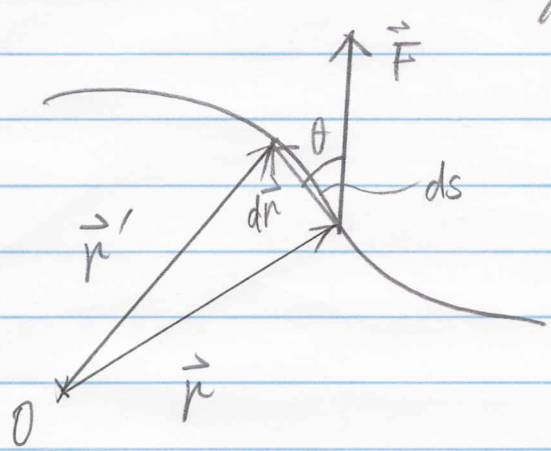
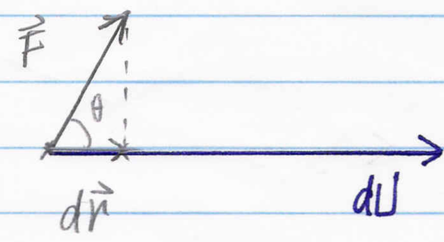
Kinetics of a Particle: Work and Energy

The Work of a Force

* Work = Force × Displacement in the direction of the force



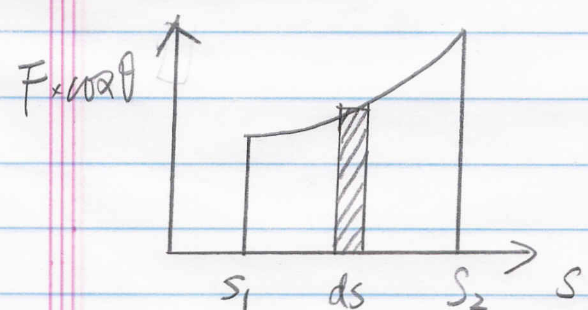


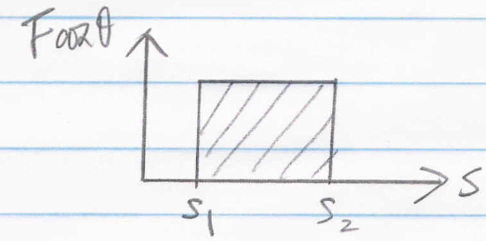
$$dU = \vec{F} \cdot d\vec{r}$$
$$= |\vec{F}| \times |d\vec{r}| \times \cos\theta$$
$$\Rightarrow \underline{dU = F \cdot ds \cdot \cos\theta}$$

* Unit: SI system: N × m = Joule = J

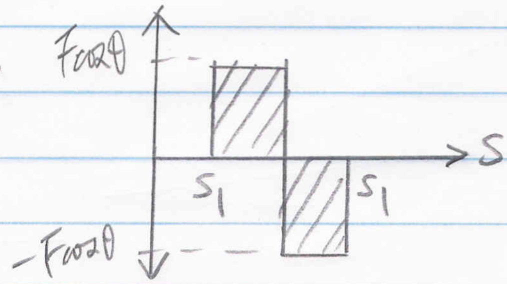
FPS system: ft × lb = foot-pound

* Variable forces





$\Sigma U = F\cos\theta\,(s_2 - s_1)$



$\Sigma U = 0$

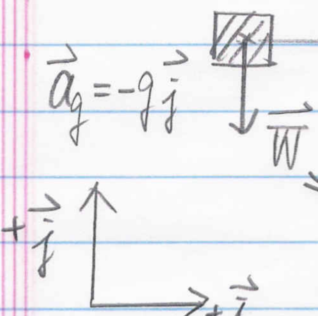
* Work done by gravity

$ds = r\vec{j} \Rightarrow$ Negative work $\Rightarrow U < 0$

$ds \Rightarrow$ Positive work $\Rightarrow U > 0$

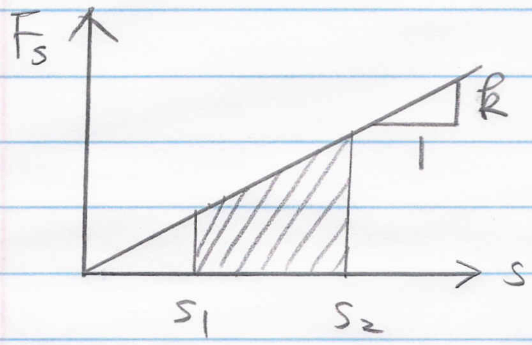
$ds = -r\vec{j}$

$\vec{a}_g = -g\vec{j}$



$\vec{W} \Rightarrow m(-g)\vec{j} = -mg\vec{j}$

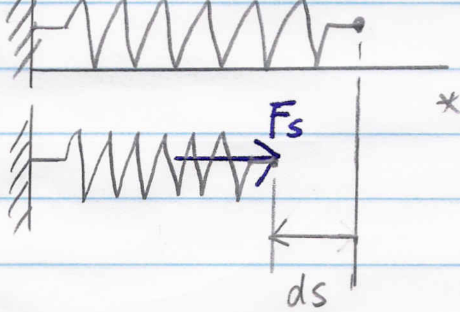
* Work done by a spring force (F_s)



$$U_{1-2} = \int_{s_1}^{s_2} F_s \cdot ds$$

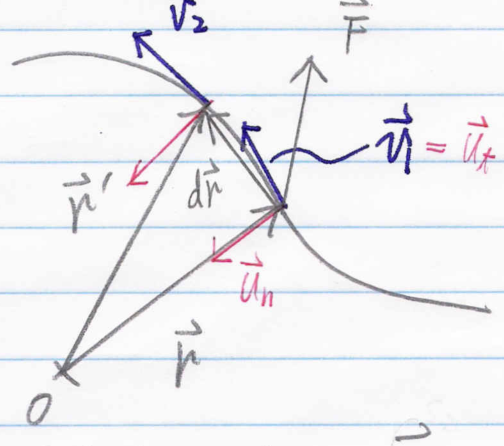
$$= \int_{s_1}^{s_2} (-ks)\, ds = \frac{-1}{2} k s^2 \Big|_{s_1}^{s_2}$$

$$\Rightarrow \boxed{U_{1-2} = -\frac{k}{2}\left(s_2^2 - s_1^2\right)} \;*$$



$* \boxed{F_s = -ks}$!

Principle of Work and Energy



$$\vec{F} = F_t\,\vec{u}_t + F_n\,\vec{u}_n$$

$$d\vec{r} = ds \times \vec{u}_t$$

$$\text{Work} = dU = \vec{F}\cdot d\vec{r} = \left(F_t\,\vec{u}_t + F_n\,\vec{u}_n\right)\cdot\left(ds \times \vec{u}_t\right)$$

$$\because\ \vec{u}_t\cdot\vec{u}_t = 1 \ \ \& \ \ \vec{u}_t\cdot\vec{u}_n = 0$$

$$\Rightarrow \boxed{dU = F_t\,ds} \ \ \& \ \ \int dU = \int F_t\cdot ds$$

$$\Rightarrow U = \int F_t\cdot ds$$

$$\text{Total work} = \sum U = \sum\int F_t\cdot ds \ \divideontimes$$

$$\because F_t = m\times a_t$$

$$\therefore \sum F_t = m\times a_t = m\times\left(\frac{v\times dv}{ds}\right)$$

$$\left(\because a = \frac{dv}{dt},\quad v = \frac{ds}{dt},\quad \therefore \frac{dv}{a} = \frac{ds}{v} \quad \text{or } a = \frac{v\cdot dv}{ds} \ \divideontimes\right)$$

$$\Rightarrow \sum F_t\,ds = m\,v\,dv$$

$$\Rightarrow \sum\int_{s_1}^{s_2} F_t\,ds = \int_{v_1}^{v_2} m\,v\,dv$$

$$\Rightarrow \sum U_{1-2} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = T_2 - T_1 = \Delta T$$

*Energy is the capacity for doing work.

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$$\boxed{T = \frac{1}{2} m v^2} = \underline{\text{kinetic energy}} \; (T > 0)$$

$$\therefore \boxed{\sum U_{1-2} + T_1 = T_2}$$

$$\because \sum U_{2-1} = T_1 - T_2 \quad \therefore \boxed{\sum U_{2-1} + T_2 = T_1}$$

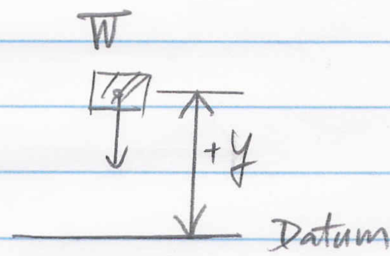
* **Remarks**: ① Kinetic energy is always positive. $(T > 0)$
② When there is no change in velocity, assuming constant mass, kinetic energy $= 0$ ※

* When many particles (mass particles) are present,

$$\sum T_1 + \sum U_{1-2} = \sum T_2$$

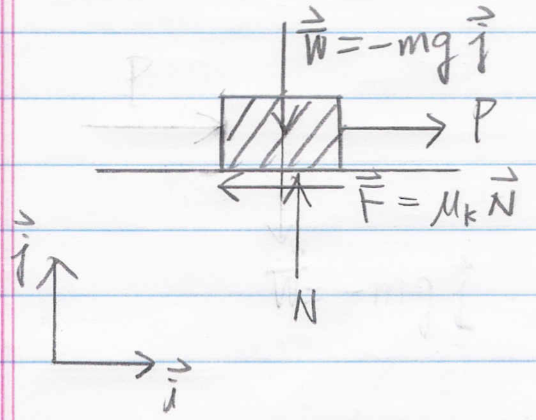
* Potential energy $(V \gtrless 0)$

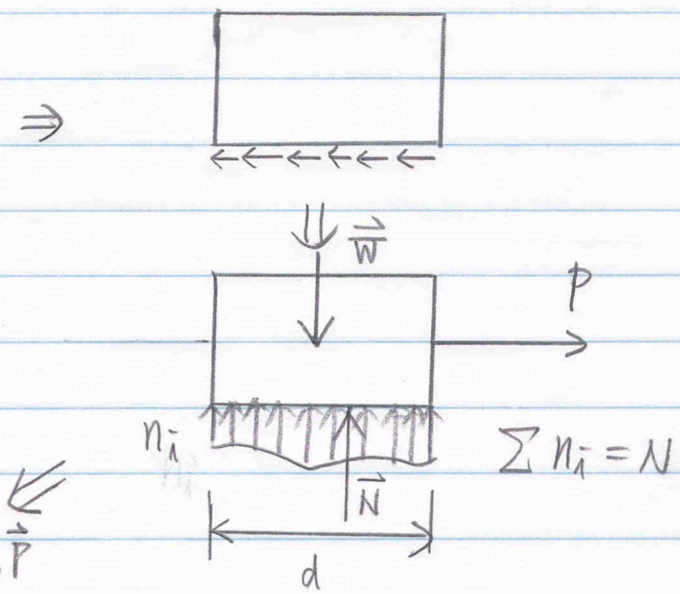
$$\boxed{V_g = m \times g \times y} = W \times y$$

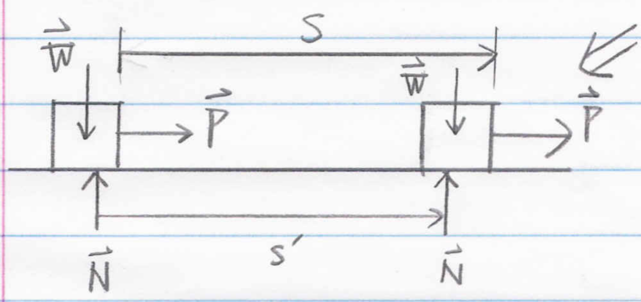


$$\Rightarrow T_1 + V_1 + \sum U_{1-2} = T_2 + V_2 \quad ※$$

* Work done by a frictional force







$\because s' < s \Rightarrow$

- $U_f = \mu_k N s' \Rightarrow$ Work done by the resultant frictional force
- $U_I = \mu_k N (s - s') \Rightarrow$ Work associated with the frictional force (internal energy)
  - $\Rightarrow$ Rigid body motion: Heat
  - Elastic body motion: Heat + Potential energy
  - Plastic body motion: Heat + Potential energy + Fracture energy

Power and Efficiency

* Power $= \dfrac{\text{Unit change of work}}{\text{Unit change of time}} = \dfrac{dU}{dt}$

Let $dU = \vec{F} \cdot d\vec{r}$, we have

$$P = \frac{dU}{dt} = \frac{\vec{F} \cdot d\vec{r}}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$$

$$= (\text{Force}) \cdot (\text{velocity})$$

* Unit: SI system: $\dfrac{\text{Joule}}{\text{second}} = \dfrac{J}{s} = \text{Watt} = W$

$$= \frac{N \cdot m}{s} \quad *$$

FPS system: $\dfrac{ft \cdot lb}{\text{second}} = \dfrac{1}{550}\ \text{horsepower} = \dfrac{hp}{550}$

or $1\ hp = 550\ ft \cdot lb/s$ *

Also, $\boxed{1\ hp = 746\ W}$ *

* Efficiency $= \dfrac{\text{Power output}}{\text{Power Input}} = \varepsilon = \dfrac{\text{Energy output}}{\text{Energy Input}} < 1$

(due to frictional forces within each mechanical system)