Principle of Impulse and Momentum

* Linear impulse and momentum:

$$\Sigma \vec{F} = \frac{d}{dt}(m\vec{v}_G) \Rightarrow \Sigma \int_{t_1}^{t_2} \vec{F}\, dt = m(\vec{v}_G)_2 - m(\vec{v}_G)_1$$

* Angular impulse and momentum:

$$\Sigma M_G = \frac{d}{dt}(I_G\, \omega) \Rightarrow \Sigma \int_{t_1}^{t_2} M_G\, dt = I_G\, \omega_2 - I_G\, \omega_1$$

$$\Rightarrow \begin{cases} \Sigma \text{Force} = \dfrac{d}{dt}(\text{Linear momentum}) = \dfrac{d}{dt}(\underline{\underline{\text{Mass}}} \times \underline{\text{Velocity}}) \\ \Sigma \text{Moment} = \dfrac{d}{dt}(\text{Angular momentum}) \\ \qquad\qquad = \dfrac{d}{dt}(\underline{\underline{\text{Mass moment of inertia}}} \times \underline{\text{Angular velocity}}) \end{cases}$$

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$$\Rightarrow \begin{cases} \Sigma(\text{Linear impulse}) = \Delta(\text{Linear momenta}) \\ \Sigma(\text{Angular impulse}) = \Delta(\text{Angular momenta}) \end{cases}$$

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Conservation of Momentum

* When $\Sigma \int_{t_1}^{t_2} \vec{F}\, dt$ is small (both $\vec{F}$ and $\int_{t_1}^{t_2} dt$ are small),

$$\Rightarrow \Sigma \int_{t_1}^{t_2} \vec{F}\, dt \cong 0 = m(\vec{v}_G)_2 - m(\vec{v}_G)_1$$

$$\Rightarrow m(\vec{v}_G)_2 = m(\vec{v}_G)_1$$

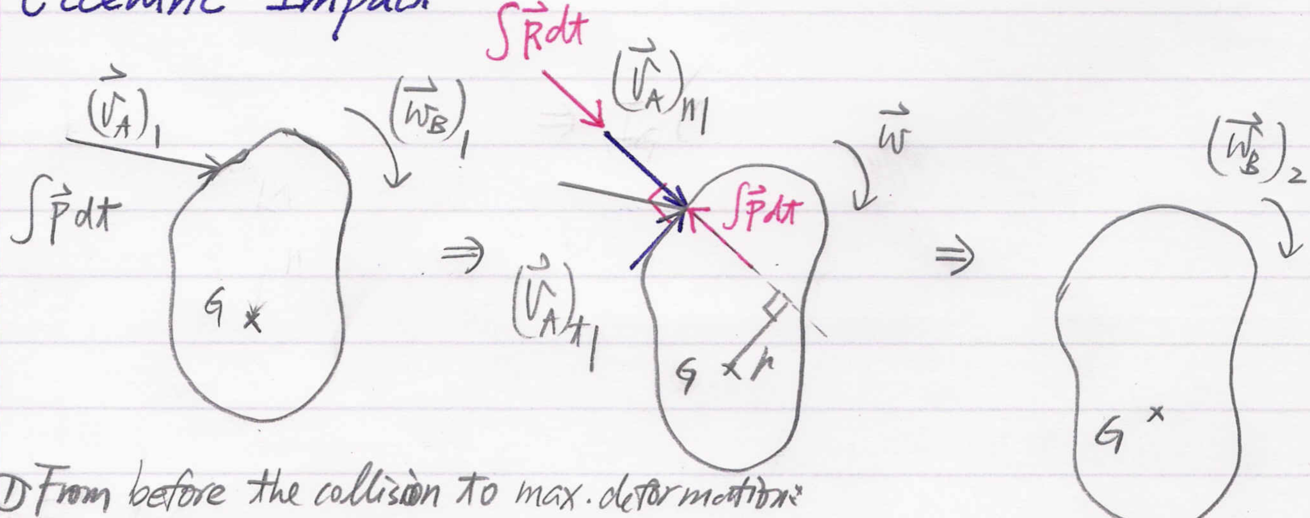
$$\Rightarrow \Sigma(m_i \vec{v}_i)_2 = \Sigma(m_i \vec{v}_i)_1$$

* When $\Sigma\int_{t_1}^{t_2} M_G\, dt$ is small (both M_G and $\int_{t_1}^{t_2} dt$ are small),

$$\Rightarrow \Sigma\int_{t_1}^{t_2} M_G\, dt \cong 0 = I_G \cdot \omega_2 - I_G \cdot \omega_1$$

$$\Rightarrow I_G \omega_2 = I_G \omega_1$$

Eccentric Impact



① From before the collision to max. deformation:

$$I_G(\omega_B)_1 + r\int P\, dt = I_G \cdot \omega$$

② From max. deformation to just after the impact:

$$I_G \omega + r\int R\, dt = I_G (\omega_B)_2$$

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$$\Rightarrow e = \frac{\int R\,dt}{\int P\,dt} = \frac{(\hat{v}_B)_2 - v}{v - (\hat{v}_B)_1} = \frac{v - (\hat{v}_A)_2}{(\hat{v}_A)_1 - v}$$

$$\Rightarrow \boxed{e = \frac{(\hat{v}_B)_2 - (\hat{v}_A)_2}{(\hat{v}_A)_1 - (\hat{v}_B)_1}}$$