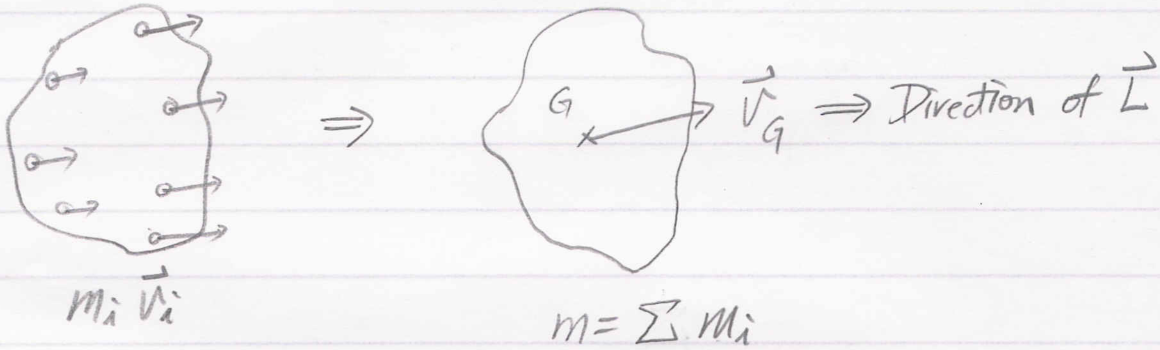
# Linear and Angular Momentum

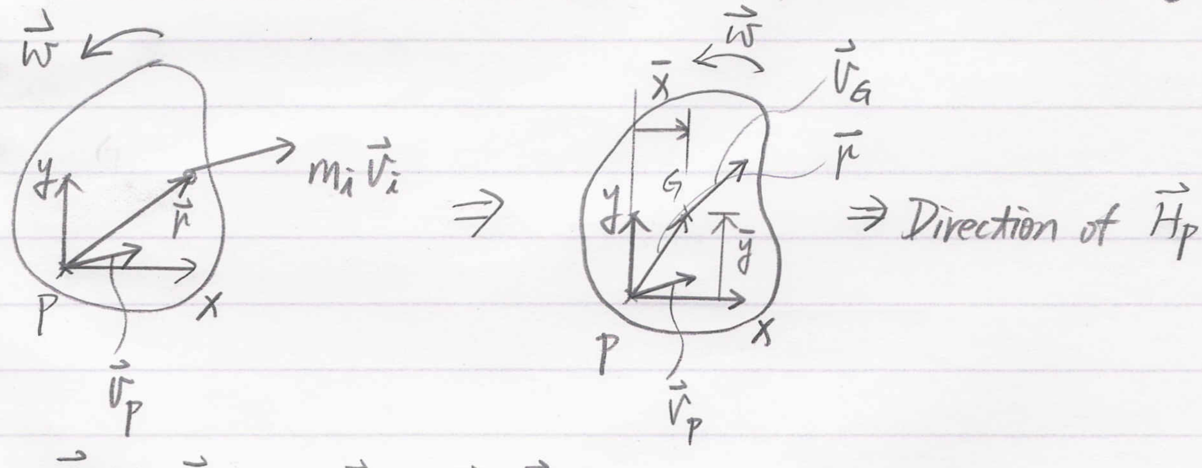
* Linear momentum:

Def: $\boxed{\vec{L} = \sum m_i \vec{v}_i = m \cdot \vec{v}_G}$ (kg·m or slug·ft)




* Angular momentum:

Def: $\boxed{(\vec{H}_P)_i = \vec{r} \times m_i \vec{v}_i}$ $\left(kg \cdot \frac{m^2}{s} \text{ or } slug \cdot \frac{ft^2}{s}\right)$




$$\vec{v}_i = \vec{v}_P + \vec{v}_{i/P} = \vec{v}_P + \vec{\omega} \times \vec{r}$$

$$(H_P)_i = -m_i y \cdot (v_P)_x + m_i x \cdot (v_P)_y + m_i \omega r^2$$

$$\Rightarrow H_P = -\bar{y} m (v_P)_x + \bar{x} m (v_P)_y + I_P \cdot \omega$$

When $P = G$, $\bar{x} = 0$, $\bar{y} = 0$

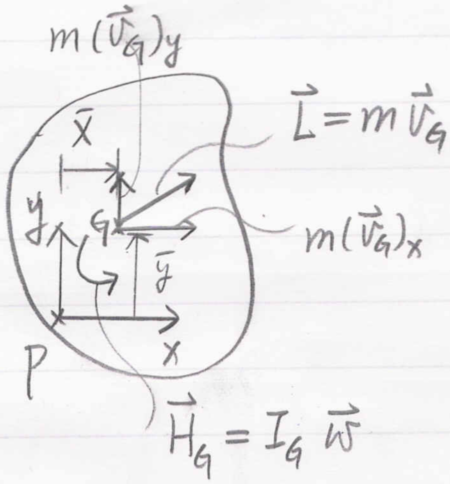
$$\Rightarrow H_P = H_G = I_P \cdot \omega$$

Since $I_P = I_G + m(\bar{x}^2 + \bar{y}^2)$

and $\vec{v}_G = \vec{v}_P + \vec{\omega} \times \vec{r}$

$$\Rightarrow (v_G)_x = (v_P)_x - \bar{y}\,\omega$$

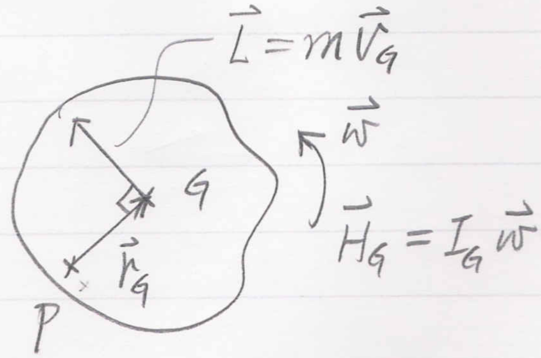
$$(v_G)_y = (v_P)_y + \bar{x}\,\omega$$




$$H_P = -\bar{y}\,m(v_G)_x + \bar{x}\,m(v_G)_y + I_G \cdot \omega$$

## General Plane Motion

$$\boxed{\begin{aligned} L &= m v_G \\ H_G &= I_G \cdot \omega \end{aligned}}$$




$H_P = I_G \cdot \omega + r_G\,(m v_G)$ (Note: $\vec{r}_G \perp \vec{v}_G$)