

Three-Dimensional Kinematics of a Rigid Body

* Finite rotations vs. Infinitesimal rotations

→ The commutative law of addition doesn't apply.

⇒ Energy consideration

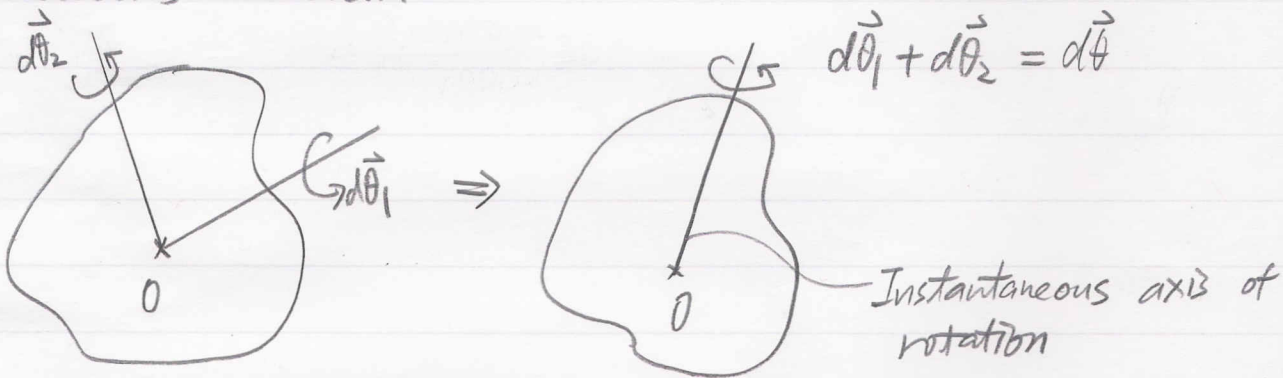
⇒ Vector operations don't apply.

When $\theta_1, \theta_2 \ll 1$, $\Rightarrow d\theta_1$ & $d\theta_2$.

⇒ The commutative law of addition applies.

⇒ Can be treated as vectors.

* Euler's theorem:



* Angular velocity:

$$\vec{\omega} = \frac{d\vec{\theta}}{dt} = \dot{\vec{\theta}}$$

$$\therefore d\vec{\theta} = d\vec{\theta}_1 + d\vec{\theta}_2$$

$$\therefore \frac{d\vec{\theta}}{dt} = \frac{d\vec{\theta}_1}{dt} + \frac{d\vec{\theta}_2}{dt} = \dot{\vec{\theta}}_1 + \dot{\vec{\theta}}_2 = \vec{\omega}_1 + \vec{\omega}_2 = \vec{\omega}$$