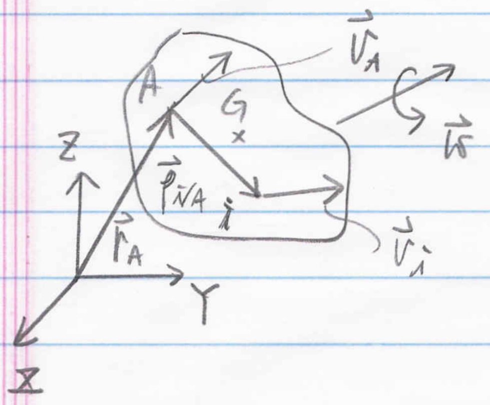
Kinetic Energy

$$T_i = \frac{1}{2} m_i v_i^2 = \frac{1}{2} m_i (\vec{v}_i \cdot \vec{v}_i)$$

$$\because \vec{v}_i = \vec{v}_A + \vec{\omega} \times \vec{\rho}_{i/A}$$



$$\therefore T = \frac{1}{2} m (\vec{v}_A \cdot \vec{v}_A) + \vec{v}_A \cdot \left(\vec{\omega} \times \int_m \vec{\rho}_{i/A} dm\right)$$

$$+ \frac{1}{2} \vec{\omega} \cdot \int_m \vec{\rho}_{i/A} \times (\vec{\omega} \times \vec{\rho}_{i/A}) dm$$

$$= T_{\text{linear velocity}} + T_{\text{angular velocity}}$$

* When the point of consideration, A, is fixed, $\boxed{\vec{v}_A = 0}$

$$\Rightarrow T_{\text{linear velocity}} = 0 \Rightarrow T = \frac{1}{2} \vec{\omega} \cdot \vec{H}_o$$

$$= \frac{1}{2} I_x \omega_x^2 + \frac{1}{2} I_y \omega_y^2 + \frac{1}{2} I_z \omega_z^2$$ #

* When point A is at the center of mass G, $\boxed{\int_m \vec{\rho}_{i/A} \, dm = 0}$

$$\Rightarrow T = \frac{1}{2} m v_G^2 + \frac{1}{2} \vec{\omega} \cdot \vec{H}_G$$

$$= \frac{1}{2} m v_G^2 + \frac{1}{2} I_x \omega_x^2 + \frac{1}{2} I_y \omega_y^2 + \frac{1}{2} I_z \omega_z^2$$ #

(Note that $\vec{H}_A = \vec{\rho}_{G/A} \times m \vec{v}_G + \vec{H}_G$.)

Equations of Motion

* Translational motion

$$\Sigma \vec{F} = m\vec{a}_G \Rightarrow \begin{cases} \Sigma F_x = m\,(a_G)_x \\ \Sigma F_y = m\,(a_G)_y \\ \Sigma F_z = m\,(a_G)_z \end{cases}$$

* Rotational motion

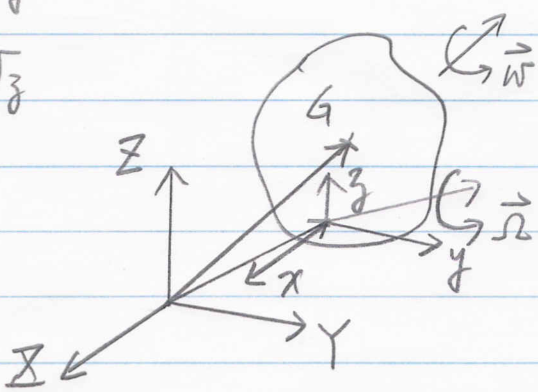
$$\Sigma \vec{M}_0 = \dot{\vec{H}}_0 \Rightarrow \begin{cases} \Sigma M_x = I_x \dot{\omega}_x - (I_y - I_z)\,\omega_y \omega_z \\ \Sigma M_y = I_y \dot{\omega}_y - (I_z - I_x)\,\omega_z \omega_x \\ \Sigma M_z = I_z \dot{\omega}_z - (I_x - I_y)\,\omega_x \omega_y \end{cases}$$

$\Rightarrow$ Euler's equations of motion.

* When choose the center of (x, y, z) at G, no local rotation.

$\Rightarrow \vec{\Omega} = \vec{\omega} = 0$

$$\Rightarrow \begin{cases} \Sigma M_x = I_x \dot{\omega}_x \\ \Sigma M_y = I_y \dot{\omega}_y \\ \Sigma M_z = I_z \dot{\omega}_z \end{cases} \Rightarrow \boxed{\Sigma \vec{M}_0 = (\dot{\vec{H}}_0)_{xyz}}$$



* When choose the center of (x, y, z) at any points other than G,

$\Rightarrow \vec{\Omega} = \vec{\omega} \neq 0$

$$\Rightarrow \boxed{\Sigma \vec{M}_0 = (\dot{\vec{H}}_0)_{xyz} + \vec{\omega} \times \vec{H}_0}$$

* When choose the center of (x, y, z) at any points other than G,

$\Rightarrow \vec{\Omega} \neq \vec{\omega} \neq 0$

$$\Rightarrow \left\{ \begin{aligned} \Sigma M_x &= I_x \dot{\omega}_x - I_y \Omega_z \omega_y + I_z \Omega_y \omega_z \\ \Sigma M_y &= I_y \dot{\omega}_y - I_z \Omega_x \omega_z + I_x \Omega_z \omega_x \\ \Sigma M_z &= I_z \dot{\omega}_z - I_x \Omega_y \omega_x + I_y \Omega_x \omega_z \end{aligned} \right\}$$

$$\Rightarrow \boxed{\Sigma \vec{M}_0 = (\dot{\vec{H}}_0)_{xyz} + \vec{\Omega} \times \vec{H}_0}$$