Thus, $\boxed{\dfrac{d}{dt}(\vec{H}_0) = \dot{\vec{H}}_0 = \vec{r} \times \Sigma \vec{F} = \Sigma M_0}$ ※

Similar to $\boxed{\Sigma \vec{F} = \dot{\vec{L}} = \dfrac{d}{dt}(m\vec{v})}$ ※

※ For a system of particles with internal forces ($\vec{f}_i$) & external forces ($\vec{F}_i$),

$$\Rightarrow \Sigma(\vec{r}_i \times \vec{F}_i) + \Sigma(\vec{r}_i \times \vec{f}_i) = \Sigma(\dot{\vec{H}}_i)_0$$

(w.r.t. point O)

## Principle of Angular Impulse and Momentum

$$\because \Sigma \vec{M}_0 = (\dot{H}_0) = \frac{d(\vec{H})_0}{dt}$$

$$\Rightarrow \Sigma \vec{M}_0 \, dt = d(\vec{H})_0$$

$$\Rightarrow \Sigma \int_{t_1}^{t_2} \vec{M}_0 \, dt = (\vec{H}_0)_2 - (\vec{H})_1 \quad \text{or}$$

$$\Rightarrow \boxed{(\vec{H}_0)_1 + \Sigma \int_{t_1}^{t_2} \vec{M}_0 \, dt = (\vec{H}_0)_2}$$

where $(\vec{H}_0) = \vec{r} \times \vec{F} \Rightarrow$ angular momentum.

$\Sigma \int \vec{M}_0 \, dt \Rightarrow$ angular impulse.

Also, $\boxed{\Sigma \int \vec{M}_0 \, dt = \Sigma \int (\vec{r} \times \vec{F}) \, dt}$

* Recall the previous three cases with either $v_x = r_x = 0$ or $v_y = r_y = 0$ or $v_z = r_z = 0$. The governing equations are

$$\begin{cases} m\vec{v}_1 + \sum\int_{t_1}^{t_2} \vec{F}\,dt = m\vec{v}_2 \quad \text{on} \begin{cases} x\text{-}y \ (v_z = 0) \\ y\text{-}z \ (v_x = 0) \\ z\text{-}x \ (v_y = 0) \end{cases} \\ \longrightarrow \text{Principle of linear impulse \& momentum} \\ (\vec{H}_0)_1 + \sum\int_{t_1}^{t_2} \vec{M}_0\,dt = (\vec{H}_0)_2 \quad \text{on} \begin{cases} z\text{-axis} \\ x\text{-axis} \\ y\text{-axis} \end{cases} \\ \longrightarrow \text{Principle of angular impulse \& momentum} \end{cases}$$

## Conservation of Angular Momentum

When $\sum\int_{t_1}^{t_2} \vec{M}_0\,dt = \sum\int_{t_1}^{t_2} (\vec{r}\times\vec{F})\,dt = 0$,

$$\Rightarrow \boxed{(\vec{H}_0)_1 = (\vec{H}_0)_2}$$

$$\Rightarrow \vec{r}\times\vec{F} = 0 \Rightarrow \underbrace{\vec{r} = 0}_{\text{central forces only!}} \text{ or } \underbrace{\vec{F} = 0}_{\text{Trivial case!}}$$

$$\Rightarrow \boxed{\sum(\vec{H}_0)_1 = \sum(\vec{H}_0)_2}$$

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