Prob. 12-71

Find $v(t=2^s)$ & $a(t=2^s)$.

$$\vec{v}=\frac{d\vec{r}}{dt}=(9t^2-2)\vec{i}-(2t^{-\frac{1}{2}}+1)\vec{j}+(6t)\vec{k} \quad m/s$$

$$@\ t=2^s,\quad \vec{v}=34\vec{i}-2.414\vec{j}+12\vec{k} \quad m/s$$

$$\Rightarrow v=|\vec{v}|=\left[34^2+(-2.414)^2+12^2\right]^{1/2}=\underline{\underline{36.1 \quad m/s}} \#$$

$$\vec{a}=\frac{d\vec{v}}{dt}=(18t)\vec{i}+(t^{-\frac{3}{2}})\vec{j}+6\vec{k} \quad m/s^2$$

$$@\ t=2^s,\quad \vec{a}=36\vec{i}+0.3536\vec{j}+6\vec{k} \quad m/s^2$$

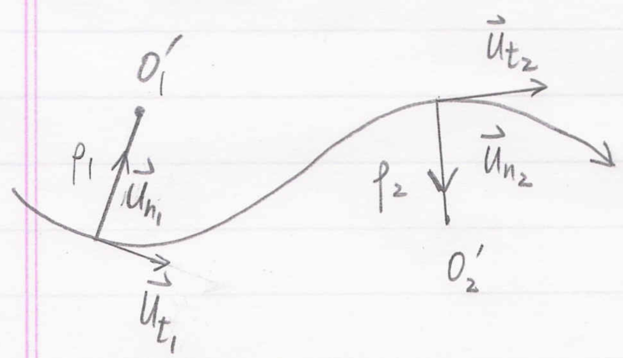
$$\Rightarrow a=|\vec{a}|=\underline{\underline{36.5 \quad m/s^2}} \#$$

Curvilinear Motion : Normal and Tangential Components

* 1. Path is a curve.
2. Can be described by a radius of curvature ρ & center of curvature O'
3. Positive direction of the normal axis is always on the concave side.

(4. The normal direction $\vec{n}$ & tangiential direction $\vec{t}$ form the embracing or osculating plane.)

Velocity

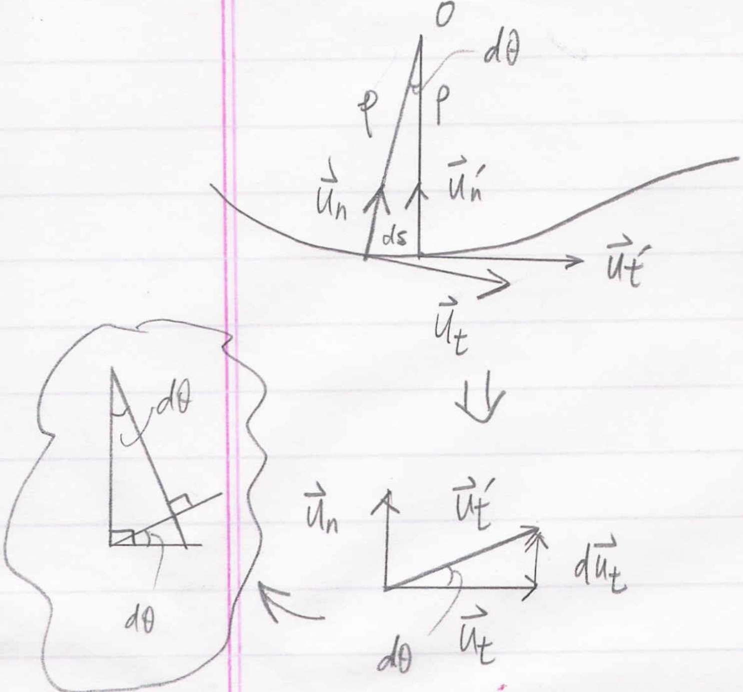


$s=s(t)$,

$\because v=\frac{ds}{dt} \Rightarrow$

$$\boxed{\begin{aligned}\vec{v}&=v\,\vec{u}_t\\ \vec{v}&=\frac{ds}{dt}\cdot\vec{u}_t\end{aligned}}$$

Acceleration



$$\vec{a} = \frac{d}{dt}\vec{v} = \frac{d}{dt}(v\,\vec{u}_t)$$

$$\Rightarrow \vec{a} = \dot{v}\,\vec{u}_t + v\,\dot{\vec{u}}_t$$

$\because \vec{u}_t' = \vec{u}_t + d\vec{u}_t$

$\because |\vec{u}_t| = |\vec{u}_n| = 1$

$\therefore d\vec{u}_t = \underbrace{1 \times d\theta}_{\text{magnitude}} \times \underbrace{\vec{u}_n}_{\text{direction}}$

$$\Rightarrow \boxed{\dot{\vec{u}}_t = \dot{\theta}\cdot\vec{u}_n} \quad \text{— ①}$$

From geometry, $\Rightarrow ds = \rho\, d\theta$

$$\Rightarrow \boxed{\dot{s} = \rho\dot{\theta} \quad \text{or} \quad \frac{\dot{s}}{\rho} = \dot{\theta}} \quad \text{— ②}$$

& $\boxed{\dot{s} = v}$!!

Substitute ① & ② into $\vec{a}$, we have

$$\Rightarrow \vec{a} = \dot{v}\,\vec{u}_t + v \times \frac{v}{\rho}\,\vec{u}_n$$

$$\boxed{\vec{a} = \dot{v}\,\vec{u}_t + \frac{v^2}{\rho}\vec{u}_n = a_t \cdot \vec{u}_t + a_n \times \vec{u}_n} \quad \#$$

where $a_t = \dot{v}$ & $a_n = \dfrac{v^2}{\rho}$