



CIVE.5120 Structural Stability (3-0-3)
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Buckling of Columns – I

Prof. Tzuyang Yu

Structural Engineering Research Group (SERG)
Department of Civil and Environmental Engineering
University of Massachusetts Lowell
Lowell, Massachusetts

SERG

Outline

- Imperfection in structures
- Post-buckling behavior
- Stability in MDOF (multi degree of freedom) systems
- Buckling of columns
- Summary

Imperfection in Structures

- Case 1: Spring-bar system without imperfection

- Case 2: Spring-bar system with imperfection
 - 2-1 Disturbing moment, M_0

Imperfection in Structures

- Case 2: Spring-bar system with imperfection
 - 2-2 Disturbing moment and initial rotation, θ_0

Post-buckling Behavior

- Analysis of cases 1 & 2:

Stability in MDOF Systems

- 2 DOF spring-bar system – Rotational spring:

Stability in MDOF Systems

- 2 DOF spring-bar system – Translational spring:

Buckling of Columns

- General assumptions in the classical column theory:
 - Perfectly straight column
 - No eccentricity
 - Plane remains plane
 - No shear deformation
 - Linear materials
 - Small deflection
- Pinned-ended columns
 - Vertically loaded without eccentricity
 - Vertically loaded with eccentricity



Buckling of a hollow metallic tube

Buckling of Columns



Cardington Laboratory, Building Research Establishment, UK (1996)

Pino Suarez Complex Bldg, Mexico City earthquake, Mexico (1985)

Buckling of Columns



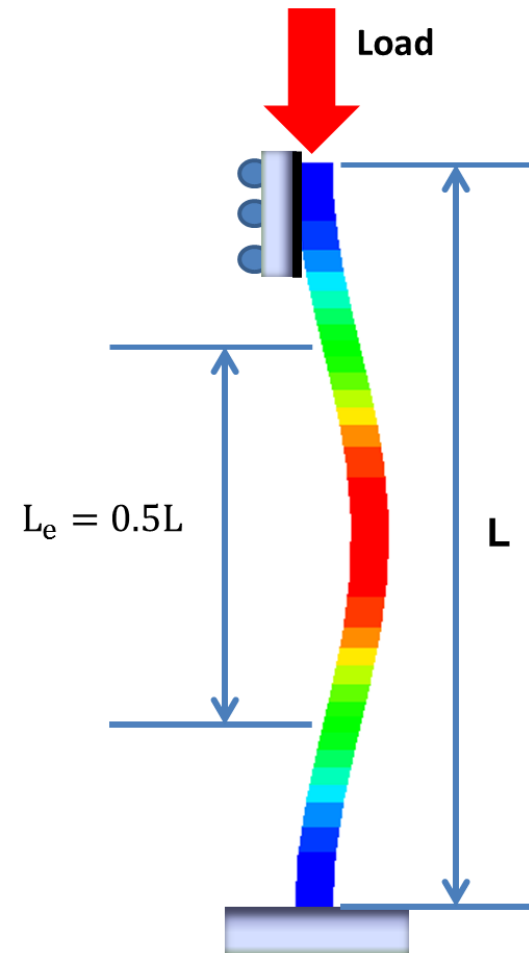
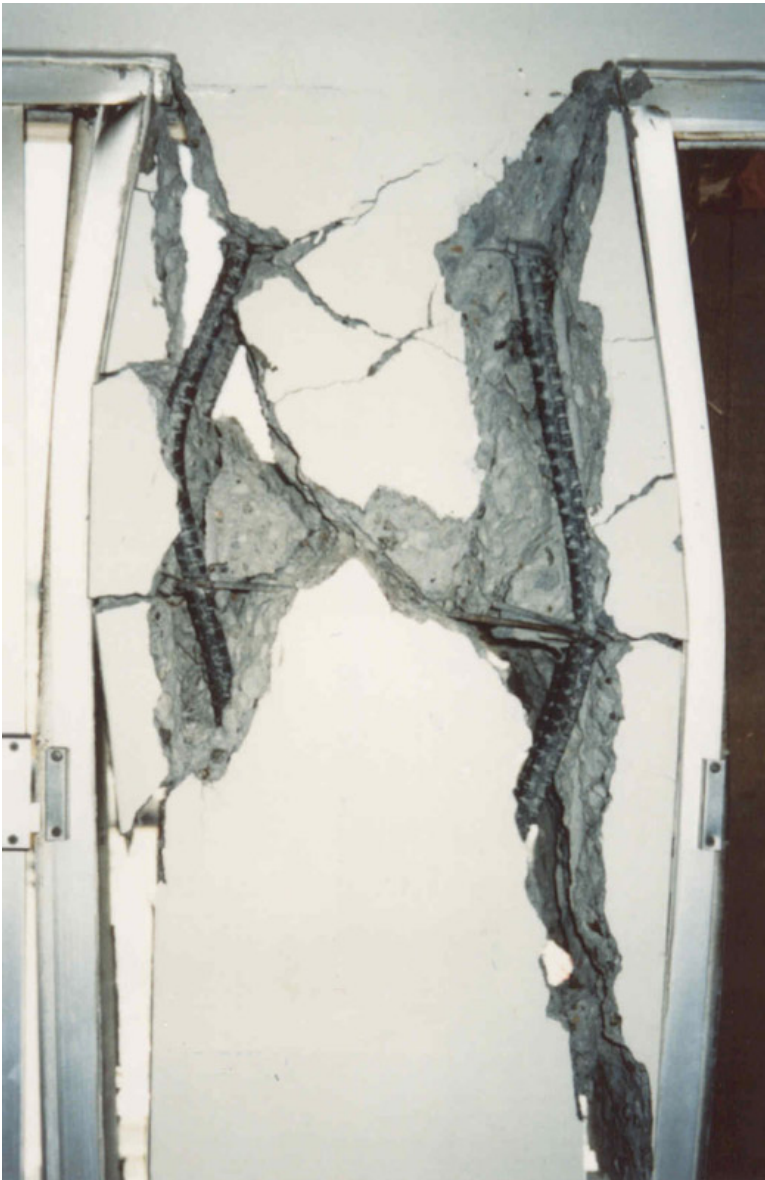
Cardington Laboratory,
Building Research
Establishment,
UK (1996)

Buckling of Columns



University of Washington, Seattle WA (03/28/09)

Buckling of Columns



Holiday Inn, Van Nuys, Northridge earthquake, California (1994)

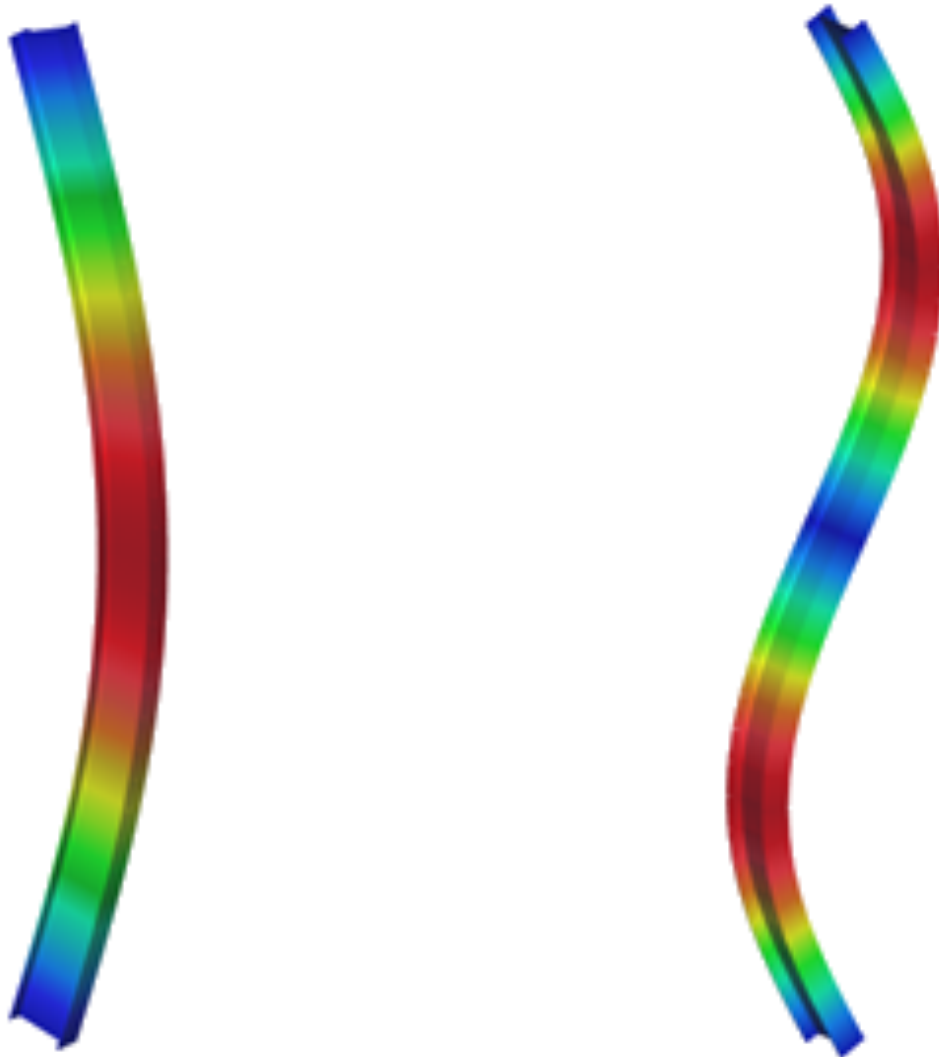
Buckling of Columns



Northridge earthquake, California (1994)

Buckling of Columns

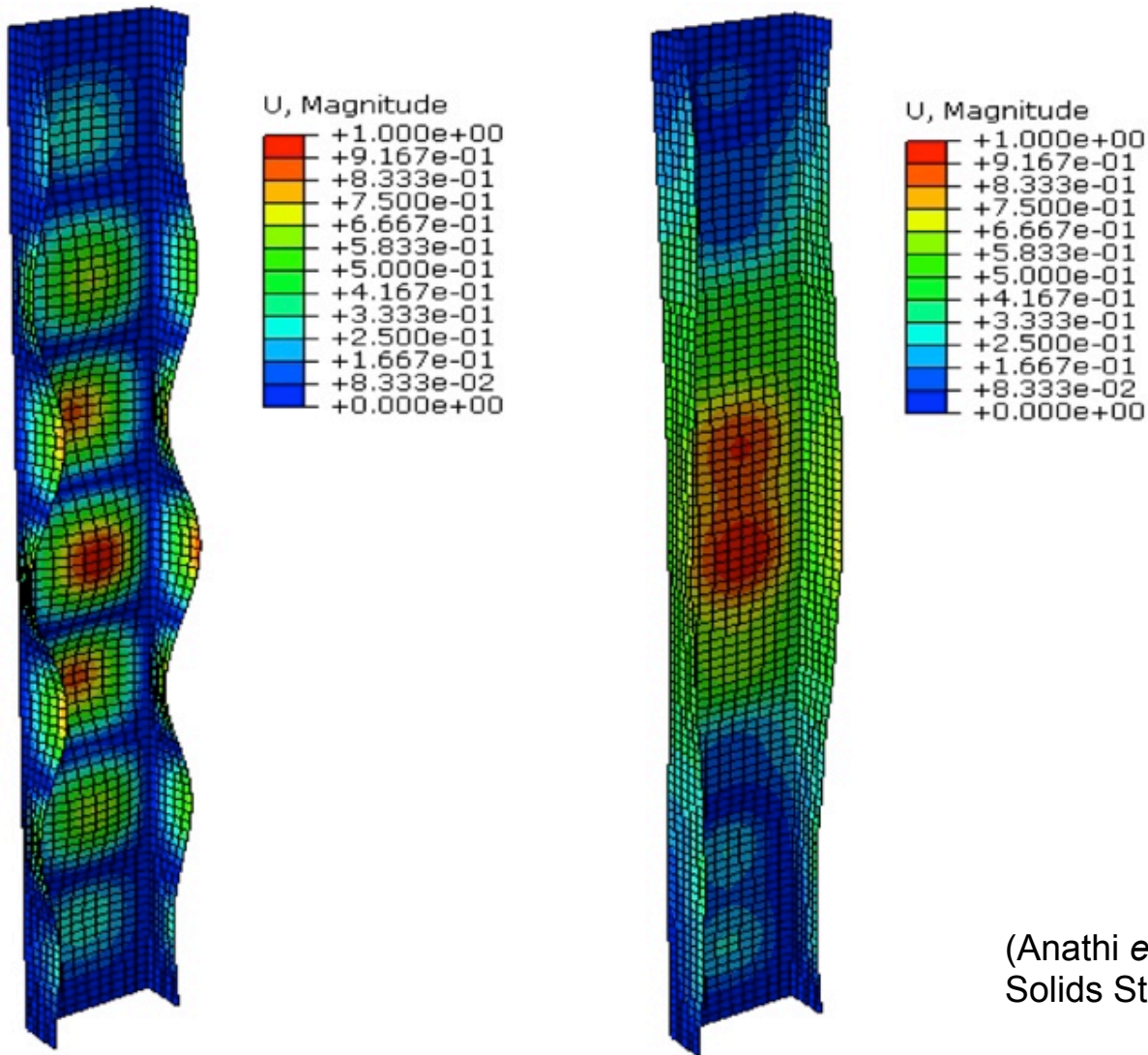
- Strong-axis vs. weak-axis



(Source: Stablab)

Buckling of Columns

- Local vs. global



(Anathi *et al.* (2015), Lat. Am. J. Solids Struct., 12(1))

Buckling of Columns

- Eccentrically-loaded column

Buckling of Columns

- Amplification factor, A_F

- Imperfection factor

Buckling of Columns

- Fixed-ended column

Buckling of Columns

- One end hinged and one end fixed column

Buckling of Columns

- One end fixed and one end guided column

Buckling of Columns

- Elastically-restrained-end column

Buckling of Columns

- Effective length factor, K

Buckled shape of column is shown by dashed line	(a) 	(b) 	(c) 	(d) 	(e) 	(f)
Theoretical K value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended K value when ideal conditions are approximated	0.65	0.80	1.2	1.0	2.10	2.0
End condition code		Rotation fixed Rotation free	Rotation fixed Rotation free	Rotation fixed Rotation free	Translation fixed Translation fixed	Translation fixed Translation free

Buckling of Columns

- Buckling/critical load and effective length

Buckling Load	$\frac{\pi^2 EI}{L^2}$	$\frac{4\pi^2 EI}{L^2}$	$\frac{2.045\pi^2 EI}{L^2}$	$\frac{\pi^2 EI}{4L^2}$	$\frac{\pi^2 EI}{L^2}$
Effective Length	L	$0.5L$	$0.699L$	$2L$	L

Buckling of Columns

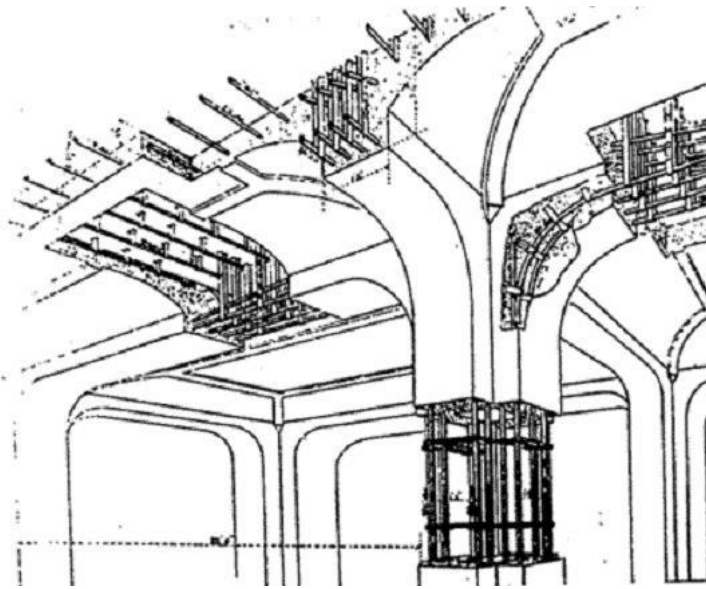
- Boundary conditions:
 - Pinned
 - Fixed
 - Guided
 - Free
 - Rotational spring



Foundation of the Golden Gate Bridge, CA

Q: What is the actual boundary condition in real structures?

Buckling of Columns



Connection of a typical reinforced concrete frame



Northeast support of Eiffel tower, Paris, France

Buckling of Columns

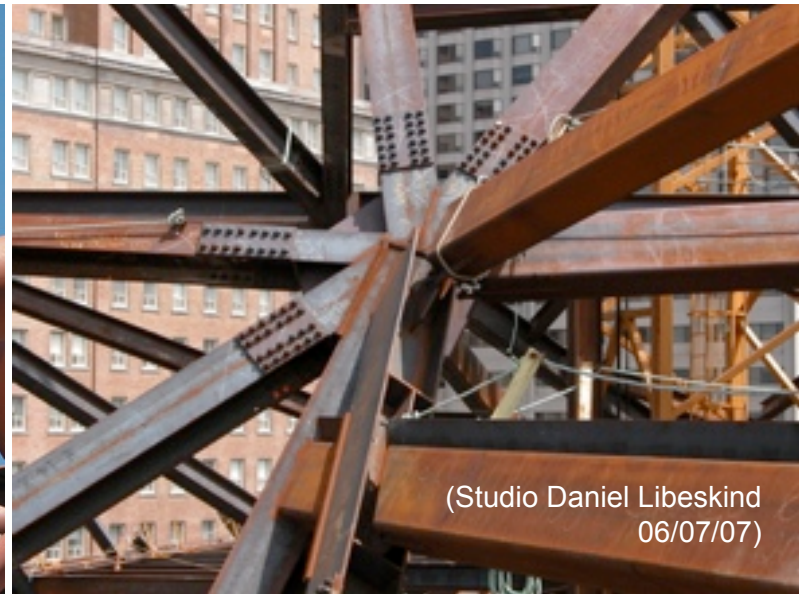


(Source: Ivo Casagrande Júnior 11/13/15)

Buckling of Columns



(Source: ArchitectureAU.com)

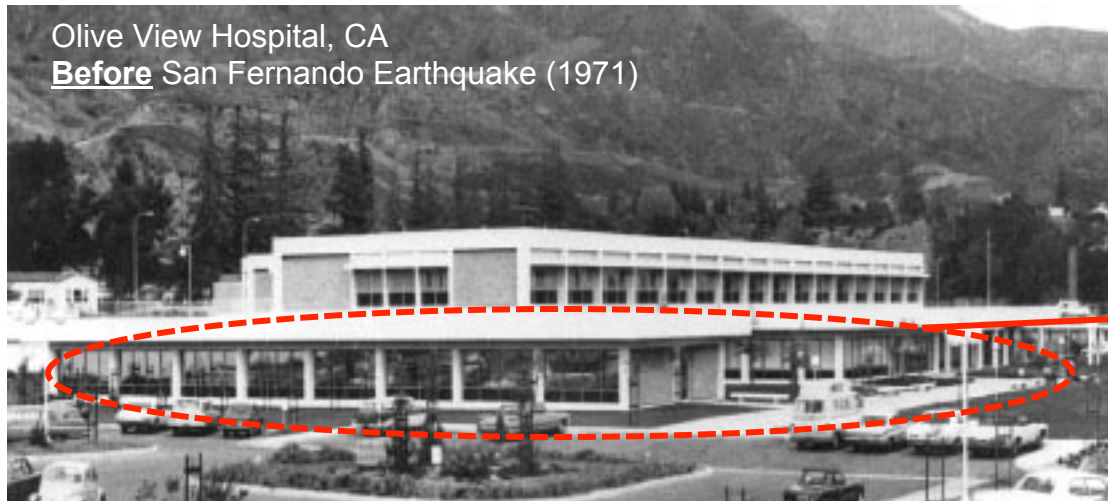


(Studio Daniel Libeskind
06/07/07)

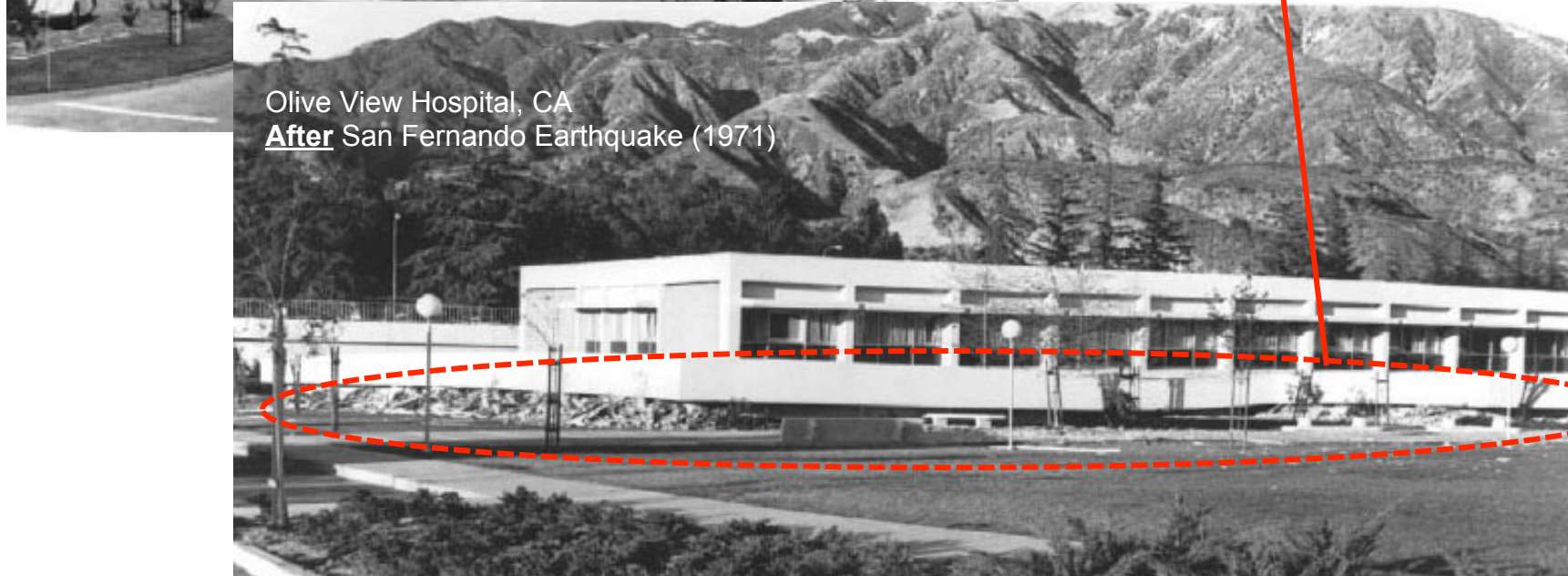
Buckling of Columns

- Effect of reinforcement in concrete structures

(Source: California Seismic Safety Commission)

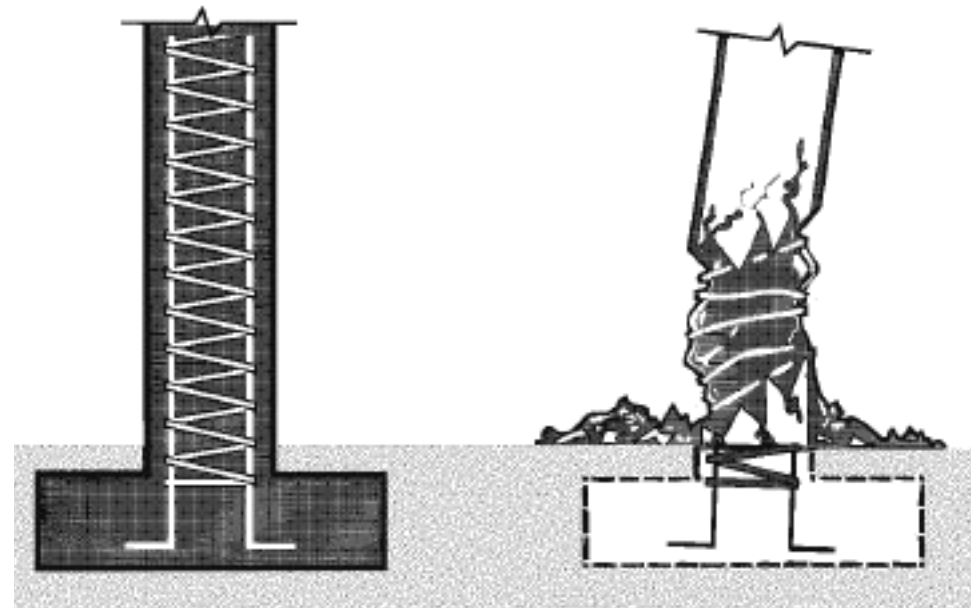
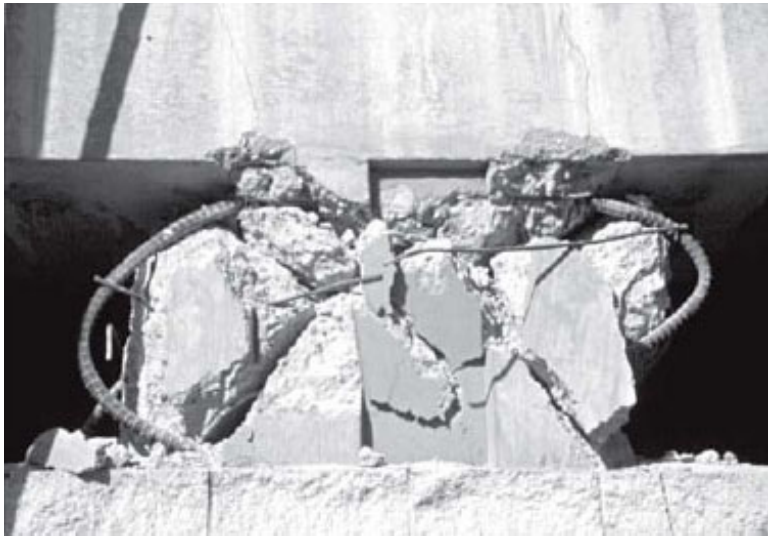


Collapsed columns
and walls resulted in
the disappearing of
the first floor

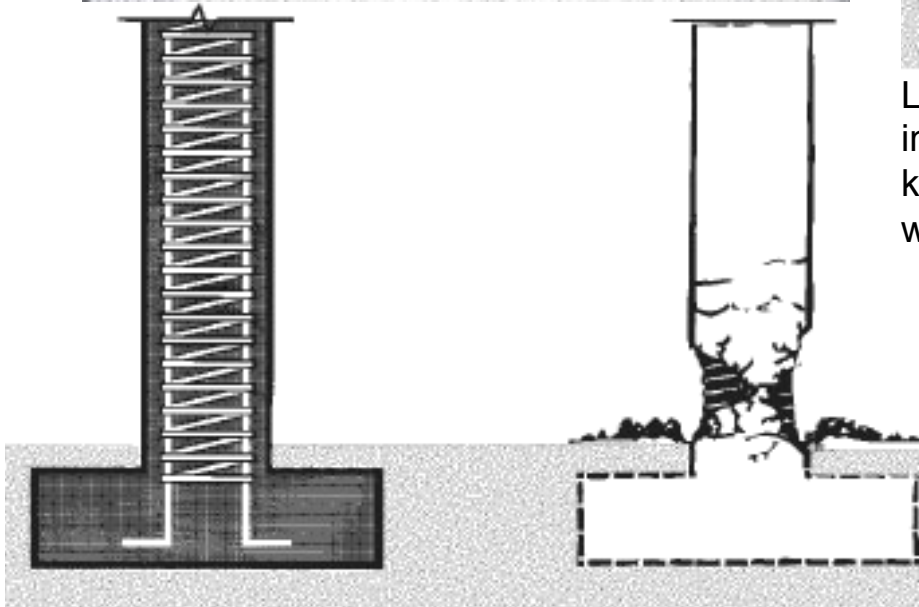


Buckling of Columns

- Effect of reinforcement in concrete structures (Source: California Seismic Safety Commission)



Lack of Reinforcing steel - If the confining reinforcing steel in a column is too widely spaced (left), it will not be able to keep the vertical reinforcing bars and the concrete in place when it is shaken by an earthquake (right).



The addition of more confining steel (left) keeps the vertical reinforcing bars from buckling and the concrete from shifting so that the building continues to be fully supported (right) even if it is damaged in an earthquake.

Buckling of Columns

- Two-axial-force column

Summary

- In theory, critical load of columns can be improved by:
 - Increasing Young's modulus of the material, or
 - Increasing the mass moment of inertia of column cross section, or
 - Reducing column length/height.
- Material imperfection in actual structures is usually inevitable.
- The buckling/critical load of structures can be determined by either the bifurcation or the energy approach, but the post-buckling behavior can only be determined by the energy approach.
- Eigenvalue analysis is carried out in determining the buckling load of MDOF systems.
- Boundary conditions (B.C.) are essential in determining the critical load of columns.
- For steel columns, temperature increase can reduce the Young's modulus of steel, hence reducing the critical load of steel columns.