CIVE.5120 Structural Stability (3-0-3)
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Buckling of Columns – II

Prof. Tzuyang Yu
Structural Engineering Research Group (SERG)
Department of Civil and Environmental Engineering
University of Massachusetts Lowell
Lowell, Massachusetts

Outline

- Differential equation approach
  - Second-order vs. fourth-order
- Slenderness ratio
- Initially crooked columns
- Inelastic columns
  - Tangent modulus theory (Engesser, 1889)
  - Reduced modulus theory (double modulus theory) (Engesser, 1895)
  - Inelastic column theory (Shanley, 1947)
- Summary

Differential Equation Approach

- Pin-ended column:
  - Second-order differential equation
  - Fourth-order differential equation

Slenderness Ratio

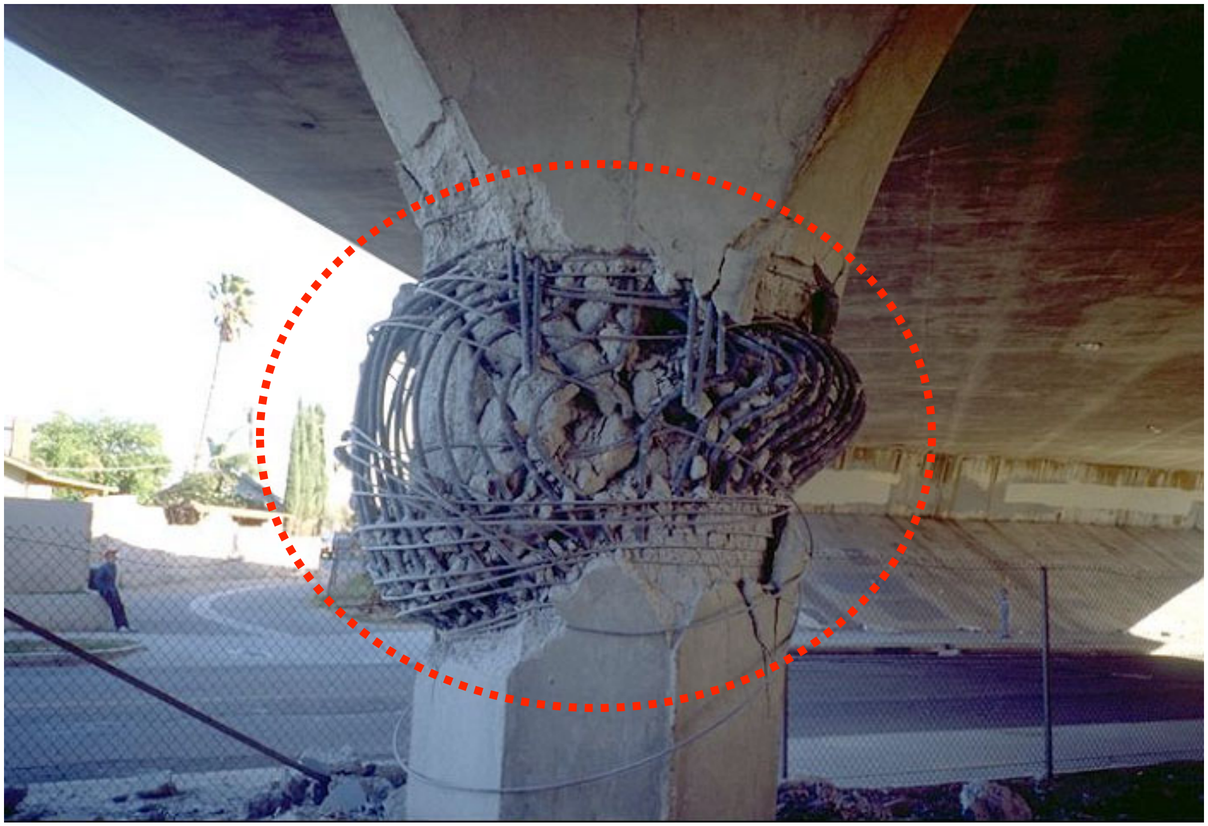
- Material strength limit:

- Short and long columns:

Initially Crooked Columns

- Pin-ended column:

Buckling of Columns



Buckling of a reinforced concrete bridge column

Buckling of a composite column

Buckling of Columns

Buckling of a steel column

Initially Crooked Columns

- Perry-Robertson formula:

Inelastic Columns

- Tangent modulus theory
  - Developed by F. Engesser (1898)
  - Assumptions:
    - Perfectly straight columns
    - Pin-ended & no eccentricity
    - Small bending deformation
    - Plane remains plane
    - No strain reversal; increasing axial force, together with bending moment, considered to cause an overall increase in axial strain
  - Predicts the smallest load for real columns; P_t
  - The tangent modulus depends only on the material property of a column.

Inelastic Columns

- Tangent modulus theory

Inelastic Columns

- Reduced (double) modulus theory
  - Proposed by F. Engesser (1895)
  - Assumptions:
    - Perfectly straight columns
    - Pin-ended & no eccentricity
    - Small bending deformation
    - Plane remains plane
    - Strain reversal considered; constant axial force during buckling (net increase of axial force = 0)

  - Predicts the largest load for real columns; P_r
  - The reduced modulus depends on both the i) material property and ii) geometry of the cross section of a column

Inelastic Columns

- Reduced modulus theory

Inelastic Columns

- Inelastic column theory
  - Proposed by F.R. Shanley (1947)
  - Assumptions:
    - Perfectly straight columns
    - Pin-ended & no eccentricity
    - Two rigid bars & a deformable cell model
    - Strain reversal considered; constant axial force during buckling (net increase of axial force ≠ 0)
  - Predicts realistic loads for real columns; P_s
  - Shanley's theory utilizes both the i) material property and ii) geometry of the cross section of a column

Inelastic Columns

- Inelastic column theory

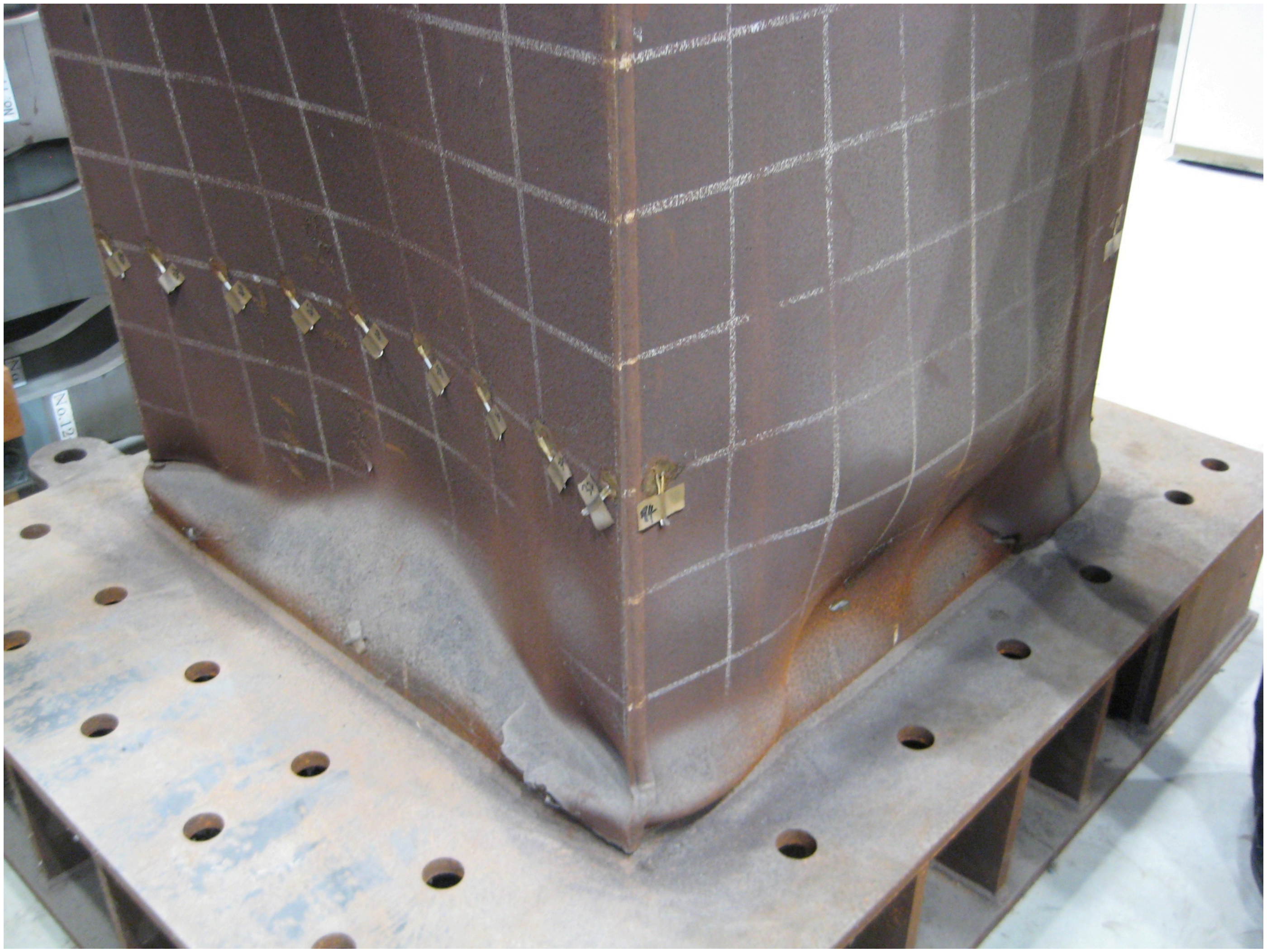
Buckling of Columns

- Rectangular columns

Buckling of a steel column, Public Works Research Institute (PWRI), Tsukuba, Japan (Source: T. Yu)

Buckling of Columns

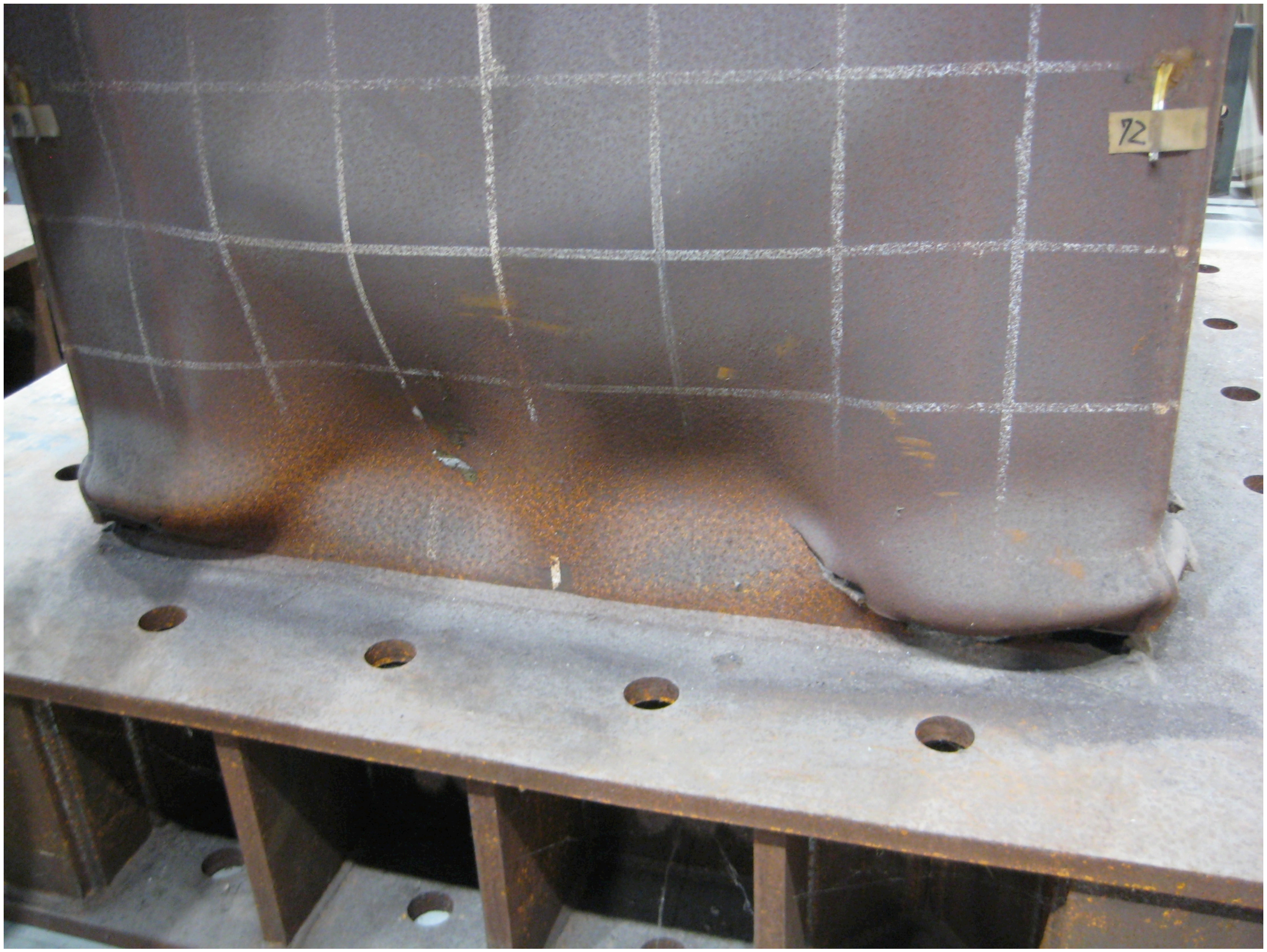
- Rectangular columns



Buckling of a steel column, Public Works Research Institute (PWRI), Tsukuba, Japan (Source: T. Yu)

Buckling of Columns

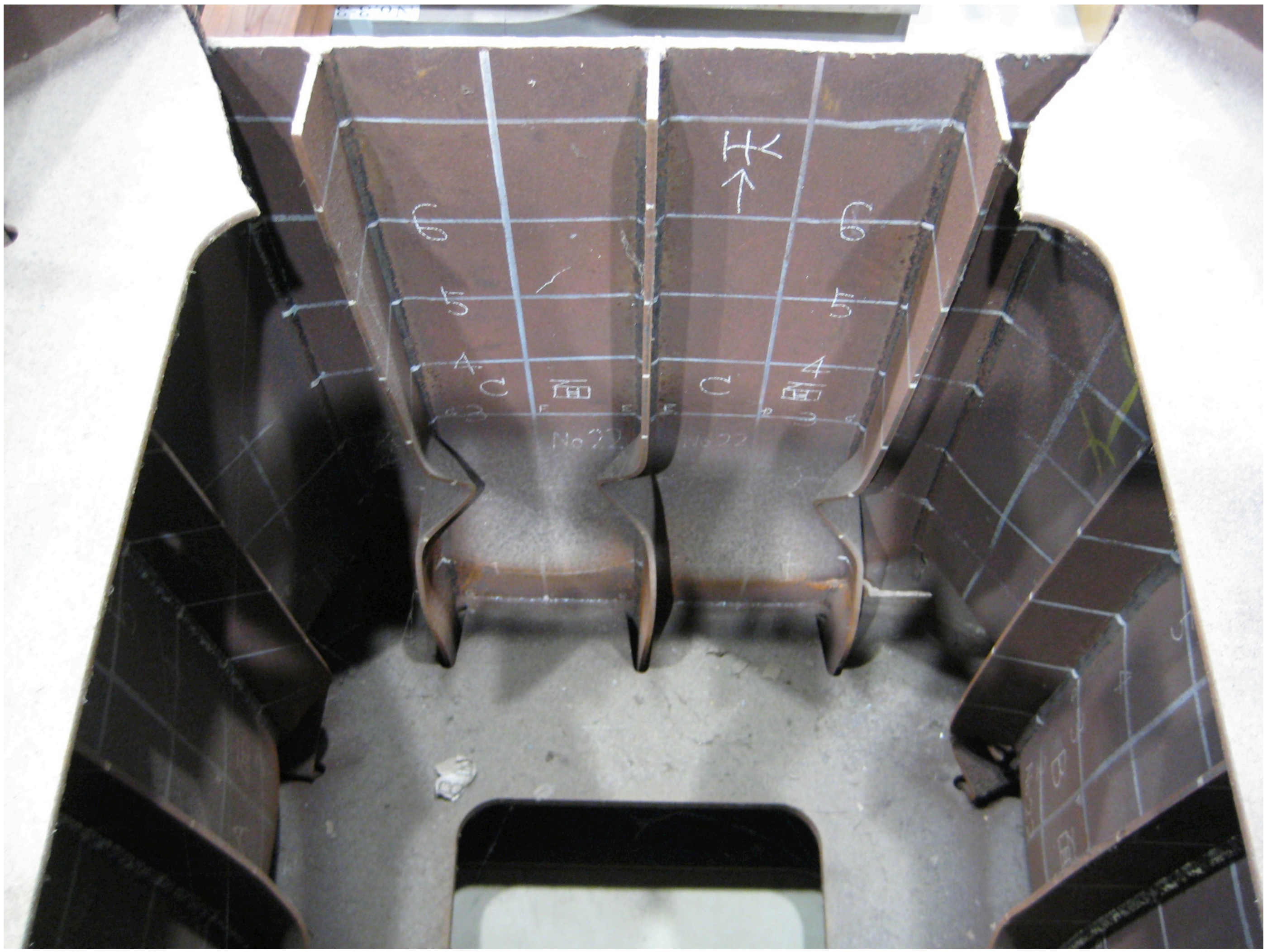
- Rectangular columns



Buckling of a steel column, Public Works Research Institute (PWRI), Tsukuba, Japan (Source: T. Yu)

Buckling of Columns

- Rectangular columns



Buckling of a steel column, Public Works Research Institute (PWRI), Tsukuba, Japan (Source: T. Yu)

Buckling of Columns

- Rectangular columns

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Buckling of Columns

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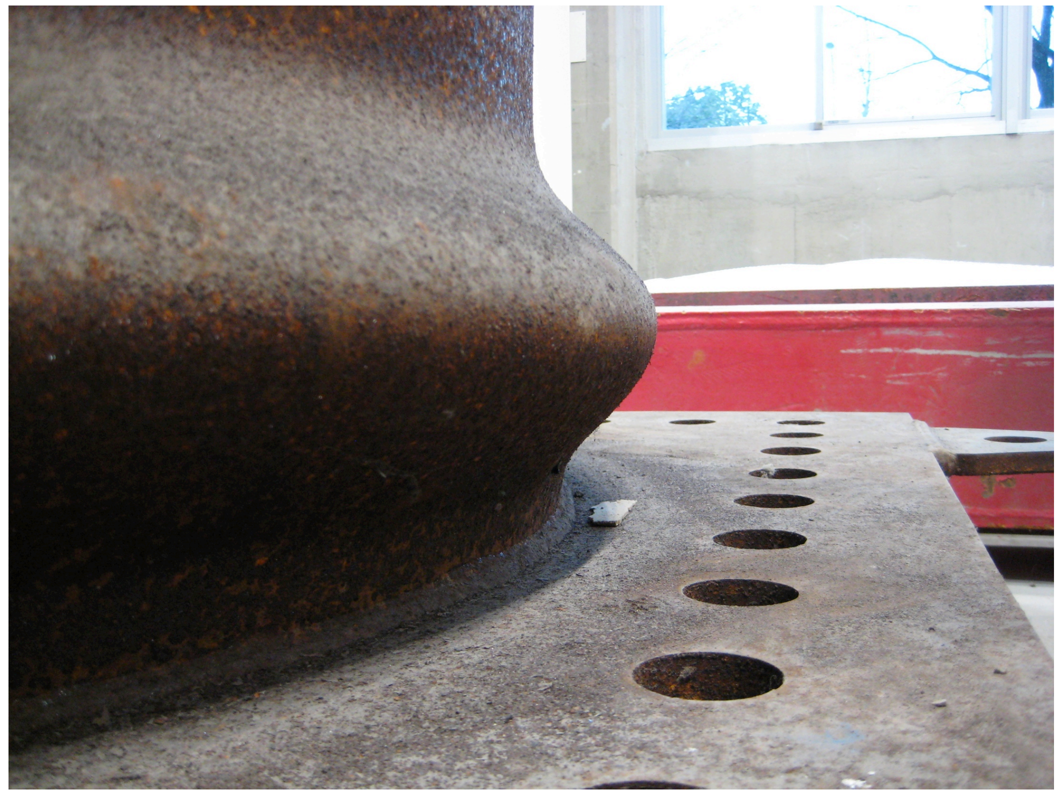

Buckling of a steel column, Public Works Research Institute (PWRI), Tsukuba, Japan (Source: T. Yu)

Summary

- Second-order D.E. approach is based on moment equilibrium; fourth-order D.E. approach is based on both force and moment equilibriums.

- Second-order D.E. approach utilizes only geometric B.C., while fourth-order D.E. approach utilizes mixed geometric and force B.Cs.

- Slenderness ratio distinguishes short and long columns.

- Initial crookness of a column amplifies the effective bending moment by producing a second-order moment in the column.

Summary

- The tangent modulus concept gives the maximum load up to which an initial straight column remains straight.
- The actual maximum load exceeds the tangent modulus load (P_t), but it cannot be as large as the reduced modulus load (P_r).
- Any load larger than P_t will cause the column to be laterally deformed.
- In the load range of $P_t < P_s < P_{max}$, there is always strain reversal present.
- In general, $P_t < P_s < P_r < P_e$.