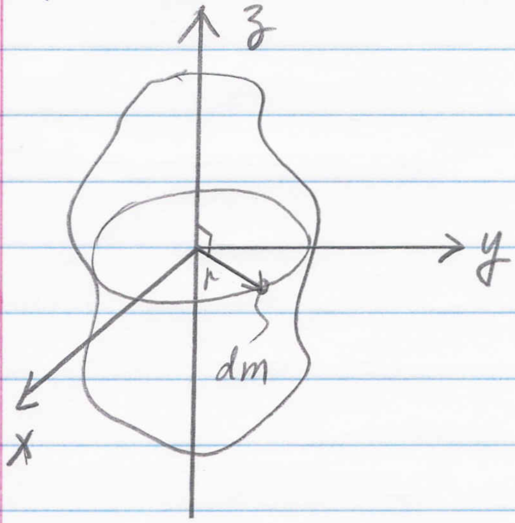
Mass Moment of Inertia

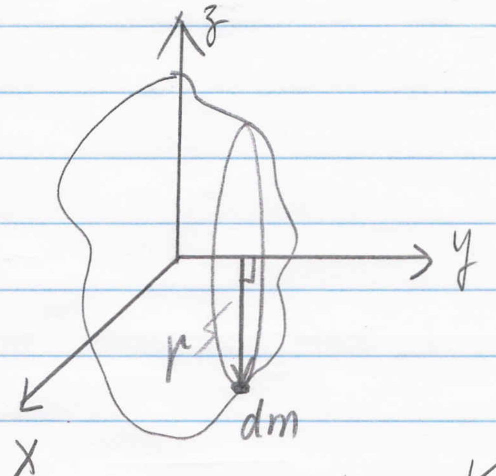


Definition:

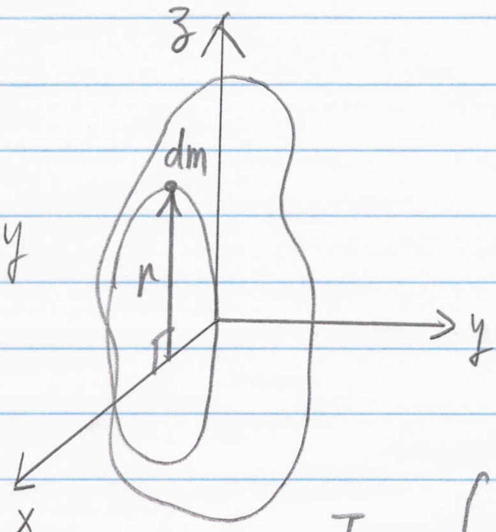
$$I_z = \int_m r^2 \, dm$$

Replace $dm = \rho \cdot dV$.

$$\Rightarrow I_z = \int_V r^2 \cdot \rho \, dV$$



$$I_y = \int_m r^2 \, dm$$



$$I_x = \int_m r^2 \, dm$$

Parallel-Axis Theorem

$$I = \int_m r^2 \, dm = \int_m \left[(d + x')^2 + y'^2 \right] dm$$

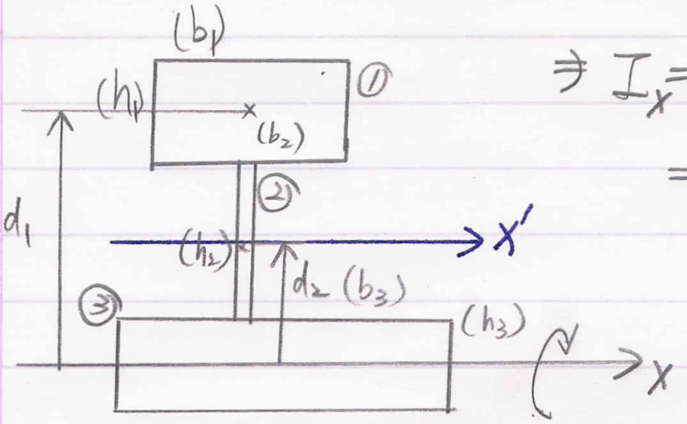
$$= \underbrace{\int_m (x'^2 + y'^2) \, dm}_{I_G} + 2d \underbrace{\int_m x' \, dm}_{0} + d^2 \underbrace{\int_m dm}_{m}$$

$$= I_G + m d^2$$

Radius of Gyration, k

$$I = mk^2 \quad \Rightarrow \quad \boxed{k = \sqrt{\frac{I}{m}}}$$

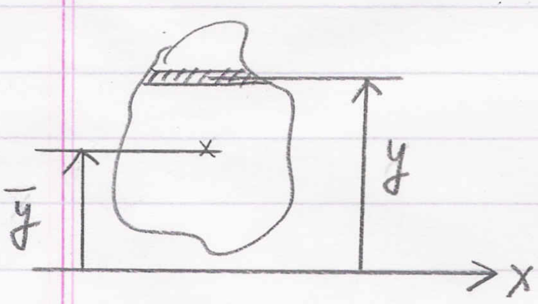
For composite bodies, $I = \sum (I_G + md^2)$



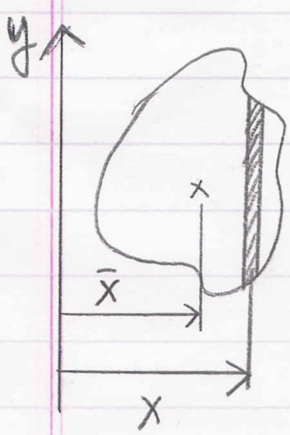
$$\Rightarrow I_x = I_1 + I_2 + I_3$$
$$= (b_1 h_1^3 + b_1 h_1 \times d_1^2)$$
$$+ (b_2 h_2^3 + b_2 h_2 \times d_2^2)$$
$$+ (b_3 h_3^3) \#$$

(Q: What about $I_{x'} = ?$)

Distance to the Mass Center



$$\boxed{\bar{y} = \frac{\sum y\, m}{\sum m}}$$



$$\boxed{\bar{x} = \frac{\sum x \cdot m}{\sum m}}$$

$$\Rightarrow \boxed{I_P = I_G + m(\bar{x}^2 + \bar{y}^2)}$$