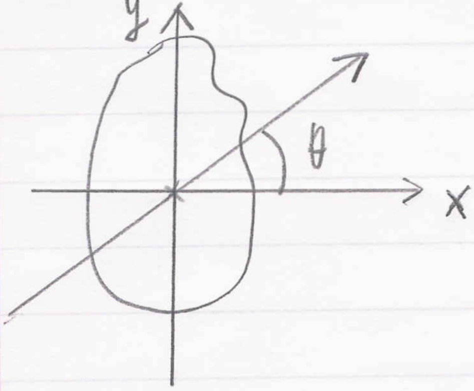
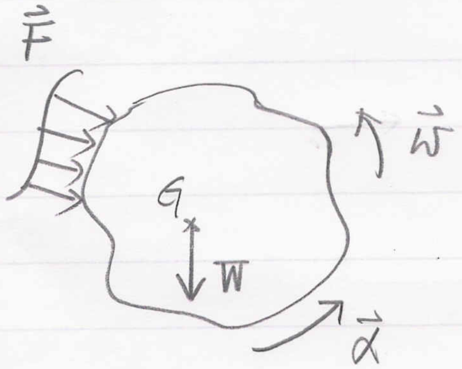
Rotation at an Inclined Angle



$$\boxed{\theta = \tan^{-1}\frac{I_y}{I_x}}$$

Planar Kinetic Equations of Motion

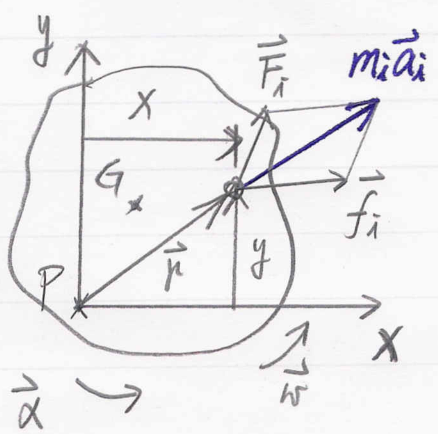
$$\boxed{\sum \vec{F} = m\,\vec{a}_G} \Rightarrow \begin{cases} \sum F_x = m\,(a_G)_x \\ \sum F_y = m\,(a_G)_y \\ \sum F_z = m\,(a_G)_z \end{cases}$$



$\Rightarrow$ Eqn. of Translational Motion #

$$\boxed{\sum M_G = I_G \cdot \alpha}$$ $\Rightarrow$ Eqn. of Rotational Motion #

For the rotational moment at any point P,



$\vec{r} \times \vec{F}_i + \vec{r} \times \vec{f}_i = \vec{r} \times (m_i \vec{a}_i)$

$\Rightarrow (\vec{M}_P)_i = \vec{r} \times (m_i \vec{a}_i)$

$\because \vec{a}_i = \vec{a}_p + \vec{\alpha} \times \vec{r} - \omega^2 \vec{r}$

where $\omega = |\vec{\omega}|$.

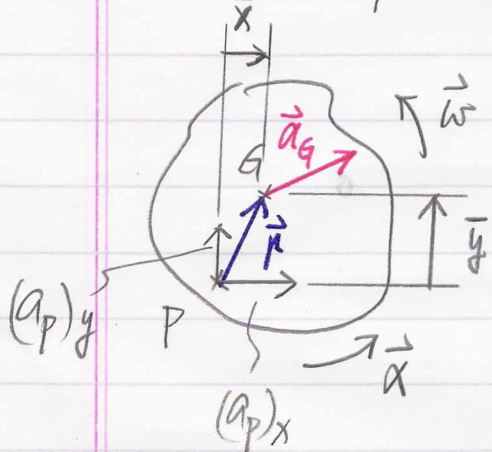
Also, $I_P = I_G + m(\bar{x}^2 + \bar{y}^2)$ or $I_G = I_P - m(\bar{x}^2 + \bar{y}^2)$

When P coincides with G, $\bar{x} = 0$ & $\bar{y} = 0$

$$\Rightarrow I_G = \int_m (x'^2 + y'^2)\, dm$$

$$\because \sum M_P = \bar{x} \cdot m (a_P)_y - \bar{y} \cdot m (a_P)_x + I_P \cdot \alpha \quad \text{— (1)}$$

where $\alpha = |\vec{\alpha}|$.



and

$$\vec{a}_G = \vec{a}_P + \vec{\alpha} \times \vec{r} - \omega^2 \vec{r}$$

$$\Rightarrow \begin{cases} (a_G)_x = (a_P)_x - \bar{y}\alpha - \bar{x}\omega^2 \\ (a_G)_y = (a_P)_y + \bar{x}\alpha - \bar{y}\omega^2 \end{cases}$$

$$\Rightarrow \begin{cases} -(a_P)_x + \bar{y}\alpha = -(a_G)_x - \bar{x}\omega^2 \\ (a_P)_y + \bar{x}\alpha = (a_G)_y + \bar{y}\omega^2 \end{cases}$$

$$\therefore \sum M_P = \bar{x} \cdot m (a_P)_y - \bar{y} \cdot m (a_P)_x + \left[I_G + m(\bar{x}^2 + \bar{y}^2) \right] \cdot \alpha$$

$$= \bar{x} \cdot m \left[(a_P)_y + \bar{x}\alpha \right] + \bar{y} \cdot m \left[-(a_P)_x + \bar{y}\alpha \right] + I_G \cdot \alpha$$

$$= \bar{x} \cdot m \left[(a_G)_y + \bar{y}\omega^2 \right] + \bar{y} \cdot m \left[-(a_G)_x - \bar{x}\omega^2 \right] + I_G \cdot \alpha$$

$$\boxed{\sum M_P = \bar{x} \cdot m (a_G)_y - \bar{y} \cdot m (a_G)_x + I_G \cdot \alpha} \quad \text{— (2)}$$