Angular Momentum

* The momentum of the particle's linear momentum about point O

$\Rightarrow \vec{L} = m\vec{v} \Rightarrow$ Linear momentum (Force)

$\vec{H}_O = \vec{r} \times m\vec{v} = \vec{r} \times \vec{L} \Rightarrow$ Angular momentum (Moment)

$$\Rightarrow \vec{L} = L_x \vec{i} + L_y \vec{j} + L_z \vec{k}$$
$$= (mv_x)\vec{i} + (mv_y)\vec{j} + (mv_z)\vec{k}$$

$$\vec{H}_O = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ r_x & r_y & r_z \\ mv_x & mv_y & mv_z \end{vmatrix}$$

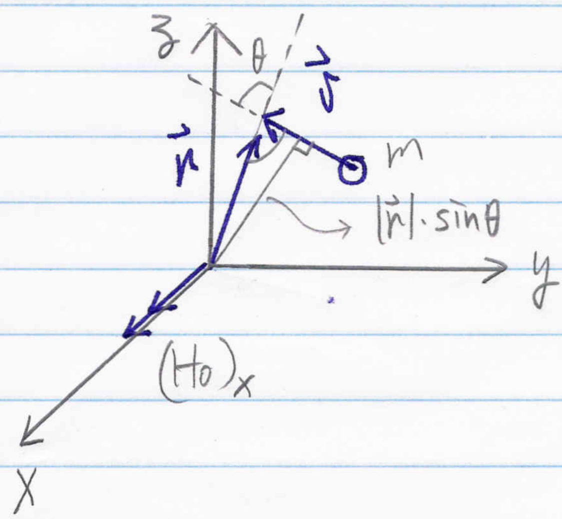
$$= (r_y \cdot mv_z - r_z \cdot mv_y)\vec{i} + (r_z \cdot mv_x - r_x \cdot mv_z)\vec{j}$$
$$+ (r_x \cdot mv_y - r_y \cdot mv_x)\vec{k}$$

* When $v_x = 0$ & $r_x = 0$, $\vec{L} = (mv_y)\vec{j} + (mv_z)\vec{k}$

$$\vec{H}_O = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & r_y & r_z \\ 0 & mv_y & mv_z \end{vmatrix} = (r_y \cdot mv_z - r_z \cdot mv_y)\vec{i}$$
$$= \vec{r} \times \vec{L} = (H_O)_x \cdot \vec{i}$$

$(\vec{r} = r_y \cdot \vec{j} + r_z \vec{k})$

$$\Rightarrow |\vec{H}_O| = |r| \cdot |L| \times \sin\theta$$

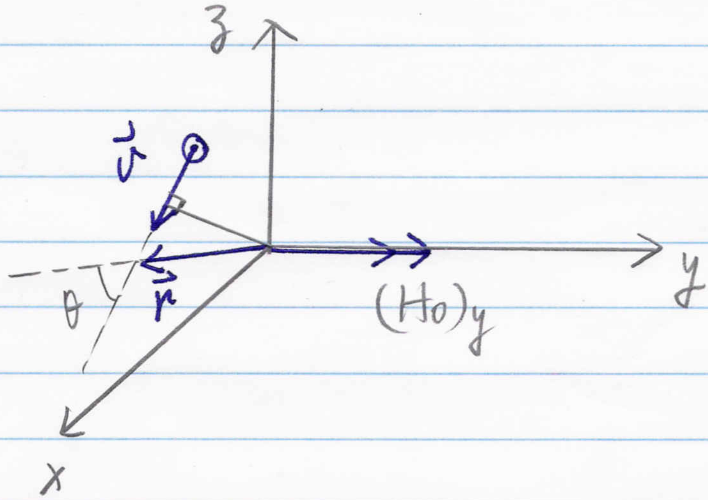


* When $v_y = 0$ & $r_y = 0$, $\vec{L} = (mv_x)\vec{i} + (mv_z)\vec{k}$

$$\vec{r} = r_x\,\vec{i} + r_z\,\vec{k}$$

$$\vec{H}_0 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ r_x & 0 & r_z \\ mv_x & 0 & mv_z \end{vmatrix} = \left(r_z \cdot mv_x - r_x - mv_z\right)\vec{j}$$

$$= (H_0)_y \cdot \vec{j}$$



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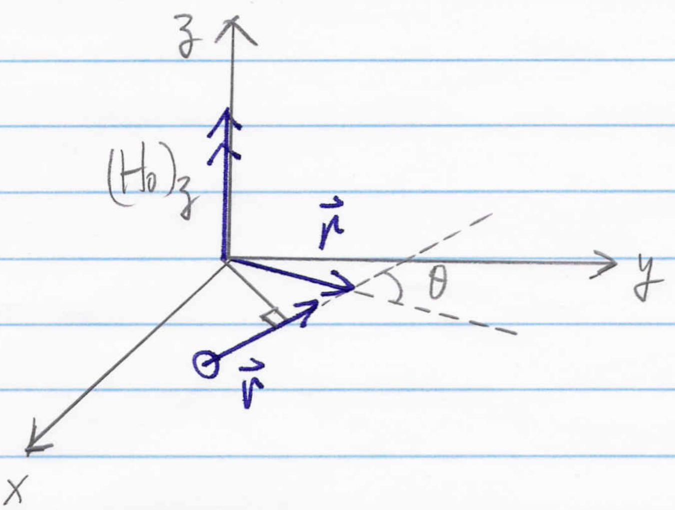
* When $v_z = 0$ & $r_z = 0$, $\vec{L} = (mv_x)\vec{i} + (mv_z)\vec{j}$

$\vec{r} = r_x\,\vec{i} + r_y\,\vec{j}$

$$\vec{H_0} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ r_x & r_y & 0 \\ mv_x & mv_y & 0 \end{vmatrix} = \left(r_x \cdot mv_y - r_y * mv_x\right)\vec{k}$$

$$= (H_0)_z \cdot \vec{k}$$



Relation Between Moment of a Force and Angular Momentum

Knowing that $\vec{H_0} = \vec{r} \times m\vec{v}$, the time derivative of $\vec{H_0}$ is found to be

$$\frac{d}{dt}(\vec{H_0}) = \frac{d}{dt}(\vec{r} \times m\vec{v})$$

$$= \dot{\vec{r}} \times m\vec{v} + \vec{r} \times m\dot{\vec{v}}$$

$\because$ $\vec{v} = \dot{\vec{r}}$ & $\dot{\vec{r}} \times \dot{\vec{r}} = 0$

$$\therefore \frac{d}{dt}(\vec{H_0}) = \dot{H_0} = \vec{r} \times m\dot{\vec{v}} = \vec{r} \times m\vec{a} = \vec{r} \times \sum\vec{F}$$

$$= \sum M_0$$ #