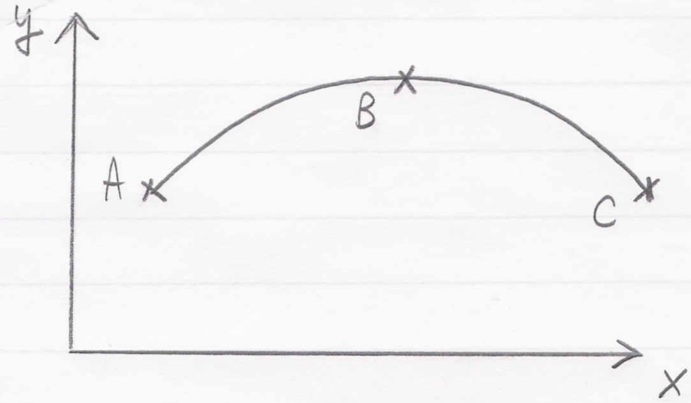
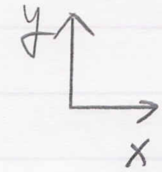
* Motion of a Projectile



$\downarrow \vec{a}_g \Rightarrow |\vec{a}_g| = g = 9.81\ m/s^2$
$= 32.2\ ft/s^2$



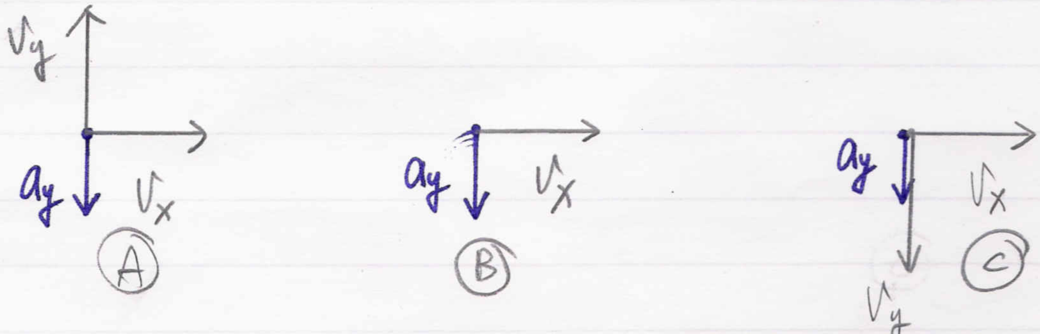
$\Rightarrow \boxed{\vec{a}_g = -g\,\vec{j}}$ #

At point A, $\begin{cases} x = x_A \\ y = y_A \end{cases}$, $\begin{cases} v_x = (v_x)_A > 0 \\ v_y = (v_y)_A > 0 \end{cases}$, $\begin{cases} a_x = (a_x)_A = 0 \\ a_y = (a_y)_A = -g \end{cases}$

At point B, $\begin{cases} x = x_B > \\ y = y_B \end{cases}$, $\begin{cases} v_x = (v_x)_B > 0 \\ v_y = (v_x)_B = 0 \end{cases}$, $\begin{cases} a_x = (a_x)_B = 0 \\ a_y = (a_y)_B = -g \end{cases}$

At point C, $\begin{cases} x = x_c \\ y = y_c \end{cases}$, $\begin{cases} v_x = (v_x)_c > 0 \\ v_y = (v_y)_c < 0 \end{cases}$, $\begin{cases} a_x = (a_x)_c = 0 \\ a_y = (a_y)_c = -g \end{cases}$

The motion from C to A can be analyzed by:



* i) a_y is always g in the $-\vec{j}$ direction; $a_x = 0$.

ii) $v_x =$ const.

iii) y usually experiences $+ \rightarrow -$ motion, limited by B.C.

x increases monotonically, only limited by the boundary condition (B.C.)

Since $a_x = 0 = \text{const.}$, we have

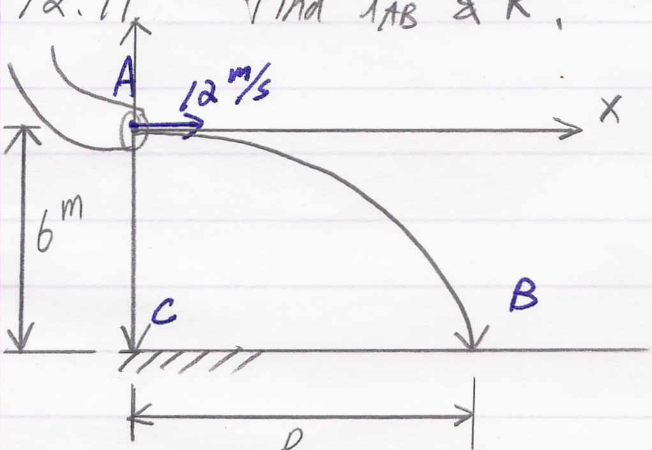
$$\begin{cases} v = v_0 + a_c t = (v_0)_x = v_x \\ x = x_0 + v_0 t + \frac{1}{2} a_c t^2 = x_0 + (v_0)_x \cdot t = x \\ v^2 = v_0^2 + 2 a_c (x - x_0) = (v_0)_x = v_x \end{cases}$$

in the horizontal direction. Also, we have $a_y = -g = \text{const.}$

$$\begin{cases} v_y = (v_0)_y - g t = v_y \\ y = y_0 + (v_0)_y t - \frac{1}{2} g t^2 = y \\ v_y^2 = (v_0)_y^2 - 2g(y - y_0) \end{cases}$$

in the vertical direction.

Example 12.11 Find t_{AB} & R.



① List the initial conditions (I.C.) of $\vec{r}$, $\vec{v}$, and $\vec{a}$.

$$\begin{cases} \vec{r} \Rightarrow x = 0, \quad y = 0 \\ \vec{v} \Rightarrow v_x = 12 \text{ m/s}, \quad v_y = 0 \text{ m/s} \\ \vec{a} \Rightarrow a_x = 0 \text{ m/s}^2, \quad a_y = -9.81 \text{ m/s}^2 \end{cases}$$

② Determine the B.C.

@ $t = t_{AB}$, $x = R^{m}$, $y = -6^{m}$

③ List the governing eqns.

$$\begin{cases} x = x_0 + (v_0)_x t_{AB} \\ y = y_0 + (v_0)_y t_{AB} - \frac{1}{2} g t_{AB}^2 \end{cases}$$

With $x_0 = y_0 = 0$, $(v_0)_x = 12$ m/s

$$\begin{cases} R = 12 t_{AB} + 0 \\ -6 = 0 + 0 - \frac{1}{2}(9.81) t_{AB}^2 \end{cases}$$

$\Rightarrow$ Find $t_{AB} = \sqrt{\dfrac{6 \times 2}{9.81}} = \underline{\underline{1.11}}$ s #

$\Rightarrow R = 12 \times t_{AB} = \underline{\underline{13.3}}$ m #