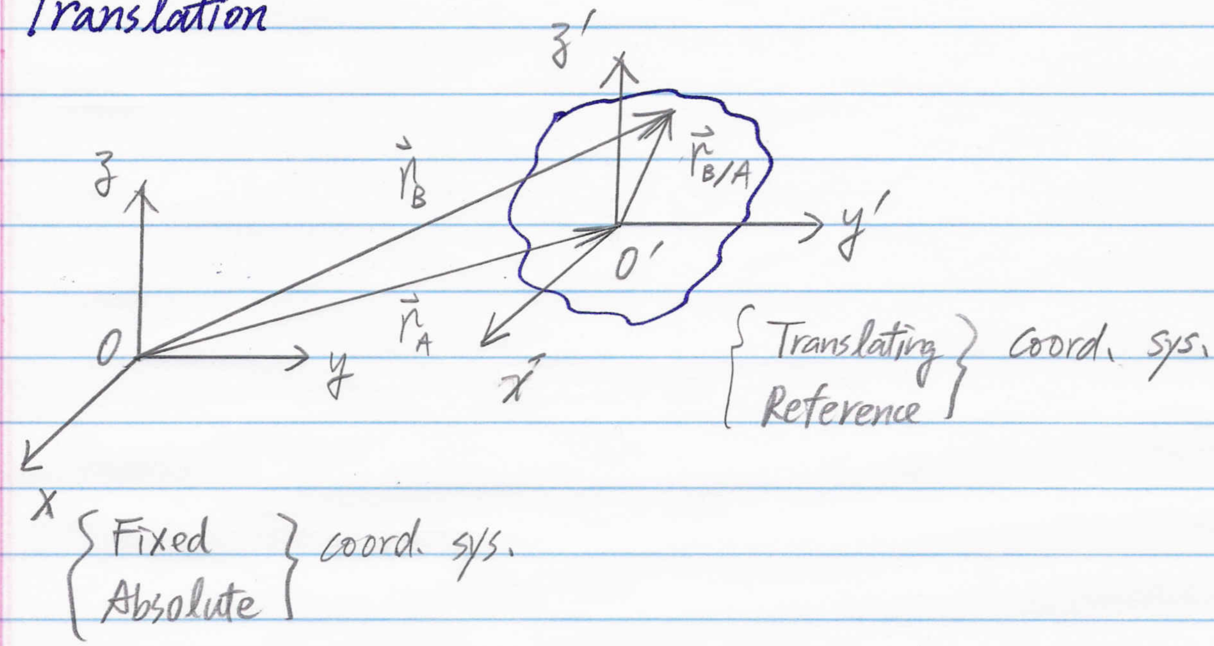
Planar Kinematics
Planar Rigid-Body Motion

* Types of rigid body planar motion:

1. Translation — Straight-line (rectilinear), curve (curvilinear)
2. Rotation — w.r.t. a fixed point (2D objects) or a fixed axis (3D objects)
3. General plane motion — Translation + Rotation

Translation



* Position

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

Velocity

$$\frac{d}{dt}(\vec{r}_B) = \frac{d}{dt}(\vec{r}_A) + \underbrace{\frac{d}{dt}(\vec{r}_{B/A})}_{=0}$$

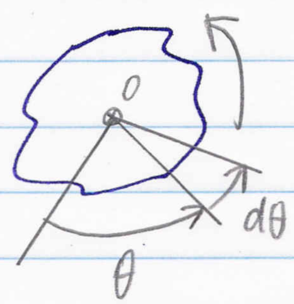
(∵ Rigid-body)

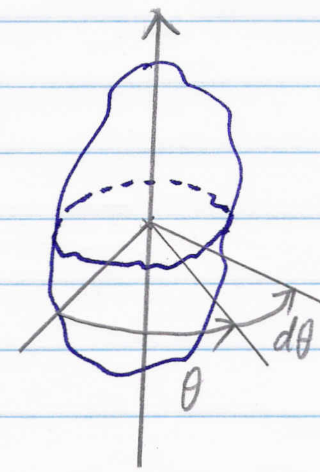
$$\Rightarrow \vec{v}_B = \vec{v}_A$$

Acceleration

$$\frac{d}{dt}(\vec{v}_B) = \frac{d}{dt}(\vec{v}_A) \Rightarrow \vec{a}_B = \vec{a}_A$$

Rotation





$d\theta \Rightarrow$ angular displacement (rad.)

$\frac{d\theta}{dt} = \omega \Rightarrow$ angular velocity (rad/s)

$\frac{d\omega}{dt} = \alpha \Rightarrow$ angular acceleration (rad/s²)

Similar to $dt = \frac{ds}{v} = \frac{dv}{a} \Rightarrow a\,ds = v\,dv$, we have

$$dt = \frac{d\theta}{\omega} = \frac{d\omega}{\alpha} \Rightarrow \boxed{\alpha\, d\theta = \omega \cdot d\omega} \text{ ※}$$

Also, for constant $\boxed{\alpha = \alpha_c}$,

$$\begin{cases} \omega = \omega_0 + \alpha_c \cdot t \\ \theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2 \\ \omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0) \end{cases}$$

(Note : Positive direction must be defined for $d\theta$ first.)

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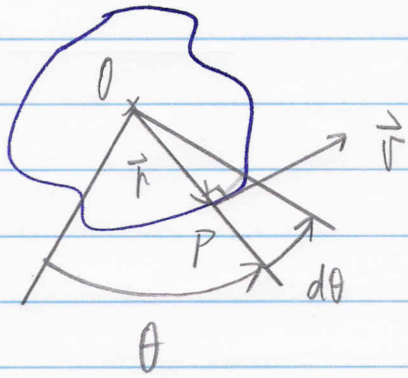


$\because \vec{v}$ is $\perp \vec{r}$

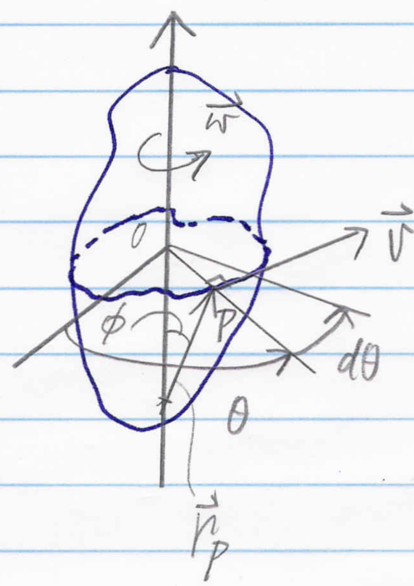
$\therefore \boxed{ds = r \cdot d\theta}$

$$\Rightarrow \frac{ds}{dt} = r \cdot \frac{d\theta}{dt}$$

$$\Rightarrow \boxed{v = r \cdot \omega}$$

$$\Rightarrow \frac{d^2 s}{dt^2} = r \cdot \frac{d^2\theta}{dt^2}$$

$$\Rightarrow \boxed{a = r \cdot \alpha = a_t}$$



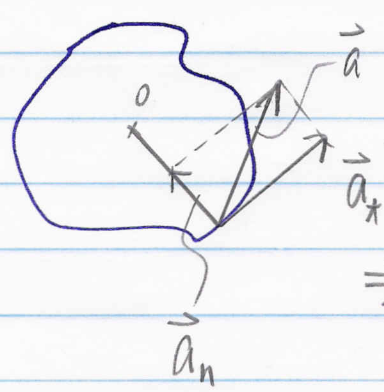
$\because \vec{v} \perp \vec{r}$

$\Rightarrow ds = r \cdot d\theta$

$$\vec{v} = \vec{\omega} \times \vec{r}_p = \frac{d\vec{r}_p}{dt}$$

$$r = r_p \cdot \sin\phi \Rightarrow v = \omega \cdot r_p \cdot \sin\phi$$

When $\phi = \frac{\pi}{2}$, $\sin\phi = 1 \Rightarrow v = \omega \cdot r$



$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d\vec{\omega}}{dt} \times \vec{r}_p + \vec{\omega} \times \frac{d\vec{r}_p}{dt}$$

$$= \vec{\alpha} \times \vec{r}_p + \vec{\omega} \times (\vec{\omega} \times \vec{r}_p)$$

$$\Rightarrow \boxed{\vec{a} = \vec{\alpha} \times \vec{r}_p - \omega^2 \cdot \vec{r}_p}$$

And

$$a = \left[a_n^2 + a_t^2 \right]^{1/2}$$