5 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$	fast	$\begin{bmatrix} .1 & .9 & 0 & 0 & 0 \\ 0 & .1 & .9 & 0 & 0 \\ 0 & 0 & .1 & .9 & 0 \\ 0 & 0 & 0 & .1 & .9 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

TABLE 4

Reward (R_k) Levels

Level	R_1	R_2	R_3	R_4
50%	337.5	225	150	100
100%	800	400	200	100
200%	2700	900	300	100

to ensure that the assumptions were met. (See the Appendix for more details on how the yield matrices were generated.) We generated 10 matrices and sorted them in ascending order of variance. That is, level 0 refers to the yield matrix with the lowest variance, and level 9 refers to the yield matrix with the highest variance. This is the only factor for which the two experiments differ. For the first experiment, the only yield-related assumptions that must be met are Assumptions 3, 4, and 5. For the second experiment, we also need to ensure that Assumptions 9 and 10 are met. The actual β_{ik} values used are reported in the Appendix.

4.2. *Results*

Table 5 summarizes the results of our experiments. The table reports the percentage improvement provided by Method 3 over Methods 1 and 2 for each experiment. The table lists summary results for the entire set of 2,970 test problems in each experiment as well as for the subset of problems for each factor level. Figures 1 through 5 provide a visual summary of this information.

The average improvement provided by Method 3 over Method 1 for Experiment 1 is 27.3 percent, and the average improvement provided by Method 3 over Method 2 for Experiment 1 is 2.5 percent. For Experiment 2, the average Method 3/Method 1 difference is 32.7 percent, while the average Method 3/Method 2 difference is 3.3 percent. (Some experiments yielded extremely large improvement values; for consistency, we omitted the highest and lowest values in the calculation of all averages.)

One of our goals is to understand whether the improvement is the result of a difference in the maintenance schedule or a difference in the production schedule. To better understand the differences between Method 3 and Method 1 (and hence Method 2) maintenance policies, we compare the proportion of time each method spends cleaning the equipment. We would expect the difference between Method 3 and Method 2 rewards to be small if the difference in cleaning proportions is small. Recall that in our model x_{ia} is the long-run proportion of time that the machine is in state i and action a is taken. Thus, the proportion of time spent cleaning is simply $\Sigma_i x_{i,K+1}$, since cleaning is designated as action $K + 1$. The percentage difference between Method 1 and 3 cleaning proportions is also reported in Table 5.

For Experiment 2, the correlation between the cleaning proportion differences and the Method 3/Method 1 reward differences for all problems is -0.21 (p-value < 0.001). This negative association suggests that by spending more time cleaning, Method 1 is able to increase yields and therefore the reward. (The correlation between the cleaning proportion differences and the Method 3/Method 1 reward differences for all Experiment 1 problems is -0.02, which is not significant.) In contrast, the correlation between the cleaning proportion differences and the Method 3/Method 2 differences for all problems is approximately 0.5 for both Experiments 1 and 2 (p-value < 0.001 for both). As one would expect, this positive association suggests that Method 2 is able to perform better as its cleaning policy gets closer to that of Method 3. Below we discuss the results for both experiments with respect to each parameter.

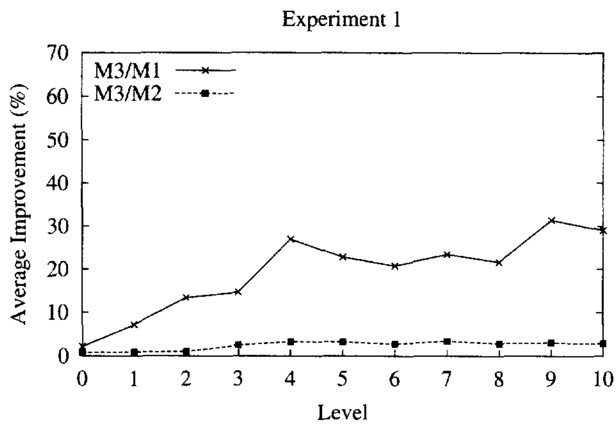
PRODUCT MIX (γ_k). The results suggest that Method 3 provides more improvement when production is spread out among two or more products (levels 4 through 10) rather than

TABLE 5
Numerical Experiment Results

		Experiment 1			Experiment 2		
		Improvement (%)			Improvement (%)		
All Problems		M3/M1[a]	M3/M2[b]	Cleaning Prop. Diff.[c]	M3/M1	M3/M2	Cleaning Prop. Diff.
Average		27.3	2.5	15.5	32.7	3.3	34.2
Maximum		8,664.0	41.9		1,002.8	37.5	
Minimum		0.0	0.0		0.2	0.0	
Factor	Level						
Product Mix (γ_k)	0	2.2	0.8	3.0	3.5	1.2	5.3
	1	7.2	0.9	3.6	13.1	0.5	8.2
	2	13.3	1.0	4.7	33.7	0.5	13.4
	3	14.6	2.5	7.3	61.9	1.5	20.2
	4	26.9	3.3	20.8	38.1	6.4	52.7
	5	22.8	3.2	23.5	31.8	5.4	47.3
	6	20.8	2.8	19.4	25.4	4.1	41.2
	7	23.4	3.4	21.6	31.3	3.6	43.8
	8	21.6	2.9	22.5	27.4	2.8	37.9
	9	31.4	3.0	20.7	48.3	3.6	43.5
	10	29.1	2.9	22.9	32.6	5.9	62.2
Equip. Deterioration (p_{ij})	slow	4.8	2.5	36.8	6.4	3.0	69.2
	med.	18.3	2.7	7.8	27.8	3.6	21.0
	fast	58.4	2.2	1.8	63.7	3.3	12.2
Cleaning Cost (C)	\$100	10.3	2.5	18.0	17.0	3.2	39.8
	\$200	14.8	2.6	15.6	24.0	3.3	34.9
	\$400	48.0	2.4	12.8	56.7	3.3	27.8
Rewards (R_k)	50%	50.3	2.1	12.2	54.0	2.2	21.1
	100%	17.3	2.2	14.7	27.9	3.2	32.8
	200%	9.6	3.1	19.5	15.2	4.4	48.6
Yield Matrix (β_{ik})	0	4.2	0.4	2.5	20.3	2.8	27.0
	1	5.1	1.3	13.7	20.6	2.6	23.2
	2	4.2	0.6	2.3	15.6	3.1	21.7
	3	18.7	1.5	9.5	28.0	2.8	30.3
	4	12.8	2.4	22.0	26.3	2.4	26.0
	5	75.5	6.2	43.6	19.5	3.0	31.9
	6	5.4	3.8	20.7	40.7	4.1	46.4
	7	29.4	1.3	10.0	43.2	3.4	39.8
	8	47.2	3.7	20.1	48.4	3.9	49.4
	9	16.7	3.1	10.3	44.2	4.1	45.9

[a] *M3/M1* is the average percentage improvement of Method 3 over Method 1 for the given factor level.
[b] *M3/M2* is the average percentage improvement of Method 3 over Method 2 for the given factor level.
[c] *Cleaning Prop. Diff.* refers to the percentage difference between the proportion of time Method 3 spends cleaning and the proportion of time Method 1 spends cleaning. A value of x means that Method 1 spends x percent *more* time cleaning than Method 3.

concentrated on one product (levels 0 through 3). This makes sense because if one product dominates the mix, then we are effectively solving a single-product problem. Method 1 will do a better job of choosing the "correct" cleaning policy (i.e., the same as Method 3), and changes in the dispatching policy will have little effect when production is concentrated in a single product.



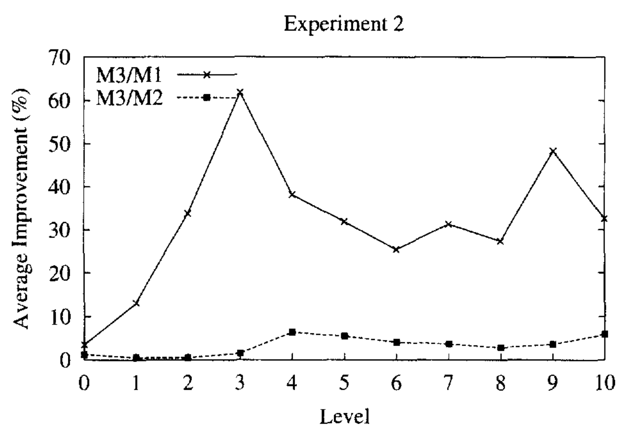


FIGURE 1. Average Improvement as a Function of Product Mix (γ_k).

EQUIPMENT DETERIORATION (p_{ij}). As the deterioration rate increases, the improvement provided by Method 3 over Method 1 also increases for both Experiment 1 and Experiment 2. However, the reward difference between Method 3 and Method 2 is highest when the deterioration rate is "med." We can see a strong association between the differences in cleaning proportions and the Method 3/Method 1 differences. This result suggests that as the deterioration rate increases, Method 1 is better able to choose the "correct" cleaning policy. However, the FCFS dispatching policy used by Method 1 has a major negative effect on yield. Method 2 is able to fine-tune the production schedule and performs almost as well as Method 3.

CLEANING COST (C). The results for Experiments 1 and 2 are quite similar. As the cleaning cost increases, the difference between Method 3 and Method 1 rewards increases. The difference between Method 3 and Method 2 is nearly the same for all levels of cleaning cost. The difference in cleaning proportions decreases as the cleaning cost increases. Apparently, as the cleaning cost increases, Method 1 is better able to choose the correct cleaning policy, but the FCFS dispatching results in lower yields. Method 2 is able to adjust the production schedule and perform reasonably well.

REWARD (R_k). The results for Experiments 1 and 2 are basically the same. As the reward differences increase (i.e., the rewards become more spread out), the Method 3/Method 1 differences decrease while the Method 3/Method 2 differences increase. It is interesting to note that as the reward differences increase, the cleaning proportion differences increase. This supports the idea that higher cleaning proportion differences are associated with lower

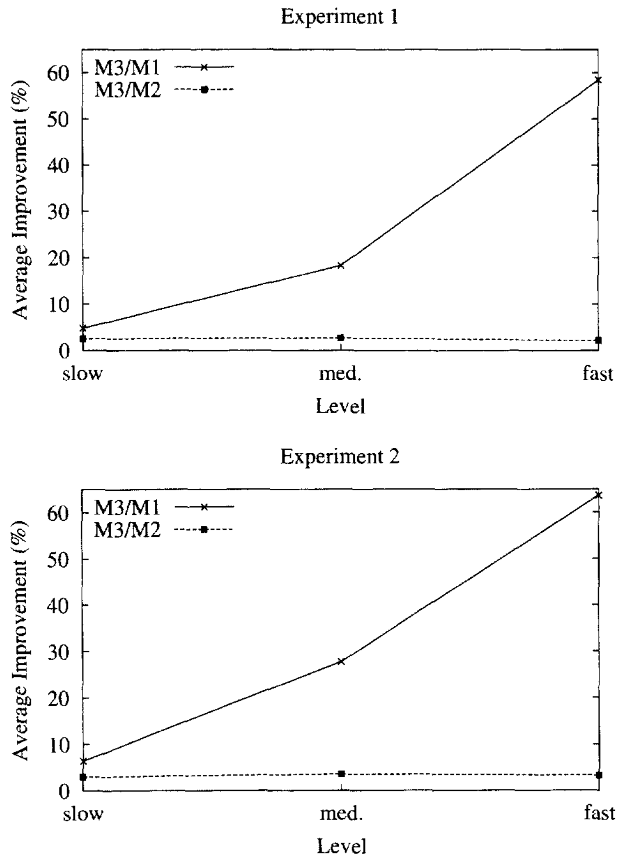


FIGURE 2. Average Improvement as a Function of Equipment Deterioration (p_{ij}).

Method 3/Method 1 differences and are associated with higher Method 3/Method 2 differences.

YIELD MATRIX (β_{ik}). No clear relationship between yield matrix variance and Method 3 improvement is evident for Experiment 1. In Experiment 2, it appears that as the variance of the yield matrix increases, the improvement provided by Method 3 also increases. As would be expected, the Method 3/Method 1 differences are larger than the Method 3/Method 2 differences, but the general trend is the same. The cleaning proportion differences also increase as the yield matrix variance increases. It seems reasonable that the Experiment 2 results would be more consistent given that we have imposed additional structure on the yield matrix.

These results help shed light on the questions posed at the beginning of the section. First, we can conclude that substantial gains can be made by simultaneously determining the maintenance and production schedules. Second, the simultaneous scheduling will have the biggest impact when there is a diverse product set. In other words, the production requirements must be spread out among several products, and the selling prices for the products must also be spread out. When the cleaning cost is high or when the equipment deterioration rate is fast, the Method 1 and Method 3 cleaning policies are likely to be similar. In such cases, the Method 3/Method 1 differences will be large, but the Method 3/Method 2 differences will be small.

Third, much of the improvement provided by simultaneous scheduling appears to be the result of finding a dispatching policy that matches the appropriate machine states and products. Even when the Method 3/Method 1 difference is quite large, Method 2 is able to

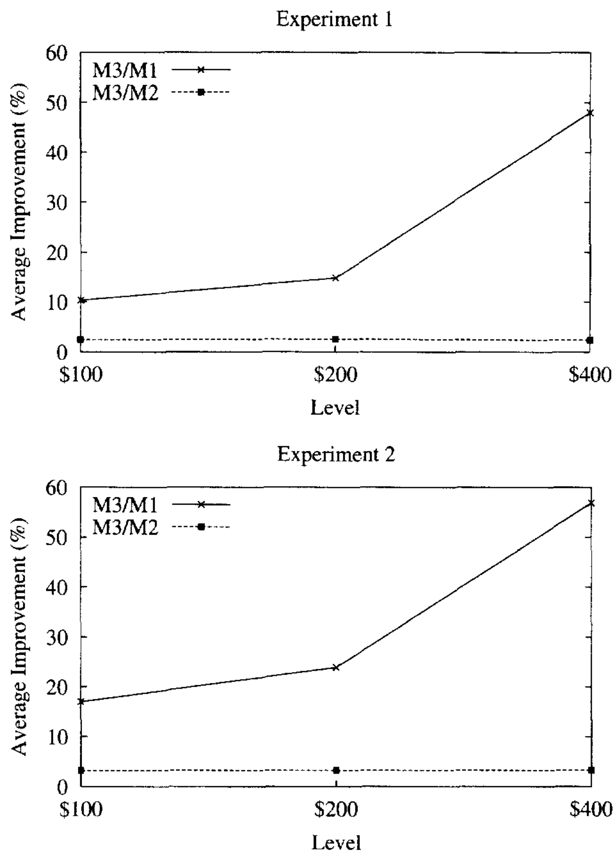


FIGURE 3. Average Improvement as a Function of Cleaning Cost (C).

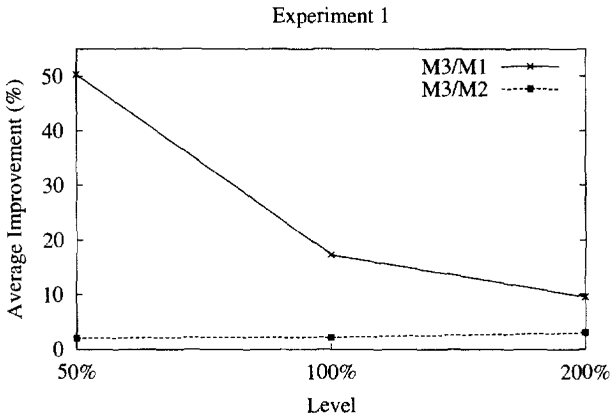
adjust the production schedule and perform well as compared to Method 3. In summary, there is a stiff penalty for ignoring the effect of the equipment condition on yield.

Finally, we can see that the improvement is higher for Experiment 2 than for Experiment 1, although not substantially. This suggests that the combined production and maintenance scheduling model is useful even if Assumptions 8, 9, and 10 are not met. The key factor appears to be the relationship between product yield and the equipment condition.

5. Conclusions

Traditionally, the problems of maintenance scheduling and production scheduling in a multiple-product environment have been treated separately. In this paper, we have demonstrated the usefulness of incorporating issues from both of these areas into a single model. Specifically, the problem of scheduling production and maintenance of a multiple-product manufacturing system with a single, deteriorating machine was modeled as a Markov decision process. The state of the machine, which changes stochastically over time, affects the yield of different product types differently. The decision maker must determine whether to clean the machine or keep producing, and, if production is picked, which product type to produce. The problem was motivated by an application in the semiconductor industry.

We showed that under reasonable assumptions about the rewards and the deterioration of the equipment, a stationary optimal policy exists. We examined the structural properties of the optimal policy and identified sufficient conditions for the policy to be monotone. We



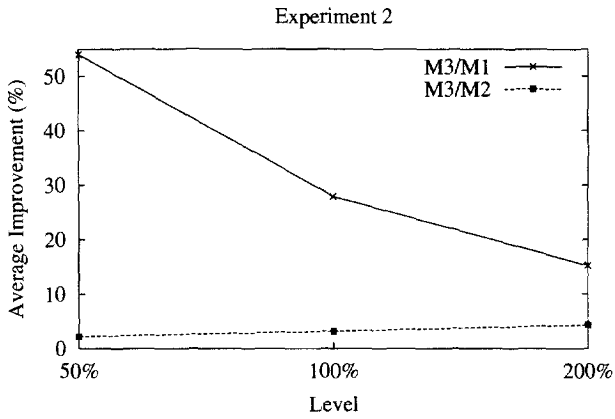


FIGURE 4. Average Improvement as a Function of Rewards (R_k).

compared the combined and traditional methods over a wide range of parameter values. The results suggest that the improvement provided by the combined method increases as the diversity of the products increases and that most of the improvement is due to changing the production schedule rather than the maintenance schedule. In the problems studied, the reward from the combined method was an average of more than 25 percent greater than the reward from the traditional method.

Many opportunities exist to build on this work. In terms of implementation, several issues must be addressed. One must first characterize the machine states and determine the effect of the equipment condition on the different products. In addition, our model implicitly assumes that all of the products are available at each decision epoch, and this may not be true in practice. In terms of future research, one could consider actions other than "produce" and "clean," such as an intermediate cleaning action. Examining how well the lessons learned here apply in a multiple-machine system and exploring the effects of imperfect information about equipment condition would also be of interest.[1]

[1] The authors thank the area editor and the referees for valuable insights and suggestions. This work was supported in part by the Alfred P. Sloan Foundation grant for the study on "Competitive Semiconductor Manufacturing."

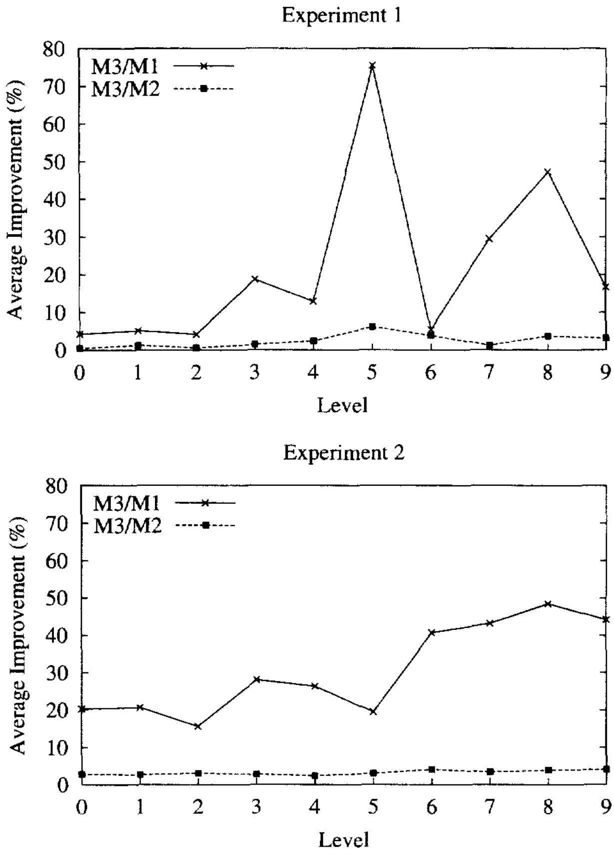


FIGURE 5. Average Improvement as a Function of Yield Matrix (β_{ik}).

Appendix

Proof of Proposition 3.1

The proof of part (*a*) of Proposition 3.1 relies on the following theorem adapted from Ross (1983) and stated without proof.

THEOREM A.1. *If there exists a bounded function $h(i)$, $i \geq 0$ and a constant g such that*

$$g + h(i) = \max_a \left\{ R_a\beta_{ia} + \sum_{j=0}^{M} p_{ij}^a h(j) \right\} \qquad i \geq 0, \tag{A1}$$

then there exists a stationary policy π^* such that

$$g = \Phi_{\pi^*}(i) = \max_\pi \{\Phi_\pi(i)\}, \qquad \textit{for all } i \geq 0,$$

and π^* is any policy that for each i, prescribes an action that maximizes the right side of (A1).

Following the examples of Ross (1983), Heyman and Sobel (1984), and Puterman (1994), the results for the average reward model are built upon a related, discounted reward problem. Define $V_\alpha(i)$ as the expected maximal discounted profit given that the initial state is i and the discount factor is α, where $0 \leq \alpha < 1$. Then $V_\alpha(i)$ satisfies the *optimality equation,*

$$V_\alpha(i) = \max_a \left\{ R_a\beta_{ia} + \alpha \sum_j p_{ij}^a V_\alpha(j) \right\}. \tag{A2}$$

For economy, we state the following lemmas without proof [refer to Sloan (1998) for proofs].

LEMMA A.1. *A stationary discounted-reward optimal policy exists.*

LEMMA A.2. *For $0 \leq \alpha < 1$, $V_\alpha(i)$ is nonincreasing in i.*

We can now apply the following theorem from Ross (1983).

THEOREM A.2. *If there exists an* $N < \infty$ *such that*

$$|V_\alpha(i) - V_\alpha(0)| \le N \qquad \text{for all } \alpha \text{ and all } i,$$

then

(i) *There exists a bounded function* $h(i)$ *and a constant* g *satisfying* (A1);
(ii) *For some sequence* $\alpha_n \to 1$, $h(i) = \lim_{n\to\infty} [V_{\alpha_n}(i) - V_{\alpha_n}(0)]$;
(iii) $\lim_{\alpha\to 1} (1 - \alpha)V_\alpha(0) = g$.

To verify that the condition of the theorem has been met, we apply Assumption 6 to rewrite (A2) as

$$V_\alpha(i) = \max\left\{-C + \alpha V_\alpha(0),\ \max_k \left[R_k\beta_{ik} + \alpha \sum_j p_{ij}V_\alpha(j)\right]\right\}. \tag{A3}$$

For $N > C$ we have

$$V_\alpha(i) \ge -C + \alpha V_\alpha(0) \ge -N + \alpha V_\alpha(0).$$

Since $V_\alpha(i)$ is nonincreasing in i, we have

$$V_\alpha(i) - V_\alpha(0) > -N \qquad \text{and} \qquad V_\alpha(0) - V_\alpha(i) > N$$

for all i and all α. In other words, $|V_\alpha(i) - V_\alpha(0)| \le N$, so the condition of Theorem A.2 holds. Therefore, an average-reward optimal policy exists by Theorem A.1. This completes the proof of part (*a*) of Proposition 3.1.

To prove part (*b*) of Proposition 3.1, we first note that $h(i)$ has the same structural properties as $V_\alpha(i)$ (Ross 1983). This is apparent from the definition of $h(i)$ in part (ii) of Theorem A.2. Thus, $h(i)$ is nonincreasing in i. Assumption 7 states that for each l, $\sum_{j=l}^{M} p_{ij}$ is nondecreasing in i. This is equivalent to the condition that $\sum_j p_{ij}f(j)$ is nonincreasing in i for all nonincreasing functions $f(\cdot)$ (Derman 1963). From (A1), it is optimal to clean the machine if

$$-C + h(0) \ge \max_k \left[R_k\beta_{ik} + \sum_j p_{ij}h(j)\right].$$

The fact that $h(j)$ is nonincreasing in j implies that $\sum_j p_{ij}h(j)$ is nonincreasing in i. Therefore there exists a threshold $\hat{\imath}$ such that the optimal policy cleans the machine when the state is i if $i \ge \hat{\imath}$ and does not clean if $i < \hat{\imath}$, where $\hat{\imath}$ is given by

$$\hat{\imath} = \min\left\{i\colon -C + h(0) \ge \max_k \left[R_k\beta_{ik} + \sum_j p_{ij}h(j)\right]\right\}.$$

This completes the proof of part (*b*) of Proposition 3.1.

Proof of Proposition 3.2

Proposition 3.2 is based on the theorem below, which has been adapted from Puterman (1994). Below, we denote the set of machine states as $\mathcal{S} = \{0, 1, \ldots, M\}$ and the set of possible actions as $\mathcal{A} = \{1, 2, \ldots, K + 1\}$.

THEOREM A.3. *If the following conditions hold, then there exists an average-reward optimal stationary policy such that the optimal action,* $a(i)$, *is nondecreasing in* i:

1. $|R_a\beta_{ia}| < \infty$ for each i and each a.
2. $V_\alpha(i) > -\infty$ for each i and for all $0 \le \alpha < 1$
3. There exists an $N < \infty$ such that $|V_\alpha(i) - V_\alpha(0)| \le N$ for each i and for all $0 \le \alpha < 1$.
4. There exists a non-negative function $M(i)$ such that
   (a) $M(i) < \infty$ for each i.
   (b) $|V_\alpha(i) - V_\alpha(0)| \ge -M(i)$ for each i and for all $0 \le \alpha < 1$.
   (c) $\sum_{j=0}^{M} p_{ij}^{a}M(j) < \infty$ for each i and each a.
5. $R_a\beta_{ia}$ is nonincreasing in i for all a.
6. For each l, $\sum_{j=l}^{M} p_{ij}^{a}$ is nondecreasing in i for all a.
7. $R_a\beta_{ia}$ is supermodular on $\mathcal{S} \times \mathcal{A}$.
8. For each l, $\sum_{j=l}^{M} p_{ij}^{a}$ is supermodular on $\mathcal{S} \times \mathcal{A}$.

For a proof of this theorem, refer to Puterman (1994). Here, we simply wish to show that the conditions of the theorem have been met. The first and second conditions are true by Assumption 2 and the fact that $0 \le \beta_{ia} \le 1$ for all i and all a. To verify the third condition, let $N = C$, as stated immediately following Theorem A.2. To verify the fourth condition, define $M(i)$ as the expected reward earned in a first passage from state i to state 0 under any stationary policy. Assumption 2, Assumption 6, and the fact that the number of states is finite, imply that every stationary policy will induce an irreducible Markov chain. Condition 4 can now be verified by applying Proposition 5 of Sennott (1989). The fifth condition is true by Assumption 3, and the sixth condition is identical to Assumption 7.

A function $f(i, a)$ is *supermodular* if

$$f(i_1, a_1) + f(i_2, a_2) \geq f(i_1, a_2) + f(i_2, a_1) \tag{A4}$$

for $i_1 \geq i_2$ and $a_1 \geq a_2$. Condition 7 can be verified by rewriting (A4) as

$$R_{a_1}\beta_{i_1,a_1} + R_{a_2}\beta_{i_2,a_2} \geq R_{a_2}\beta_{i_1,a_2} + R_{a_1}\beta_{i_2,a_1},$$

which is true for $a_1 < K + 1$ by Assumption 10. For $a_1 = K + 1$, we have

$$-C + R_{a_2}\beta_{i_2,a_2} \geq R_{a_2}\beta_{i_1,a_2} - C,$$

which is true by Assumption 3. Therefore the seventh condition holds.

To verify the eighth condition, first consider $a_1 < K + 1$. This implies that $a_2 < K + 1$ and, by Assumption 6, that $p_{ij}^{a_1} = p_{ij}^{a_2}$ for all i and j. In this case, (A4) becomes

$$\sum_{j=l}^{M} p_{i_1,j} + \sum_{j=l}^{M} p_{i_2,j} \geq \sum_{j=l}^{M} p_{i_1,j} + \sum_{j=l}^{M} p_{i_2,j},$$

which is clearly true for each l. Now consider $a_1 = K + 1$. In this case, (A4) becomes

$$\sum_{j=l}^{M} p_{i_1,j}^{K+1} + \sum_{j=l}^{M} p_{i_2,j} \geq \sum_{j=l}^{M} p_{i_1,j} + \sum_{j=l}^{M} p_{i_2,j}^{K+1},$$

which is true by Assumptions 6 and 7. Therefore condition 8 holds. We have now shown that all of the conditions of Theorem A.3 hold.

Generation of Yield Matrices (β_{ik})

The yield matrices were generated from a beta distribution using the technique described in Bratley, Fox, and Schrage (1987). The beta distribution is useful in this case because we do not have actual yield data, but clearly the values must be between 0 and 1. We used a beta distribution with a mean of 0.6 and variance of 0.13 (shape parameters $\alpha_1 = 0.5$ and $\alpha_2 = 0.33$). For Experiment 1, we generated a vector of yields for each product and sorted them in descending order to ensure that β_{ik} was nonincreasing in i (Assumption 3). For Experiment 2, we fixed base yields for each product (i.e., state 0 yield) and then generated the yields for other states, discarding those that were larger than the base yield. This ensured

TABLE A1

Yield Matrices (β_{ik}) *for Experiment* 1

Level	Matrix	Level	Matrix
0	$\begin{bmatrix} 0.9686 & 0.9598 & 0.9996 & 0.9536 \\ 0.9446 & 0.7211 & 0.9976 & 0.6809 \\ 0.7926 & 0.4685 & 0.9875 & 0.3935 \\ 0.4392 & 0.2927 & 0.0762 & 0.2620 \end{bmatrix}$	5	$\begin{bmatrix} 0.9672 & 0.9965 & 0.9041 & 1.0000 \\ 0.4011 & 0.8101 & 0.7950 & 0.9977 \\ 0.1306 & 0.6000 & 0.7700 & 0.9897 \\ 0.0091 & 0.0047 & 0.3560 & 0.9863 \end{bmatrix}$
1	$\begin{bmatrix} 0.9974 & 0.9790 & 0.5313 & 0.8859 \\ 0.8124 & 0.7077 & 0.4818 & 0.8744 \\ 0.7750 & 0.3460 & 0.2375 & 0.3668 \\ 0.3819 & 0.0003 & 0.0219 & 0.2581 \end{bmatrix}$	6	$\begin{bmatrix} 0.9932 & 0.9064 & 0.9793 & 0.9319 \\ 0.8260 & 0.0684 & 0.9449 & 0.8980 \\ 0.8228 & 0.0425 & 0.6925 & 0.8302 \\ 0.3949 & 0.0315 & 0.2423 & 0.2221 \end{bmatrix}$
2	$\begin{bmatrix} 1.0000 & 0.9997 & 0.8655 & 0.9998 \\ 0.9031 & 0.9973 & 0.6498 & 0.9548 \\ 0.8854 & 0.7204 & 0.4969 & 0.1772 \\ 0.5990 & 0.1316 & 0.2564 & 0.0793 \end{bmatrix}$	7	$\begin{bmatrix} 0.9276 & 0.8965 & 0.8463 & 0.9924 \\ 0.5520 & 0.8889 & 0.5422 & 0.7722 \\ 0.0926 & 0.5272 & 0.3033 & 0.1298 \\ 0.0546 & 0.0169 & 0.0055 & 0.0001 \end{bmatrix}$
3	$\begin{bmatrix} 0.9359 & 1.0000 & 0.9845 & 1.0000 \\ 0.4848 & 1.0000 & 0.5718 & 0.7049 \\ 0.2394 & 0.9883 & 0.3911 & 0.2700 \\ 0.0402 & 0.6110 & 0.1274 & 0.0628 \end{bmatrix}$	8	$\begin{bmatrix} 0.5895 & 0.9603 & 0.9895 & 0.9958 \\ 0.1549 & 0.6240 & 0.9843 & 0.9566 \\ 0.0985 & 0.5147 & 0.4725 & 0.7569 \\ 0.0020 & 0.0026 & 0.0003 & 0.0310 \end{bmatrix}$
4	$\begin{bmatrix} 0.9933 & 0.9997 & 0.9999 & 0.9917 \\ 0.5240 & 0.9660 & 0.9997 & 0.9376 \\ 0.5209 & 0.9215 & 0.3127 & 0.3246 \\ 0.1040 & 0.5330 & 0.1977 & 0.0023 \end{bmatrix}$	9	$\begin{bmatrix} 0.9960 & 0.9991 & 0.8928 & 0.5334 \\ 0.9552 & 0.9789 & 0.3312 & 0.5045 \\ 0.9525 & 0.5244 & 0.0105 & 0.0013 \\ 0.0023 & 0.0613 & 0.0005 & 0.0009 \end{bmatrix}$

TABLE A2

Yield Matrices (β_{ik}) *for Experiment* 2

Level	Matrix	Level	Matrix
0	$\begin{bmatrix} 0.9000 & 0.9500 & 0.9750 & 1.0000 \\ 0.8557 & 0.9262 & 0.9557 & 1.0000 \\ 0.6146 & 0.8898 & 0.9463 & 0.9999 \\ 0.0456 & 0.7136 & 0.8435 & 0.9827 \end{bmatrix}$	5	$\begin{bmatrix} 0.9000 & 0.9500 & 0.9750 & 1.0000 \\ 0.7099 & 0.8628 & 0.9660 & 0.9999 \\ 0.5549 & 0.7358 & 0.9215 & 0.9984 \\ 0.0570 & 0.3253 & 0.8804 & 0.9977 \end{bmatrix}$
1	$\begin{bmatrix} 0.9000 & 0.9500 & 0.9750 & 1.0000 \\ 0.8265 & 0.9327 & 0.9702 & 0.9983 \\ 0.6878 & 0.8955 & 0.9339 & 0.9839 \\ 0.0000 & 0.7142 & 0.9210 & 0.9734 \end{bmatrix}$	6	$\begin{bmatrix} 0.9000 & 0.9500 & 0.9750 & 1.0000 \\ 0.6346 & 0.8966 & 0.9668 & 1.0000 \\ 0.3038 & 0.7960 & 0.9598 & 1.0000 \\ 0.0270 & 0.6844 & 0.9489 & 0.9897 \end{bmatrix}$
2	$\begin{bmatrix} 0.9000 & 0.9500 & 0.9750 & 1.0000 \\ 0.8199 & 0.9179 & 0.9446 & 0.9991 \\ 0.6991 & 0.8906 & 0.9390 & 0.9968 \\ 0.0626 & 0.3369 & 0.9019 & 0.9604 \end{bmatrix}$	7	$\begin{bmatrix} 0.9000 & 0.9500 & 0.9750 & 1.0000 \\ 0.4009 & 0.8758 & 0.9538 & 0.9936 \\ 0.3439 & 0.8418 & 0.9501 & 0.9903 \\ 0.0217 & 0.7084 & 0.8344 & 0.9285 \end{bmatrix}$
3	$\begin{bmatrix} 0.9000 & 0.9500 & 0.9750 & 1.0000 \\ 0.6576 & 0.9422 & 0.9699 & 1.0000 \\ 0.4611 & 0.8650 & 0.9459 & 0.9993 \\ 0.0054 & 0.6753 & 0.8434 & 0.9369 \end{bmatrix}$	8	$\begin{bmatrix} 0.9000 & 0.9500 & 0.9750 & 1.0000 \\ 0.5443 & 0.8967 & 0.9629 & 0.9999 \\ 0.2456 & 0.8576 & 0.9532 & 0.9999 \\ 0.0504 & 0.6803 & 0.9485 & 0.9996 \end{bmatrix}$
4	$\begin{bmatrix} 0.9000 & 0.9500 & 0.9750 & 1.0000 \\ 0.6421 & 0.6991 & 0.8327 & 0.9549 \\ 0.3199 & 0.5879 & 0.7545 & 0.9361 \\ 0.0339 & 0.4417 & 0.6652 & 0.8725 \end{bmatrix}$	9	$\begin{bmatrix} 0.9000 & 0.9500 & 0.9750 & 1.0000 \\ 0.4111 & 0.9279 & 0.9734 & 0.9989 \\ 0.3292 & 0.8535 & 0.9570 & 0.9968 \\ 0.0321 & 0.6132 & 0.9151 & 0.9843 \end{bmatrix}$

that β_{ia} was nonincreasing in i (Assumption 3) and nondecreasing in a (Assumption 9). We then tested to see whether Assumption 10 had been met. If not, we discarded the entire matrix and started over.

Yield Matrix (β_{ik}) *Values*

Tables A1 and A2 report the yield values used in Experiments 1 and 2. By Assumption 4, $\beta_{Ma} = 0$ for $a \neq K + 1$, and by Assumption 5, $\beta_{i,K+1} = 1$ for all i. Thus, we report only β_{ik} values for $i = 0, 1, \ldots, M - 1$ and $k = 1, 2, \ldots, K$ in Tables A1 and A2.

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