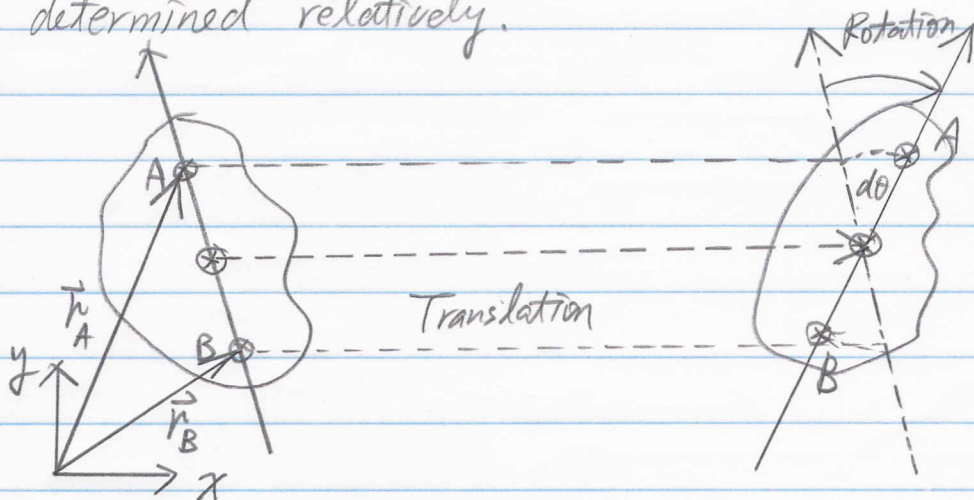


Relative-Motion Analysis: Velocity

* The motion of particles in a rigid body can be determined relatively.



Position $\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$

$$\Rightarrow d\vec{r}_B = d\vec{r}_A + d\vec{r}_{B/A} \Rightarrow \text{Displacement.}$$

Velocity $\frac{d\vec{r}_B}{dt} = \frac{d\vec{r}_A}{dt} + \frac{d\vec{r}_{B/A}}{dt}$

$$\because d\vec{r}_{B/A} = r_{B/A} \times d\theta \quad \& \quad \frac{d\theta}{dt} = \omega \quad \& \quad \frac{d\omega}{dt} = \alpha$$

$$\Rightarrow \vec{v}_B = \vec{v}_A + \underbrace{\left(\vec{r}_{B/A} \cdot \omega \right)}_{\vec{v}_{B/A}} = \vec{v}_A + \vec{v}_{B/A}$$

Also, $\vec{v} = \vec{\omega} \times \vec{r} \quad \therefore \vec{v}_{B/A} = \vec{\omega} \times \vec{r}_{B/A}$

$$\Rightarrow \boxed{\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}}$$

Instantaneous Center of Zero Velocity

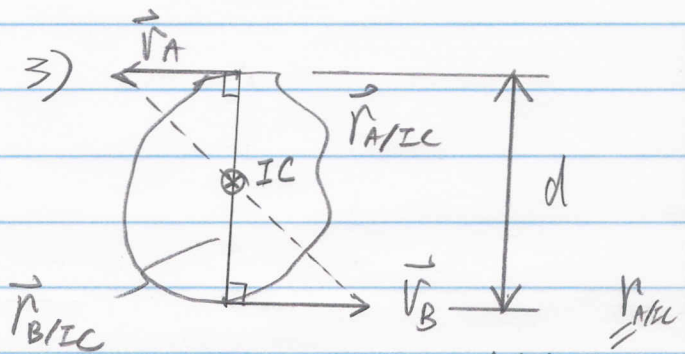
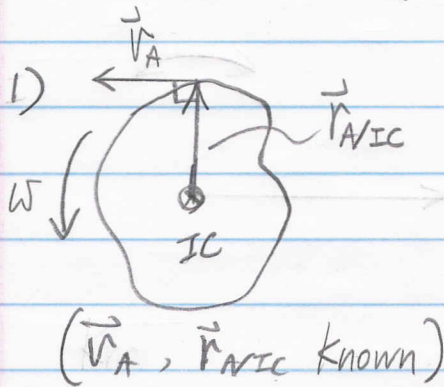
$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$$

When $\vec{v}_A = 0$, $\vec{v}_B = \vec{\omega} \times \vec{r}_{B/A}$ @ $A = IC$ of zero velocity

⇒ The locus of IC points is called a centrode.

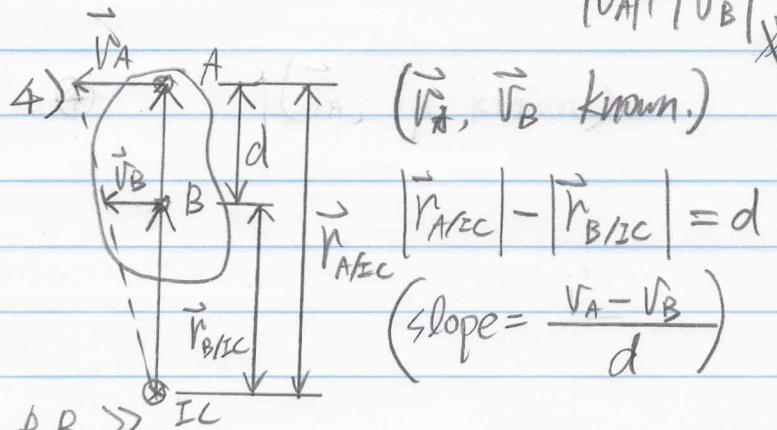
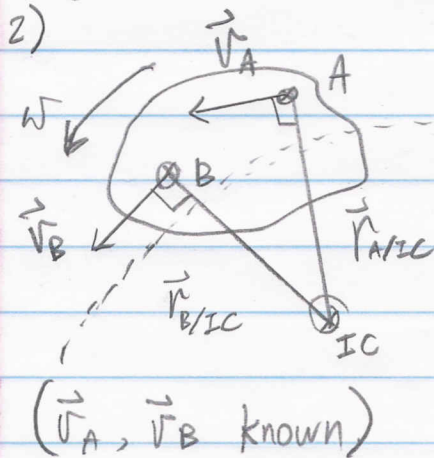
* How to find IC ?

⇒ Conditions: $\left\{ \begin{array}{l} \textcircled{1} \vec{v}_{IC} = 0 \\ \textcircled{2} \vec{v}_i = \vec{\omega} \times \vec{r}_{i/IC} \end{array} \right\} \Rightarrow \vec{\omega}$



Knowing d , $|\vec{r}_{A/IC}| = \frac{|\vec{v}_A| \times d}{|\vec{v}_A| + |\vec{v}_B|}$

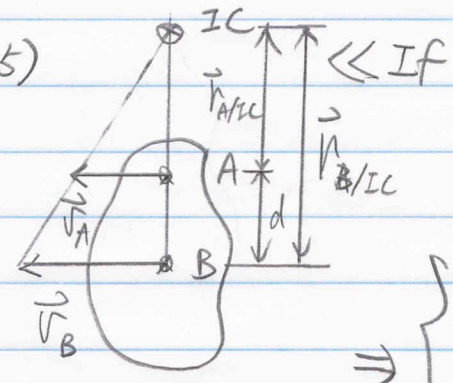
$r_{B/IC} = |\vec{r}_{B/IC}| = \frac{|\vec{v}_B| \times d}{|\vec{v}_A| + |\vec{v}_B|}$



« If $v_A > v_B$,

⇒ IC is below the f.B. »

$$\Rightarrow \begin{cases} |\vec{r}_{A/IC}| = r_{A/IC} = \frac{v_A}{v_A - v_B} \times d \\ |\vec{r}_{B/IC}| = r_{B/IC} = \frac{v_B}{v_A - v_B} \times d \end{cases}$$

5)  $\ll \text{If } v_B > v_A \Rightarrow \text{IC is above the R.B.} \gg$

Slope = $\frac{v_B - v_A}{d}$

$$\Rightarrow \begin{cases} r_{B/IC} = \frac{v_B}{v_B - v_A} \times d \\ r_{A/IC} = \frac{v_A}{v_B - v_A} \times d \end{cases}$$

* Remarks about IC:

- ① IC of zero velocity is only instantaneous,
 \Rightarrow Not necessarily true at next time instant.
- ② IC does not have zero acceleration,
 typically