ENGN2070-207 Dynamics
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Lecture 2 | Fundamentals

1. Differentiation

* $y = f(x)$, $\dfrac{dy}{dx} = f'(x) = \lim\limits_{x \to a} \dfrac{f(x) - f(a)}{x - a}$

If $a = x \quad \Delta x \Rightarrow \dfrac{dy}{dx} = \lim\limits_{\Delta x \to 0} \dfrac{f(x) - f(x - \Delta x)}{\Delta x} = f'(x)$ #

* $\dfrac{\Delta y}{\Delta x} = ? \Rightarrow \dfrac{f(x) - f(a)}{\Delta x} = \dfrac{f(x) - f(x - \Delta x)}{\Delta x}$ #

* Useful differentiation formulae :

$\dfrac{d}{dx}(x^n) = n x^{n-1}$ for $n \in I$ & $n > 0$

$\dfrac{d}{dx} \sin x = \cos x$, $\dfrac{d}{dx} \sin ax = a \cdot \cos x$

$\dfrac{d}{dx} \cos x = -\sin x$, $\dfrac{d}{dx} (\sin x)^2 = 2 \cdot \sin x \cos x$

$\dfrac{d}{dx} (\sin ax)^2 = 2a \cdot \sin ax \cdot \cos ax$

$\dfrac{d}{dx} \tan x = \sec^2 x$, $\dfrac{d}{dx} \cot x = -\csc^2 x$

$\dfrac{d}{dx} \sec x = \sec x \tan x$, $\dfrac{d}{dx} \csc x = -\csc x \cdot \cot x$

$\dfrac{d}{dx}(e^x) = e^x$, $\dfrac{d}{dx}(e^{ax}) = a \cdot e^x$

$\dfrac{d}{dx}(e^{ax})^2 = 2a \cdot e^x$

2、 Integration :

$$\int_a^b f(x)\,dx = f(b) - f(a)$$

$$\int x^a\,dx = \frac{x^{a+1}}{a+1} + C \quad , \quad (a \neq -1)$$

$$\left[\int_c^d x^a\,dx = \frac{d^{a+1}}{a+1} - \frac{c^{a+1}}{a+1}\right]$$

$$\int x^{-1}\,dx = \ln|x| + C$$

$$\int \sin x\,dx = -\cos x + C \quad , \quad \int_a^b \sin x\,dx = (-\cos b) - (-\cos a)$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int e^x\,dx = e^x + C \quad , \quad \int e^{ax}\,dx = \frac{1}{a}e^{ax} + C$$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\arctan\frac{x}{a} + C = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$$

$$\int [f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$$

$$\int a f(x)\,dx = a\int f(x)\,dx \quad , \quad \left[\begin{array}{l} \text{if } f(x) = x^2, \\ \int a f(x)\,dx = \frac{a}{3}x^3 + a\cdot C \end{array}\right]$$

$$\boxed{\int u\,dv = uv - \int v\,du}$$ (Integration by Parts)

* $$\int f(g(x)) \cdot g'(x) \cdot dx = \int f(u)\,du \quad \text{if } u = g(x)$$

$$\because g'(x) = \frac{du}{dx}$$

$$\therefore \int f(g(x))\,dx = \frac{1}{g'(x)} \cdot \int f(u)\,du = \frac{dx}{du} \int f(u)\,du$$

$$\int f(x)\,dx = \int f(g(u)) \cdot g'(u)\,du \quad \text{if } x = g(u)$$

— Change of Base in integration

* Approximated integration :

① $$\int_a^b f(x)\,dx = \frac{b-a}{2n}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)\right]$$

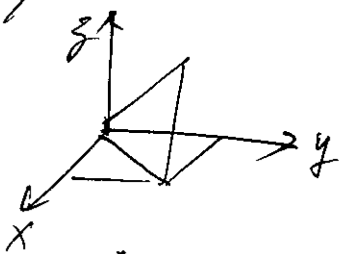
— Trapezoidal Rule

② $$\int_a^b f(x)\,dx = \frac{b-a}{3n}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right]$$

— Simpson's Rule

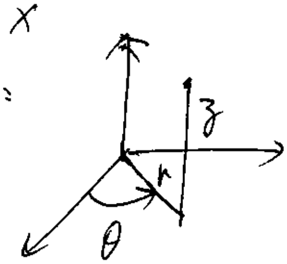
3. Coordinate Systems:

① Cartesian:



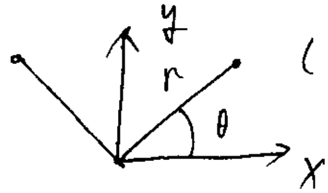
(x, y, z)

② Cylindrical:



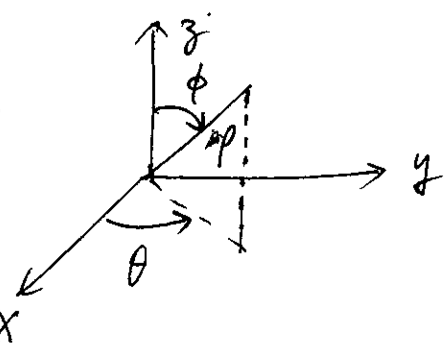
(r, θ, z)

③ Polar:



(r, θ)

④ Spherical:



(ρ, ϕ, θ)

4. Concept of motion:

- Displacement: $s = s(t) = \int v\,dt = \iint a\,dt \cdot dt$
- Velocity: $v = v(t) = \frac{ds}{dt} = \int a\,dt$
- Acceleration: $a = a(t) = \frac{dv}{dt} = \frac{d^2 s}{dt}$

5. Time Domain vs. Space Domain:

$$v = \frac{ds}{dt} \Rightarrow dt = \frac{ds}{v} \quad , \quad a = \frac{dv}{dt} \Rightarrow dt = \frac{dv}{a}$$

$$\Rightarrow \frac{ds}{v} = \frac{dv}{a} \Rightarrow \boxed{a\,ds = v\,dv} *$$