◎ Preface

* Dynamics is a discipline regarding the motion of objects.

⇒ Q1 : How do objects move?

[Ans] : Kinematics → { Dynamic parameters ($s(t)$, $v(t)$, $a(t)$) + Coordinate Systems (Cartesian, cylindrical, Spherical, polar, local) }

Q2 : Why do objects move?

[Ans] : Kinetics → { Force equilibrium, Conservation of energy, Conservation of momentum }

* Dynamic parameters in coordinate systems :

	Rectilinear	Angular	Cartesian	Cylindrical	Spherical
• Displacement	s	θ	(x, y, z)	(r, θ, z)	(r, θ, φ)
• Velocity	v	ω	(v_x, v_y, v_z)	(v_r, v_θ, v_z)	$(v_r, v_\theta, v_\varphi)$
• Acceleration	a	α	(a_x, a_y, a_z)	(a_r, a_θ, a_z)	$(a_r, a_\theta, a_\varphi)$

where
$$\begin{cases} v_r = \dot{r} \\ v_\theta = r\dot{\theta} \\ v_z = \dot{z} \end{cases} \qquad \begin{cases} v_r = \dot{r} \\ v_\theta = r\dot{\theta} \\ v_\varphi = r\dot{\varphi} \end{cases}$$

$$\begin{cases} a_r = \ddot{r} - r\dot{\theta}^2 \\ a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} \\ a_z = \ddot{z} \end{cases} \qquad \begin{cases} a_r = \ddot{r} - r\dot{\theta}^2 - r\dot{\varphi}^2\sin^2\theta \\ a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\dot{\varphi}^2\sin\theta\cos\theta \\ a_\varphi = r\ddot{\varphi} + 2\dot{r}\dot{\varphi}\sin\theta + 2r\dot{\theta}\dot{\varphi}\cos\theta \end{cases}$$

$$\dot{r} = \frac{dr}{dt}, \quad \dot{\theta} = \frac{d\theta}{dt}, \quad \dot{\varphi} = \frac{d\varphi}{dt}$$

$$* \begin{cases} \text{Time derivative} : \ y = \dfrac{dx}{dt} = \dot{x} \\ \text{Time difference} : \ y = \dfrac{\Delta X}{\Delta t} \end{cases}$$

(a) <u>Rectilinear Kinematics</u>

$s = s(t) =$ path

$v = v(t) =$ hodograph / velocity diagram $= \dfrac{ds}{dt} = \dot{s}$

$a = a(t) =$ acceleration diagram $= \dfrac{dv}{dt} = \dot{v} = \ddot{s}$

Or $s = \int v\, dt = \iint a\, dt\, dt$

$v = \int a\, dt$

$$\Rightarrow \boxed{v = \frac{ds}{dt}} \ \text{—(1)} \quad \& \quad \boxed{a = \frac{dv}{dt} = \frac{d^2 s}{dt^2}} \ \text{—(2)}$$

* Constant velocity ($v = v_c$)

$$s = \int_{t_0}^{t_1} ds = \int_{t_0}^{t_1} v_c\, dt \Rightarrow \boxed{s_1 - s_0 = v_c (t_1 - t_0)} \quad \text{— (3)}$$

* Constant acceleration ($a = a_c$)

$$v = \int_{t_0}^{t_1} dv = \int_{t_0}^{t_1} a_c\, dt \Rightarrow \boxed{v_1 - v_0 = a_c (t_1 - t_0)} \quad \text{— (4)}$$

Let $t_0 = 0$, Eq. (4) becomes $v_1 = v_0 + a_c t_1$.

Or, generally, $\underline{\underline{v = v_0 + a_c t}}$

Substitute it into Eq. (1).

$\because V = \dfrac{ds}{dt} = V_0 + a_c t$

$\therefore S = \displaystyle\int_{t_0}^{t_1} ds = \int_{t_0}^{t_1} (V_0 + a_c t)\, dt$

Again, let $t_0 = 0$. It becomes

$$S_1 - S_0 = V_0 t + \frac{1}{2} a_c t^2 \Rightarrow S_1 = S_0 + V_0 t + \frac{1}{2} a_c t^2$$

Or, generally, $\boxed{S = S_0 + V_0 t + \frac{1}{2} a_c t^2}$ —— (5)

Combine Eqs. (1) and (2).

$$V = \frac{ds}{dt} \Rightarrow V dt = ds \Rightarrow dt = \frac{ds}{V}$$

$$a = \frac{dV}{dt} \Rightarrow a dt = dV \Rightarrow dt = \frac{dV}{a}$$

$\because dt = dt \quad \therefore \dfrac{ds}{V} = \dfrac{dV}{a}$ or $\underline{\underline{a ds = V dV}}$

$$\Rightarrow \int_{t_0}^{t_1} a ds = \int_{t_0}^{t_1} V dV$$

If $a = a_c$, $a_c (S_1 - S_0) = \frac{1}{2}(V_1^2 - V_0^2)$

Or $\boxed{V_1^2 - V_0^2 = 2 a_c (S_1 - S_0)}$ —— (6)

* Space-time conversion :

$$\begin{cases} \text{Time domain} = S(t),\ V(t),\ a(t) \\ \text{Space domain} = S(s),\ V(s),\ a(s) \end{cases}$$