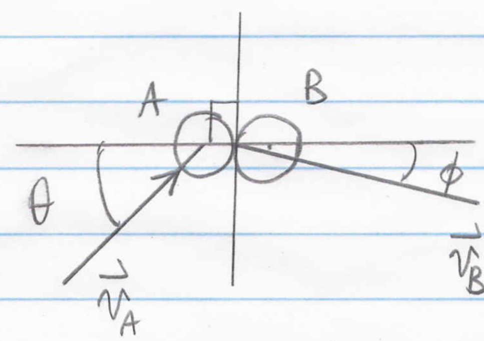
Impact

* Occurs when two mass particles collide with each other during a very short period of time.

* Defined by ① the line of impact
② the plane of contact } ⇒ the angle of impact, and
③ the initial momenta.

* When the angle of impact $\begin{cases} = 0 \Rightarrow \text{central impact} \\ \neq 0 \Rightarrow \text{oblique impact} \end{cases}$



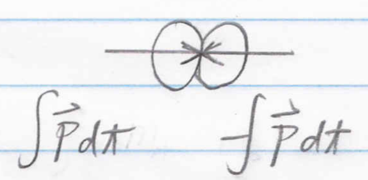
* Assume <u>elastic</u> bodies (non-rigid)

Initial momentum $\begin{cases} \vec{L}_{A1} = m_A (\vec{v}_A)_1 \\ \vec{L}_{B_1} = m_B (\vec{v}_B)_1 \end{cases}$

* <u>Stages of impact</u>:

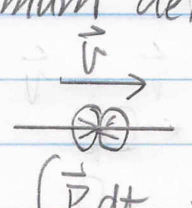
$t < t_0$ ① Before impact : $\Delta\vec{v}_{12} = \vec{v}_1 - \vec{v}_2 \neq 0$ or $\Delta\vec{v}_{21} = \vec{v}_2 - \vec{v}_1 \neq 0$

$t_0 < t < t_1$ ② Deformation impulse : $I_D = \int_{t_1}^{t_2} \vec{P}\,dt = m_A(\vec{v}_A)_1 - m_A\vec{v}$



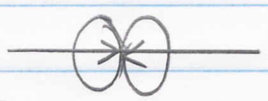
$\ll (v_A)_1 > (v_B)_1 \gg$

$t < t < t_2$ ③ Maximum deformation :



$$\int_{t_1}^{t_2} \vec{P}\,dt = (m_A + m_B) \cdot \vec{v}$$

$t_2 < t < t_3$ ④ Restitution impulse : $I_R = \int_{t_2}^{t_3} \vec{R}\, dt = m_A \vec{v} - m_A (\vec{v}_A)_2$



$\int \vec{R}\, dt \qquad \int \vec{R}\, dt$

$t_3 < t < t_4$ ⑤ After impact : Final momentum : $\begin{cases} \vec{L}_{A_2} = m_A \cdot (\vec{v}_A)_2 \\ \vec{L}_{B_2} = m_B \cdot (\vec{v}_B)_2 \end{cases}$

$\ll (\vec{v}_B)_2 > (\vec{v}_A)_2 \gg$

* <u>Analysis of impact:</u>

i) Conservation of Linear Momentum : $\sum m_i (\vec{v}_i)_1 = \sum m_i (\vec{v}_i)_2$

$$\Rightarrow m_A (\vec{v}_A)_1 + m_B (\vec{v}_B)_1 = m_A \underbrace{(\vec{v}_A)_2 + m_B (\vec{v}_B)_2}_{\text{Unknowns}} \quad \text{——— (1)}$$

*(I.Cs are known!)

ii) Principle of impulse and momentum :

<u>For particle A</u> :

$$(\overset{+}{\rightarrow}) \quad m_A (v_A)_1 - \int P\, dt = m_A \cdot v$$

<u>For particle B</u> :

$$(\overset{+}{\rightarrow}) \quad m_A v - \int R\, dt = m_A (v_A)_2$$

iii) Coefficient of restitution:

<u>For particle A</u>:

$$e = \frac{\int R\, dt}{\int P\, dt} = \frac{v - (v_A)_2}{(v_A)_1 - v}$$