* Linear momentum and linear impulse can be evaluated w.r.t. individual coordinate component.

* Consider a system of particles.

$$\sum \vec{F}_i = \sum m_i \frac{d\vec{v}_i}{dt}$$

$$\Rightarrow \sum m_i (\vec{v}_i)_1 + \sum \int_{t_1}^{t_2} \vec{F}_i \, dt = \sum m_i (\vec{v}_i)_2$$

Let $m = \sum m_i$, $\sum \vec{v}_i = \vec{v}_G$

$$\Rightarrow m(\vec{v}_G)_1 + \sum \int_{t_1}^{t_2} \vec{F}_i \, dt = m(\vec{v}_G)_2$$

Conservation of Linear Momentum

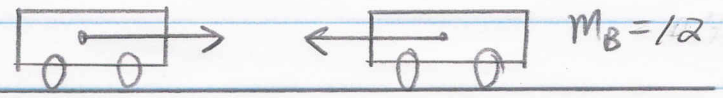
$$\boxed{\sum m_i (\vec{v}_i)_1 = \sum m_i (\vec{v}_i)_2}$$

If $\sum m_i \vec{v}_i = m \vec{v}_G$, $(\vec{v}_G)_1 = (\vec{v}_G)_2$ *

* Changes in $\vec{v}$ is due to $\vec{a}$ $\Rightarrow$ Large $\vec{a}$ leads to $\Delta m\vec{v}$

$$\begin{cases} \vec{a} = \dfrac{d\vec{v}}{dt} \Rightarrow \Delta m\vec{v} \uparrow \text{ only if } \vec{a} \text{ is significant.} \\ \vec{a}_{avg} = \dfrac{\Delta \vec{v}}{\Delta t} \Rightarrow \Delta m\vec{v} \uparrow \text{ even if } \vec{a}_{avg} \text{ is small.} \end{cases}$$

Example 15-33 $\vec{v}_A = 1.5\ m/s$ $\vec{v}_B = 0.75\ m/s$

$m_A = 15\,Mg$  $m_B = 12\ Mg$

① $\because \sum m v_1 = \sum m v_2$

$\Rightarrow (15\times10^3)\times1.5 - (12\times10^3)\times0.75 = [(12+15)\times10^3]\times v_2$

$\Rightarrow \underline{v_2 = 0.5\ m/s}$ *

② Kinetic energy: $T_1 = \frac{1}{2}(15\times10^3)\times(1.5)^2 + \frac{1}{2}(12\times10^3)\times(0.75)^2 = 20.25\ kJ$

$T_2 = \frac{1}{2}(27\times10^3)\times(0.5)^2 = 3.375\ kJ$

$\Rightarrow \Delta T = T_1 - T_2 = 20.25 - 3.375 = \underline{16.9\ kJ}$ *

(Note: This ΔT is due to shock, noise, and heat during the coupling of two masses.)

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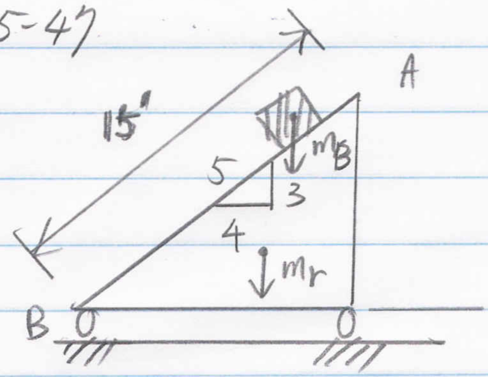
Example 15-47



$m_r = \frac{120 \text{ lb}}{32.2 \text{ ft/s}^2}$, $W_r = 120$ lb

$m_B = \frac{80 \text{ lb}}{32.2 \text{ ft/s}^2}$, $W_B = 80$ lb

(1) $\because T_1 + V_1 = T_2 + V_2$

$\Rightarrow 0 + 80 \times \left(\frac{3}{5} \times 15\right) = \frac{1}{2}\left(\frac{80}{32.2}\right) \times v_B^2 + \frac{1}{2}\left(\frac{120}{32.2}\right) \times v_r^2$

?

Also,

(2) $(\xrightarrow{+}) \quad \Sigma m v_1 = \Sigma m v_2$

$\Rightarrow 0 + 0 = \left(\frac{120}{32.2}\right) \times v_r - \left(\frac{80}{32.2}\right)(v_B)_x$

$\Rightarrow (v_B)_x = 1.5\, v_r$

(3) And $\vec{v}_B = \vec{v}_r + \vec{v}_{B/r}$

$\Rightarrow \begin{cases} (\xrightarrow{+}) & -(v_B)_x = v_r - \frac{4}{5} v_{B/r} \quad -① \\ (+\uparrow) & -(v_B)_y = 0 - \frac{3}{5} v_{B/r} \quad -② \Rightarrow (v_B)_y = \frac{3}{5} v_{B/r} \end{cases}$

① provides $-(1.5+1)v_r = -\frac{4}{5} v_{B/r}$

$\Rightarrow v_{B/r} = \frac{5}{4} \times (2.5\, v_r) \Rightarrow (v_B)_y = \frac{3}{5} \times \frac{5}{4} \times (2.5)\, v_r$

$= 1.875\, v_r$

$\Rightarrow v_B^2 = (v_B)_x^2 + (v_B)_y^2 = 5.7656\, v_r^2$

$\Rightarrow \underline{v_r = 8.93 \text{ ft/s}} \ (\rightarrow) \ \divideontimes \quad \Rightarrow \underline{v_B = 21.44 \text{ ft/s}} \ (\swarrow) \ \divideontimes$