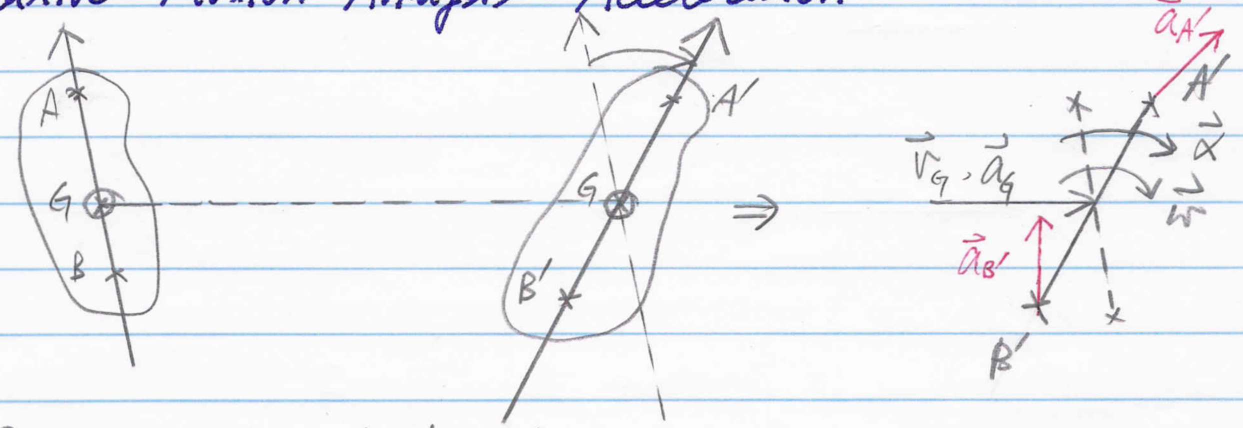
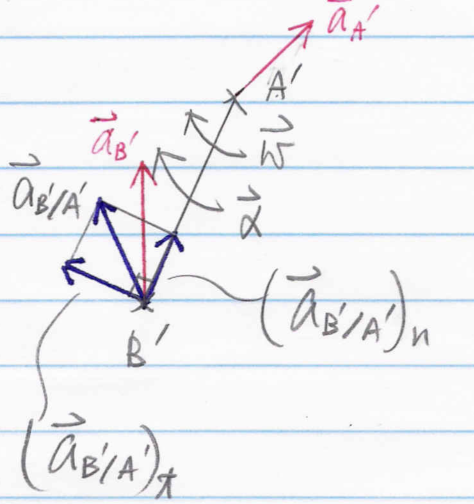
# Relative-Motion Analysis : Acceleration




* Since it is a rigid body,
  $\vec{v}_G = \vec{v}_A = \vec{v}_B$ & $\vec{a}_G = \vec{a}_A = \vec{a}_B$
  in the translation motion.

*




Angular rotation creates $\vec{a}_{B/A}$.
$(\vec{\alpha})$

$$\Rightarrow \vec{a}_{B'} = \vec{a}_{A'} + \vec{a}_{B'/A'}$$
$$= \vec{a}_{A'} + (\vec{a}_{B'/A'})_t + (\vec{a}_{B'/A'})_n$$

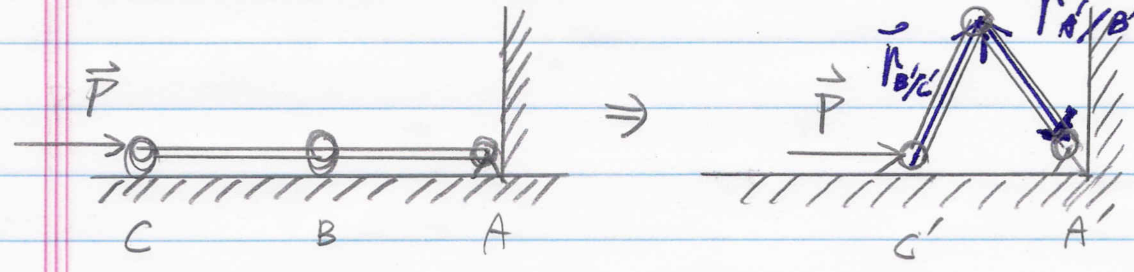
where $\begin{cases} (a_{B'/A'})_t = \alpha \cdot r_{B'/A'} \\ (a_{B'/A'})_n = \omega^2 r_{B'/A'} \end{cases}$

* From #37, we know that $\vec{a} = \vec{\alpha} \times \vec{r}_P - \omega^2 \vec{r}_P$
  or $\vec{a} = \vec{a}_n + \vec{a}_t$ ※

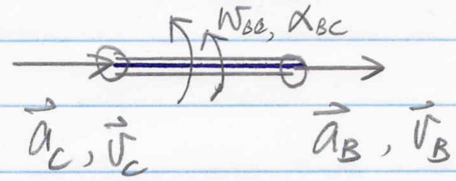
Thus, in general,

$$\boxed{\vec{a}_B = \underbrace{\vec{a}_A}_{\text{Translation}} + \underbrace{(\vec{\alpha} \times \vec{r}_{B/A} - \omega^2 \vec{r}_{B/A})}_{\text{Rotation}}}$$ #

## * The crankshaft problem



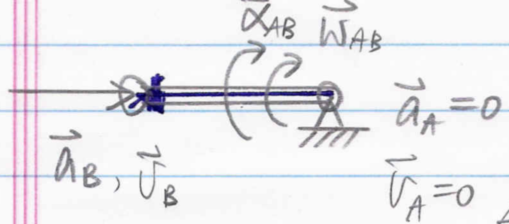

i) For member $\overline{BC}$,




$$\vec{a}_B = \vec{a}_C + \vec{a}_{B/C}$$
$$= \vec{a}_C + \vec{\alpha}_{BC} \times \vec{r}_{B/C} - \omega_{BC}^2 \vec{r}_{B/C}$$

Also, $\vec{v}_B = \vec{v}_C + \vec{\omega}_{BC} \times \vec{r}_{B/C}$ ※

ii) For member $\overline{AB}$,




$$\vec{a}_B = \vec{a}_{B/A}$$
$$= \vec{\alpha}_{AB} \times \vec{r}_B - \omega_{AB}^2 \vec{r}_B$$

Also, $\vec{v}_B = \vec{\omega}_{AB} \times \vec{r}_B$ ※

### * Remarks:

① Find IC of zero velocity (= zero acceleration) if there is any.

② Establish $\vec{a}_A = \vec{a}_{IC} + \vec{a}_{A/IC} = \vec{a}_{A/IC}$

$$= \vec{\alpha}_{A/IC} \times \vec{r}_A - \omega_{A/IC}^2 \vec{r}_A$$

③ Extend the relation from point A to another point by using boundary condition.

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# Relative-Motion Analysis using Rotating Axes

Knowing that,

$$\vec{v}_B = \vec{v}_A + \frac{d\vec{r}_{B/A}}{dt}$$

$$\Rightarrow \boxed{\vec{v}_B = \vec{v}_A + \Omega \times \vec{r}_{B/A} + (\vec{v}_{B/A})_{xyz}}$$

where $\Omega$ = angular velocity, measured from the fixed coord. sys.

$(\vec{v}_{B/A})_{xyz}$ = velocity of B w.r.t. A as measured by an observer attached to the ref. coord. sys.

Furthermore,

$$\boxed{\vec{a}_B = \vec{a}_A + \dot{\Omega} \times \vec{r}_{B/A} + \Omega \times (\Omega \times \vec{r}_{B/A}) + 2\Omega \times (\vec{v}_{B/A})_{xyz} + (\vec{a}_{B/A})_{xyz}}$$