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# **2D-FINITE DIFFERENCE TIME DOMAIN (FDTD) NUMERICAL SIMULATION (REBAR SIZE DETECTION IN FREE SPACE)**

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# Outline

- Introduction
- Objective
- Literature Review
- FDTD Numerical Simulation
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# Introduction

- Deterioration of aging civil infrastructure due to corrosion
- Federal Highway Administration (FHWA) estimated the direct cost of corrosion between 1999~2001 to be \$276 billion per year (~3.1 % of 1998GDP ).
- Nondestructive Evaluation (NDE) techniques (e.g., Eddy Current, ultrasonic) have been developed to solve this problem.

# Introduction

- Radar NDE has proved to be successful among these techniques in detecting damages in corroded civil infrastructures.
- Radar NDE involves the generation and sending of electromagnetic (EM) waves which interact with the target through its dielectric properties and geometry.

# Objective

- To detect the size (diameter) of a steel rebar by:
  - observing changes in the return signal of the rebar.
  - comparing the input and the return signal of the rebar.
  - developing mathematical models for predicting the size of rebar.

# Literature Review

- Simulated Transient Electromagnetic Response for the Inspection of GFRP-Wrapped Concrete Cylinders Using Radar NDE. Yu et. al., 2013 [1]
- Analysis of the Electromagnetic Signature of Reinforced Concrete Structures for Nondestructive Evaluation of Corrosion Damage. Roqueta et. al.,2012 [2]
- Effectiveness of 2-D and 2.5-D FDTD Ground Penetrating Radar Modeling for Bridge-Deck Deterioration Evaluated by 3-D FDTD. Belli et. al.,2009 [3]

# FDTD Numerical Simulation

- Maxwell's Curl Equations
  - Four Mathematical equations that govern the radiation of EM waves (e.g., radar signals) by relating the Electric Field,  $E$  (v/m), and the magnetic field,  $B$  (wb/m<sup>2</sup>), to the charge,  $\rho$  (C/m<sup>3</sup>), and current,  $J$  (A/m<sup>3</sup>), densities that specify the fields.
  - For source-free problems in linear and isotropic media, it is given by [4]:

$$\nabla \times H = \partial / \partial t D \dots\dots\dots(1) \qquad \nabla \times D = 0 \dots\dots\dots(3)$$

$$\nabla \times E = -\partial / \partial t B \dots\dots\dots(2) \qquad \nabla \times B = 0 \dots\dots\dots(4)$$

$H$  = magnetic field strength (A/m) ,  $D$  = electric displacement (C/m<sup>2</sup>)

# FDTD Numerical Simulation

- FDTD Methods
  - Maxwell's Curl equations were evaluated in both space and time domains numerically by finite difference methods. Discretization is based on Yee's algorithm [5].
  - Two dimensional code written in Matlab (Matrix Laboratory ® ) by [1] was used in this study.
  - Three fields were simulated; electric field in x and y direction ( $E_x$  and  $E_y$ ) and magnetic field in the z direction ( $H_z$ ).



# FDTD Numerical Simulation

- Input signal: **Modulated Gaussian** signal with a carrier frequency ( $f_c$ ) of **2.81GHz**.
- Computational domain: **300 grids x 900 grids** with spatial increment ( $\Delta x = \Delta y$ ) of **0.00025m** and  **$\Delta t = 4.1696 \times 10^{-4}$  ns**.
- Six numerical simulation cases were studied.

# FDTD Numerical Simulation

- Computational domain

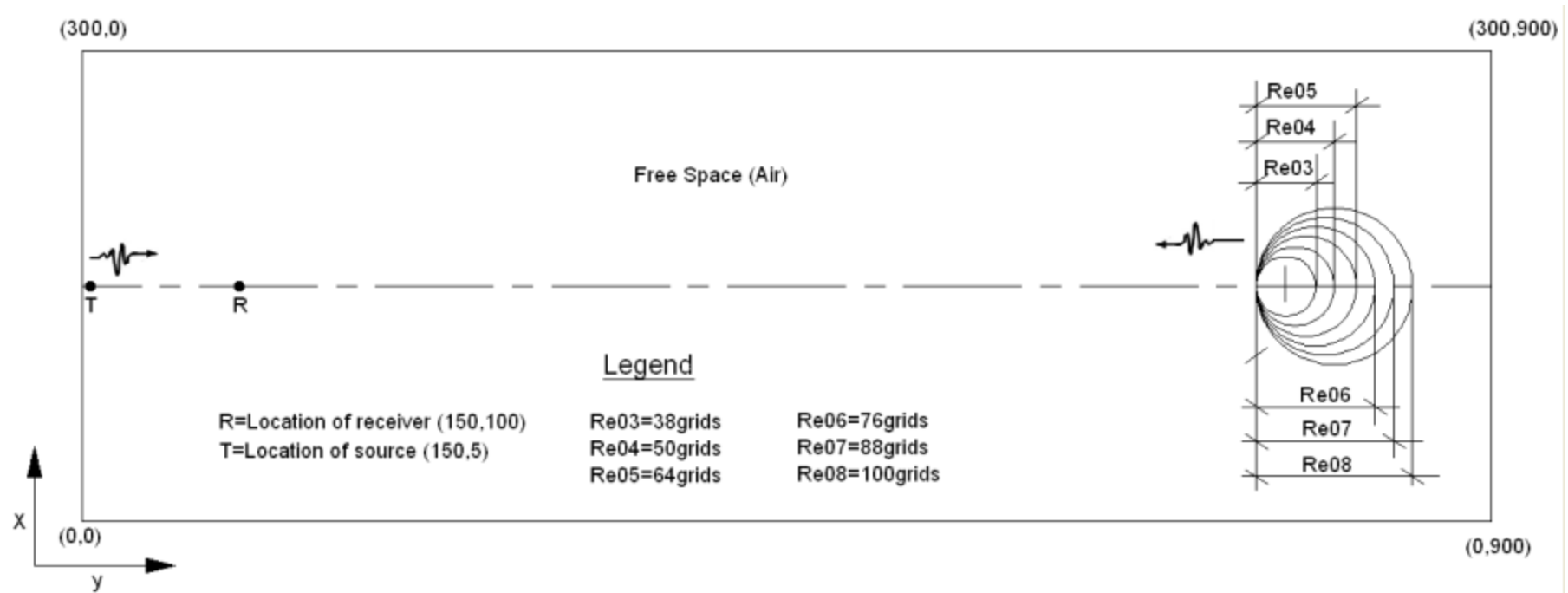


Figure 1. Configuration of the rebars and the computational domain.

# FDTD Numerical Simulation

- Table1: Summary of numerical simulation cases

| Case | Standard Rebar Diameter ( $D_s$ ) (mm) | Rebar Diameter (D) grids (mm) | Coordinate of Center (x,y) | Remarks |
|------|--|-------------------------------|----------------------------|---------|
| Re03 | 9.525                                  | 38 (9.5mm)                    | (150,769)                  | #3      |
| Re04 | 12.7                                   | 50 (12.5mm)                   | (150,775)                  | #4      |
| Re05 | 15.875                                 | 64 (16mm)                     | (150,782)                  | #5      |
| Re06 | 19.05                                  | 76 (19mm)                     | (150,788)                  | #6      |
| Re07 | 22.225                                 | 88 (22mm)                     | (150,794)                  | #7      |
| Re08 | 25.4                                   | 100 (25mm)                    | (150,800)                  | #8      |

- Dielectric constant and conductivity of steel were numerically chosen as **3000**.

# Results & Discussions

- Incident signal

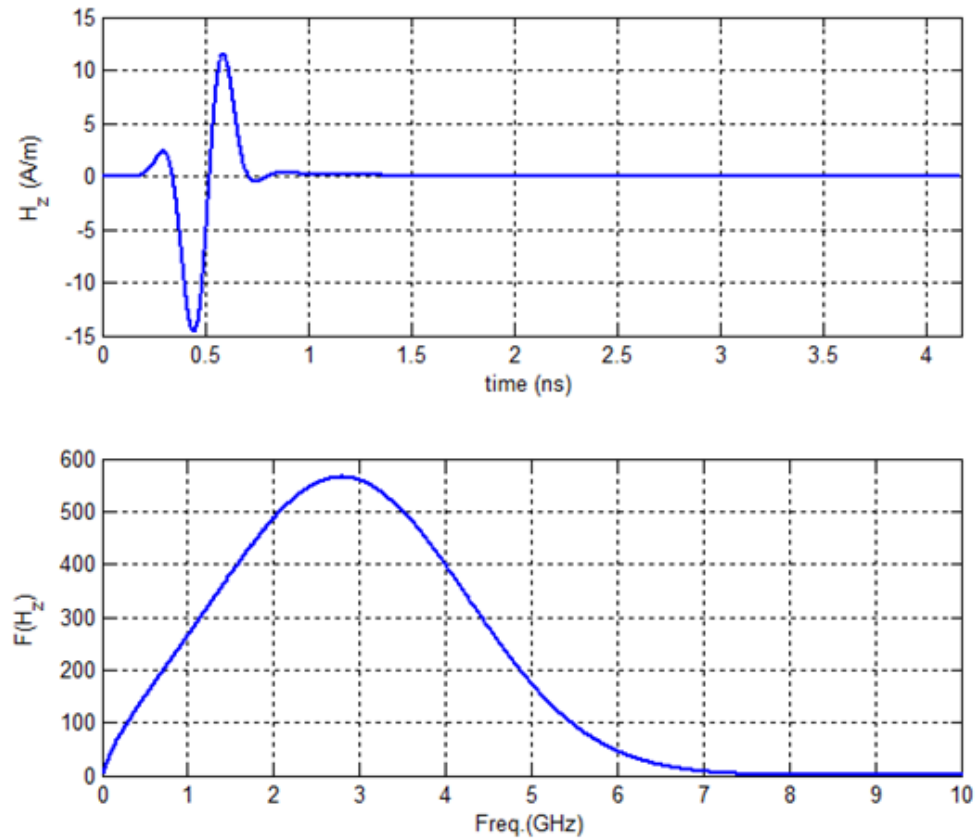


Figure 2. Time and frequency domains representations of the incident signal ( 2.81GHz modulated Gaussian input signal).

# Results & Discussions

- Total response of rebar to EM waves.
- Total response = Incident signal + Net response

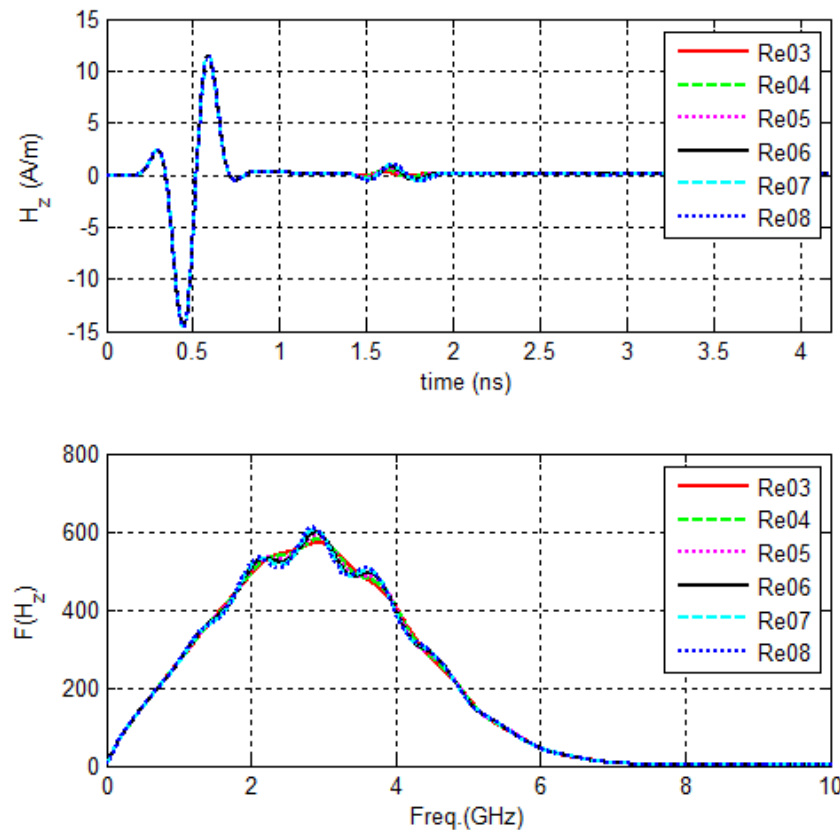


Figure 3. Total time and frequency domains response of rebar to EM waves.

# Results & Discussions

- Net response of rebar to EM waves.

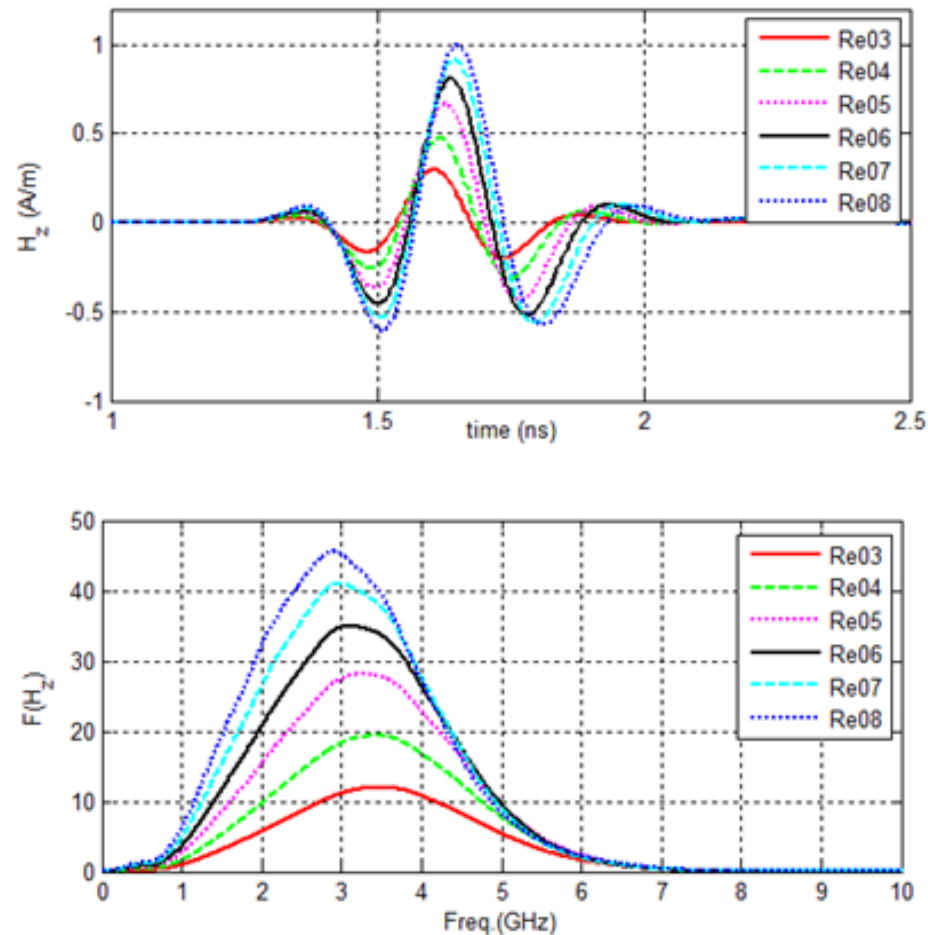


Figure 3. Net time and frequency domains response of rebar to EM waves.

# Results & Discussions

- Relationship between diameter of rebar ( $D$ ) and maximum peak amplitude,  $(H_z)_{max}$ , in net time domain response.

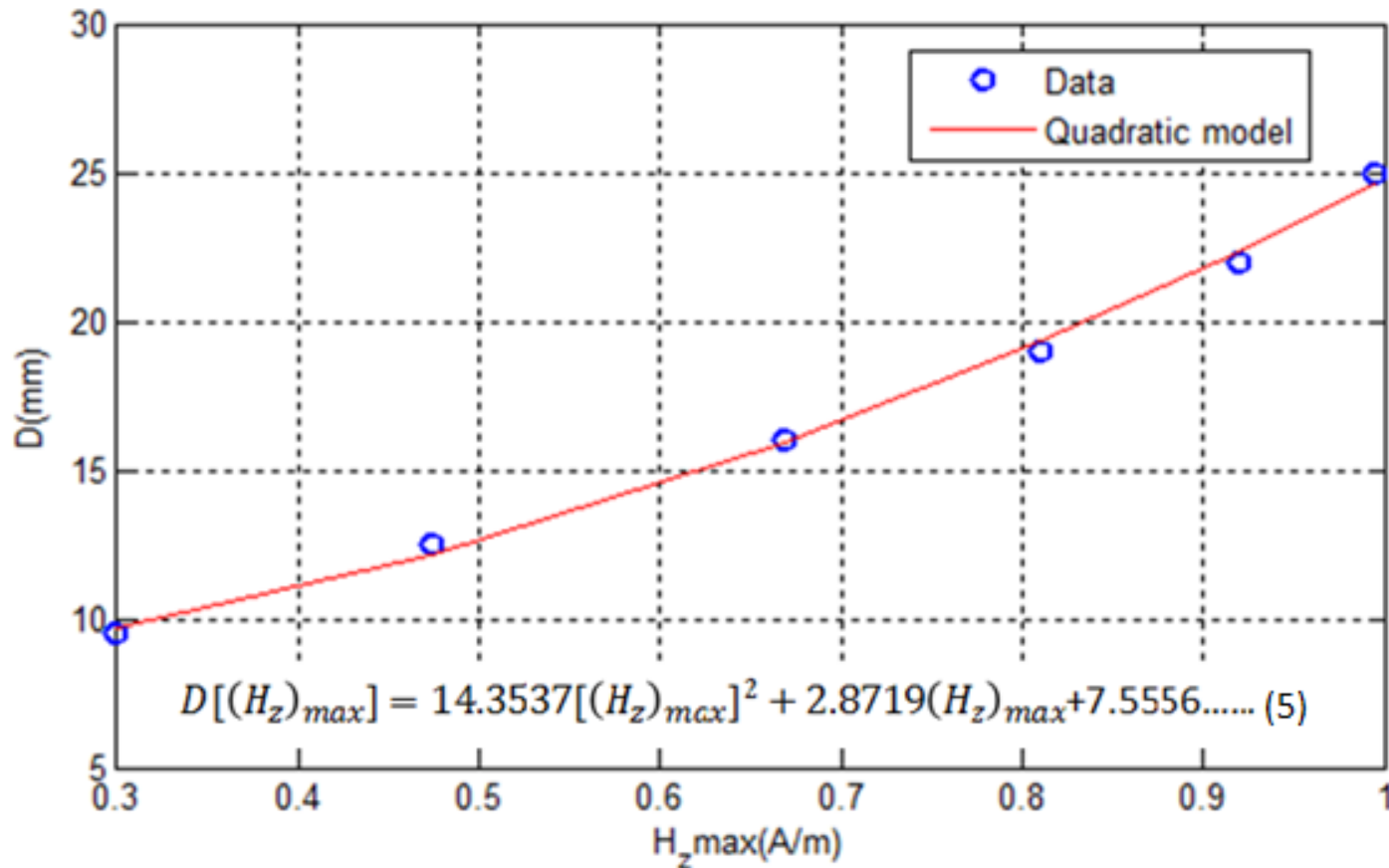


Figure 4. Diameter of rebar versus maximum peak amplitude in net time domain response.

# Results & Discussions

- Findings from Figure 4.
  - Linear increase of rebar diameter does not correspond to large increase in maximum peak amplitude in net time domain response.
  - Large difference between two maximum peak amplitudes does not suggest large increase of rebar diameter. For example when the maximum peak amplitude increases from 0.29 A/m to 0.47 A/m (62%), the rebar diameter increases from 9.5mm to 12.5mm (31.5%).
  - The coefficients of Equation 5 give the best fit of the non-linear model to the rebar diameter data.



# Results & Discussions

- Relationship between rebar diameter (D) and frequency shift ( $\Delta f$ )
- $\Delta f = f_{peak} - f_c$ .
- $f_{peak}$  = Peak frequency in net frequency response
- $f_c$  = Carrier frequency of input signal

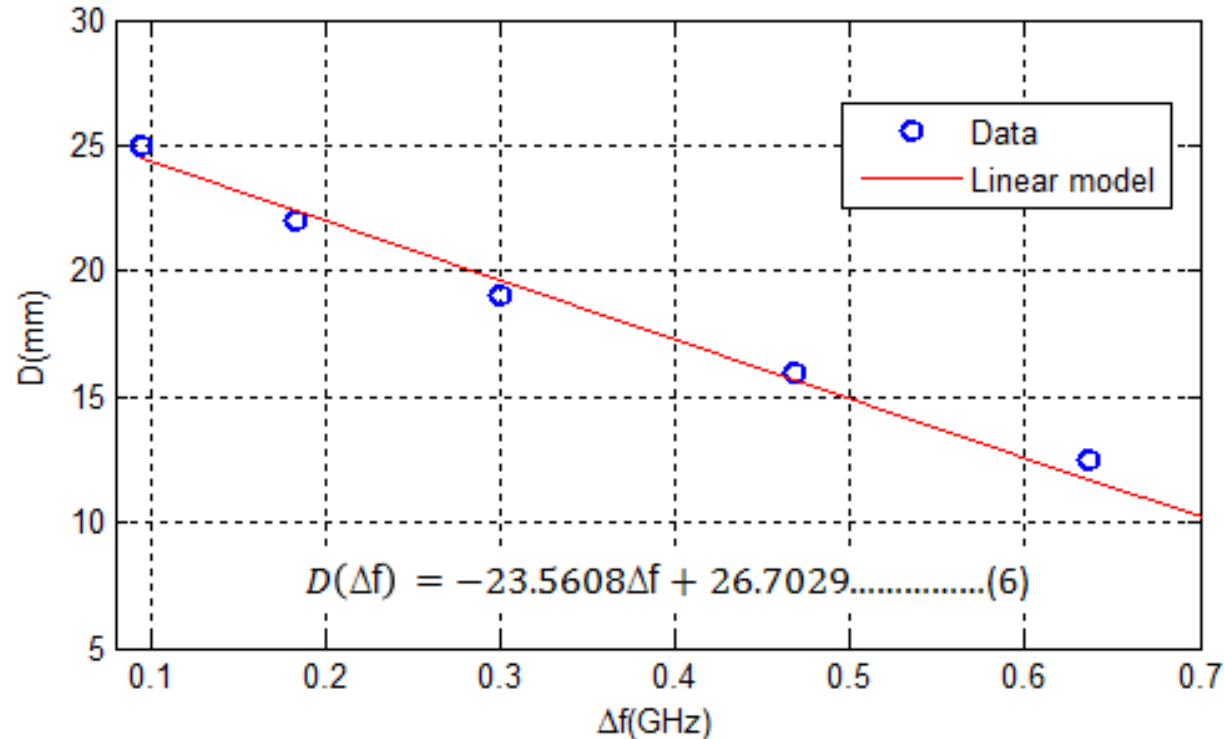


Figure 5. Rebar diameter versus frequency shift

# Results & Discussions

- Findings from Figure 5.
  - Linear increase of rebar diameter corresponds to a decrease in frequency shift.
  - Larger frequency shift suggests smaller rebar diameter and vice versa.
  - The coefficients of Equation 6 give the best fit of the linear model to the rebar diameter data.

# Ongoing and Future

- Rebar size detection in concrete (**ongoing**)
- Rebar size detection in concrete with rust layer (**future**).
- Coupling of rust thickness and crack density (**future**)

# References

- [1] Tzuyang Yu, Burack Boyaci, H. Felix Wu. Simulated Transient Electromagnetic Response for the Inspection of GFRP-Wrapped Concrete Cylinders Using Radar NDE. *Journal of the American Society for Non-destructive Testing* RNDE-24(3):125-153 (2013).
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- [4] Jin A. Kong. *Electromagnetic Wave Theory*. EMW Publishing, Cambridge, MA (2000).
- [5] Kane S. Yee. Numerical Solution of Initial Boundary Value Problems Involving Maxwell's Equations In Isotropic Media. *IEE Transactions on Antennas and Propagation* AP-14(3):302-307 (1966)