CIVE.5120 Structural Stability (3-0-3)
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Buckling of Beam-Columns – I

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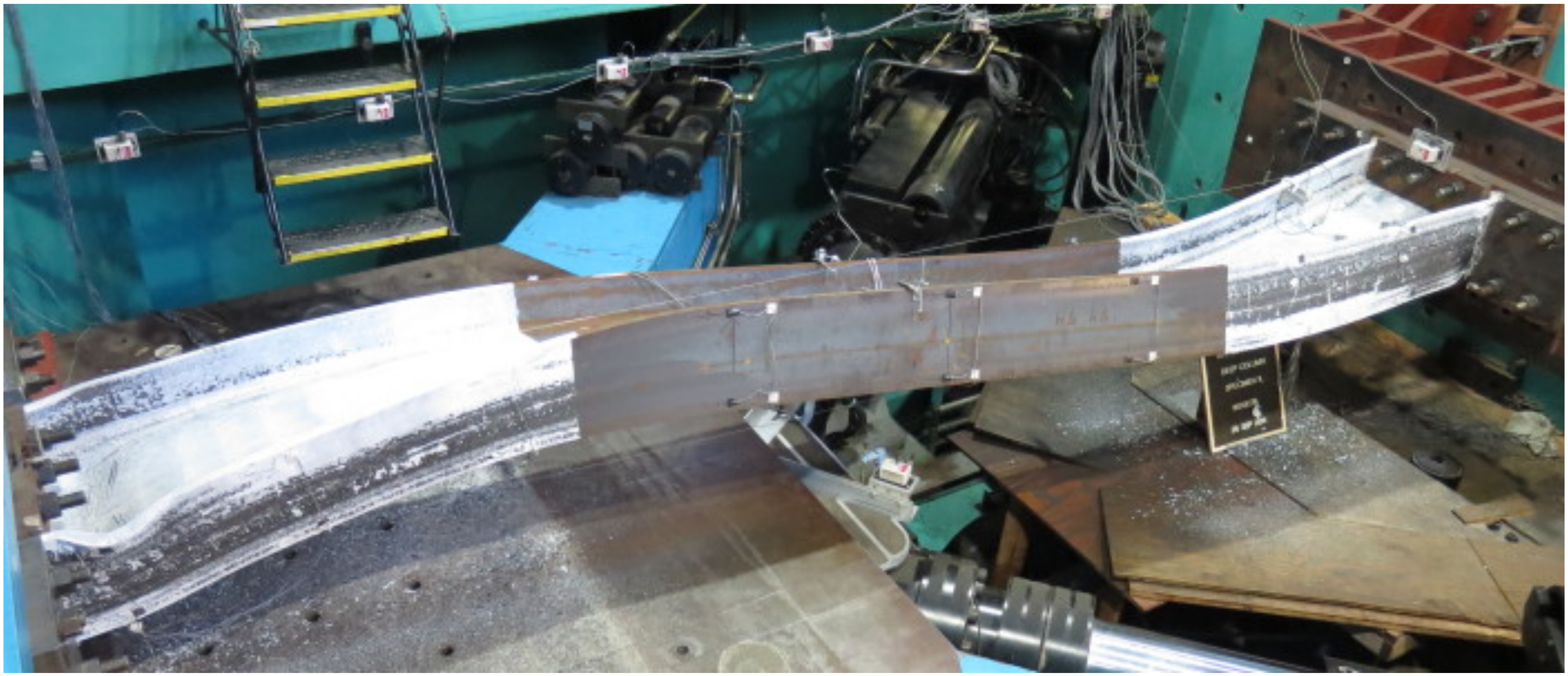
Beam-Columns – I



(Source: Structural Stability Research Council)

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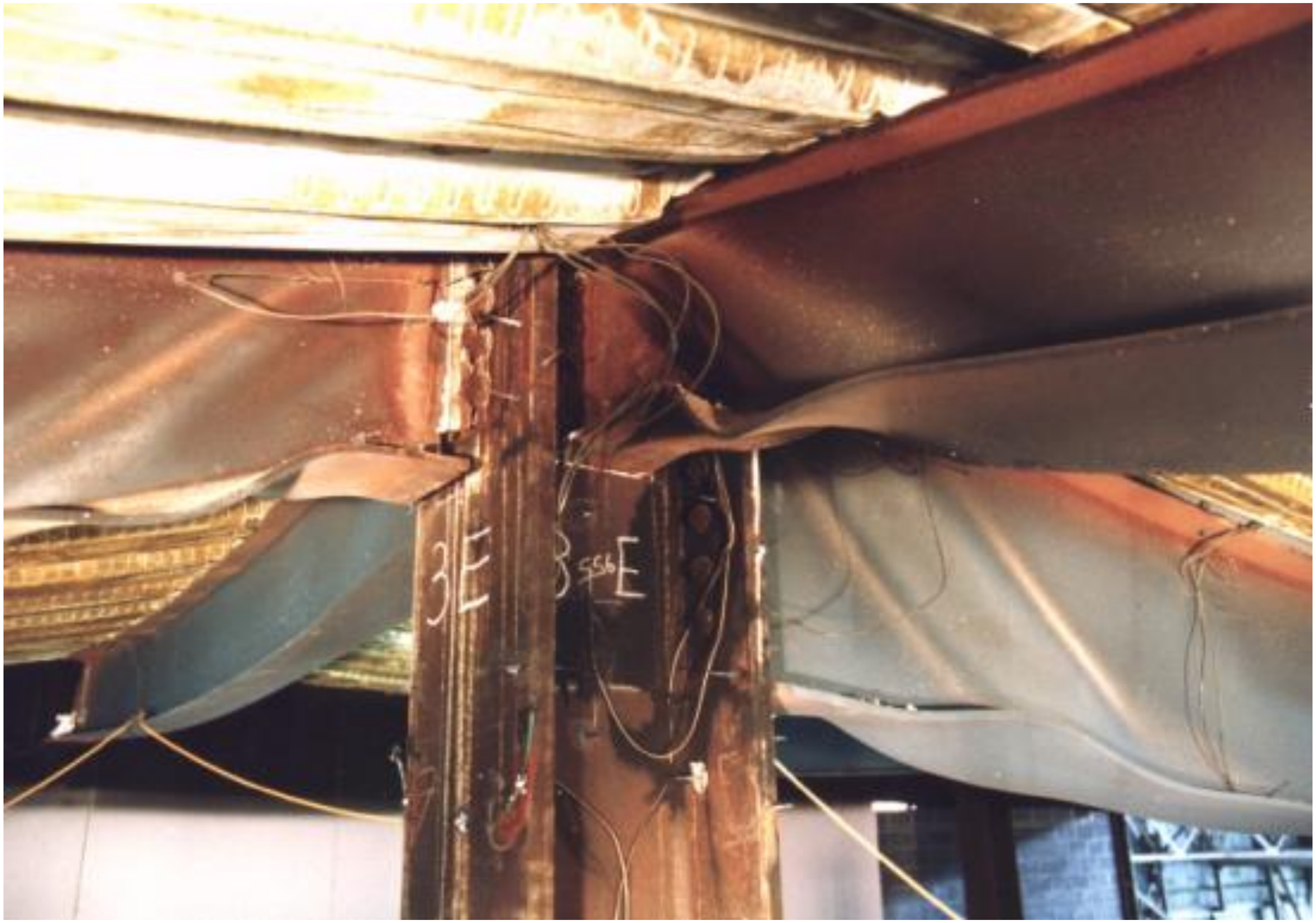

Buckling of a steel beam-column

Beam-Columns – I

Buckling of a steel beam-column

Outline

- Beam-columns with various loading conditions
  - Uniformly distributed lateral load, $w(x)$
  - A concentrated lateral load, $Q(x=a)$
  - End moments, M_A and M_B
    - single curvature
    - double curvature

- Summary

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- **Uniformly distributed lateral load:** ***w(x)***
  - 2nd-order D.E. approach
  - Governing equation:
  - Solution: $y = y_c + y_p$

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– Maximum deflection: y_{max}

– Primary and secondary effects:

– Critical load:

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– Interactions among P, $w(x)$, and y_{max}:

- At constant P, $w(x)$ and y_{max} are proportional to each other.

- At constant $w(x)$, P and y_{max} are **NOT** proportional to each other.

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– Maximum internal bending moment: M_{max}

– Primary and secondary effects:

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- Theoretical and design values:
  - Maximum deflection: y_{max}
  - Maximum internal bending moment: M_{max}

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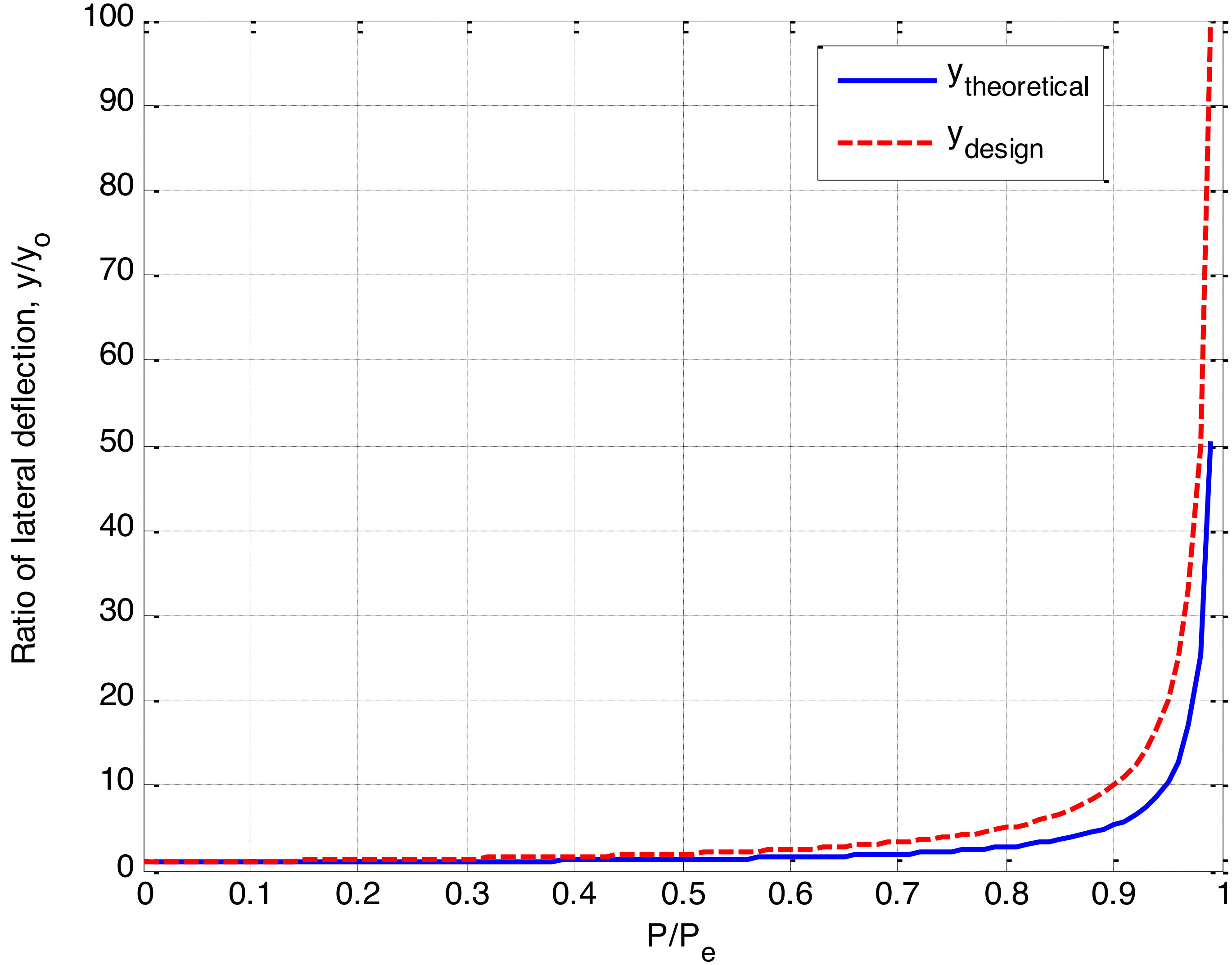

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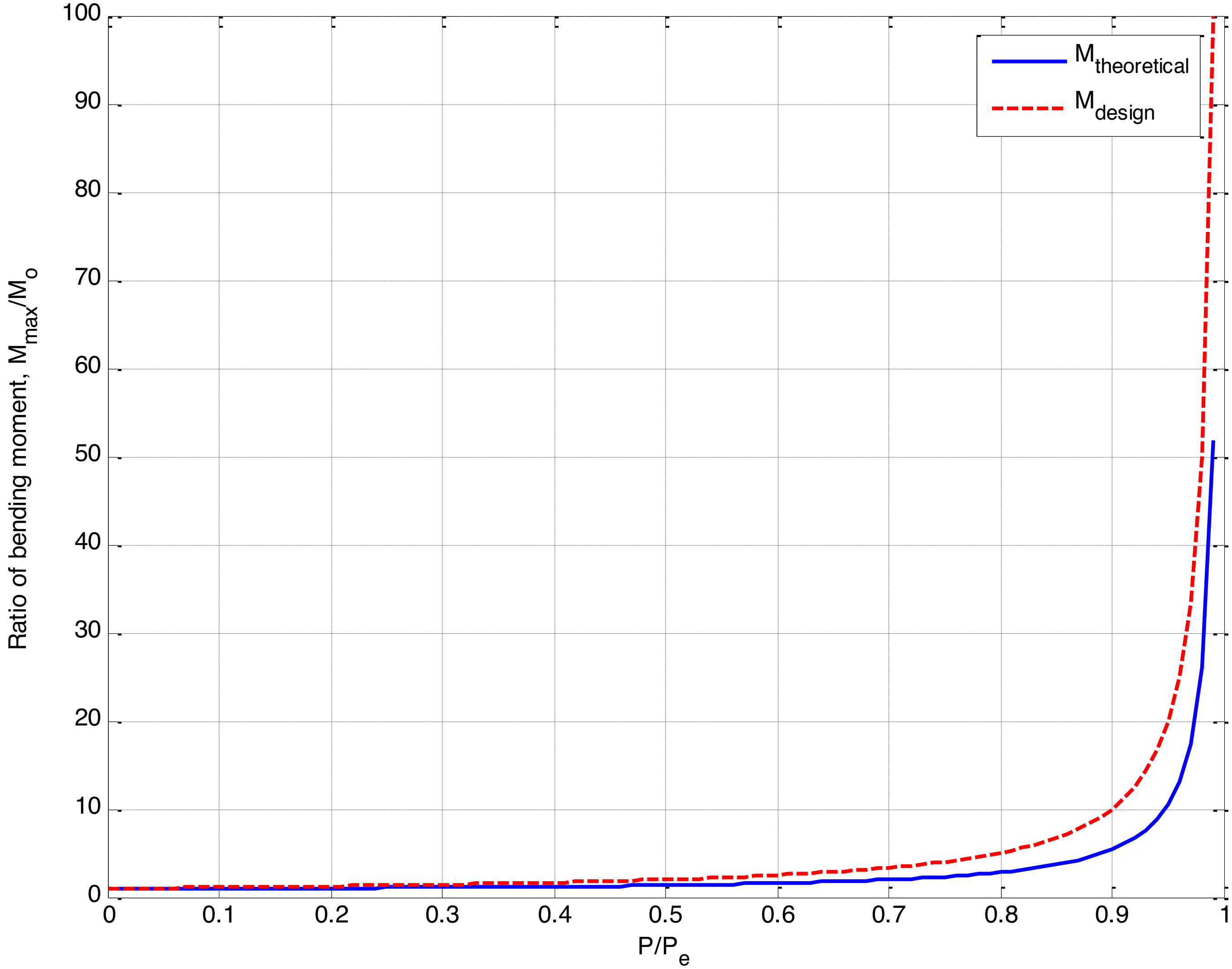

Beam-Columns – I

- Note: Taylor series expansion

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \quad \text{for all } x$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots \quad \text{for all } x$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots \quad \text{for all } x$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \cdots \quad \text{for } |x| < \frac{\pi}{2}$$

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- **A concentrated lateral load:** ***$Q(x=a)$***
  - 2^{nd}-order D.E. approach
  - Governing equation:
  - Solution: $y = y_c + y_p$

Beam-Columns – I

– Maximum deflection: y_{max}

– Primary and secondary effects:

– Critical load, $\boldsymbol{P_{cr}}$

Beam-Columns – I

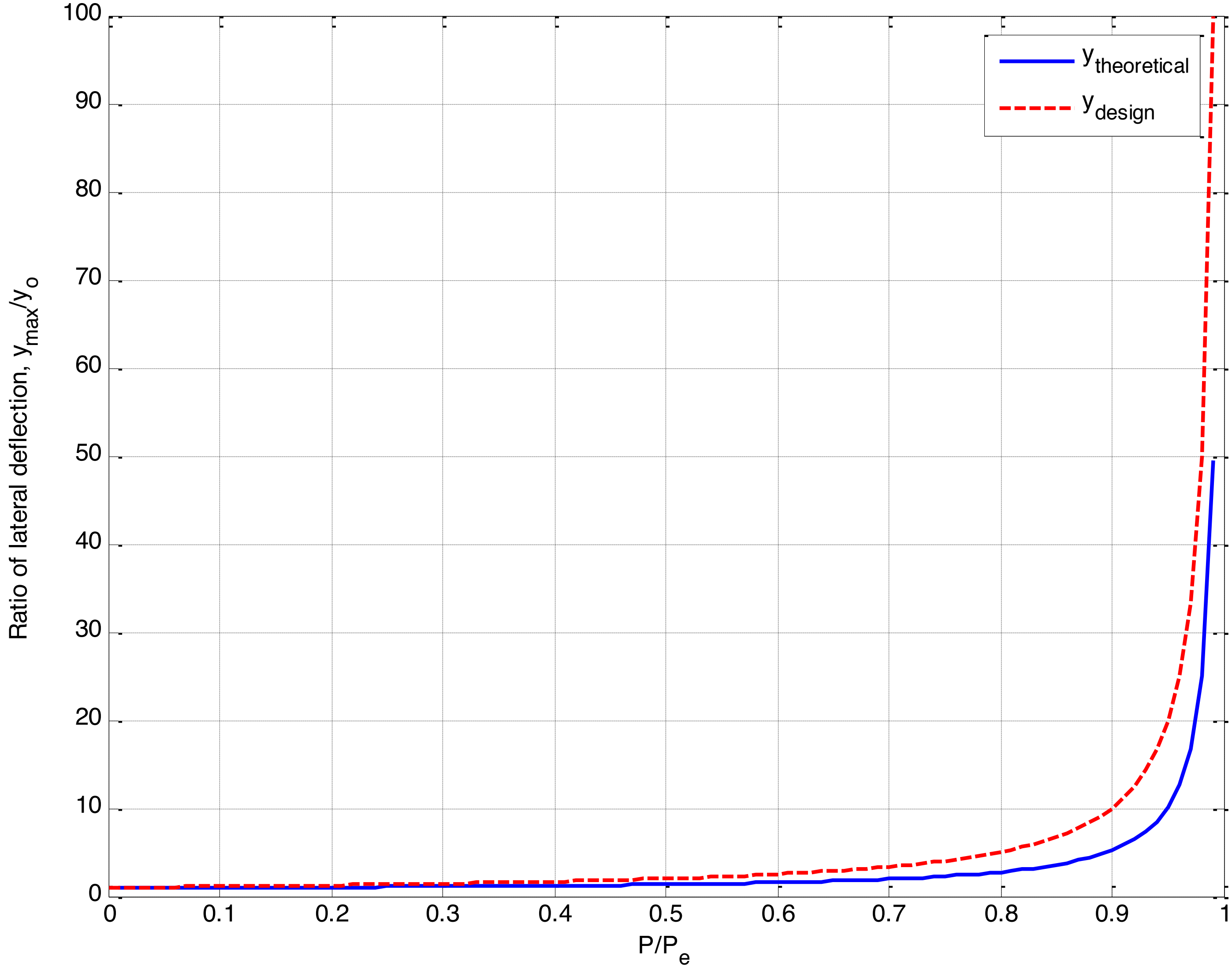
– Maximum internal bending moment: M_{max}

– Primary and secondary effects:

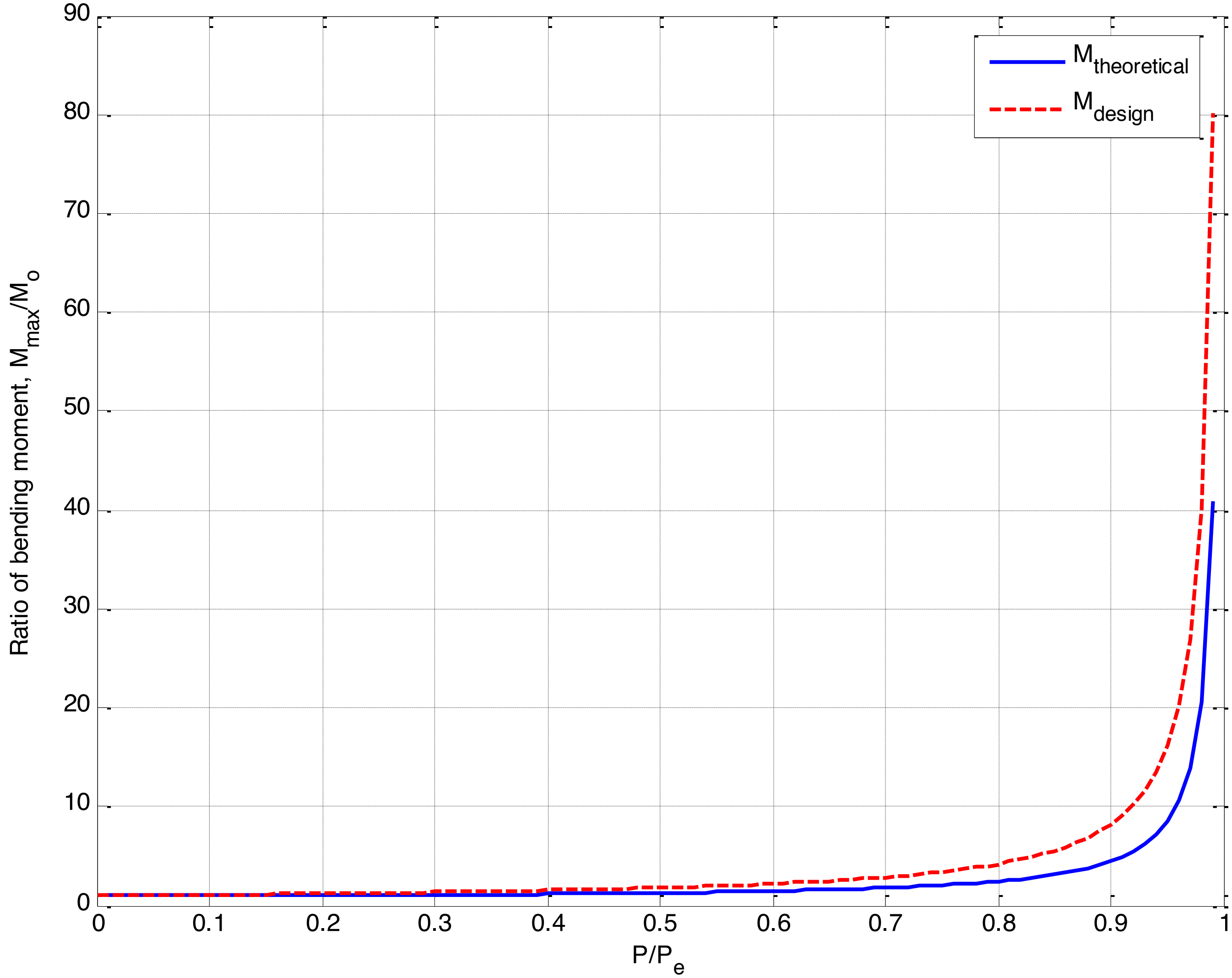
Beam-Columns – I

- Theoretical and design values:
  - Maximum deflection: y_{max}
  - Maximum internal bending moment: M_{max}

Beam-Columns – I



Beam-Columns – I



Beam-Columns – I

- **End moments: M_A and M_B**
  - 2nd-order D.E. approach
  - Governing equation:
  - Solution: $y = y_c + y_p$

Beam-Columns – I

– Maximum deflection: y_{max}

– Primary and secondary effects:

– Critical load, $\boldsymbol{P_{cr}}$

Beam-Columns – I

- Maximum internal bending moment: M_{max}

- Primary and secondary effects:

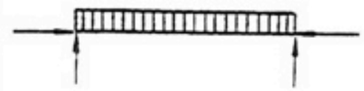
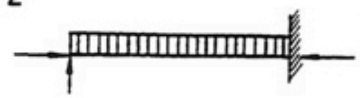
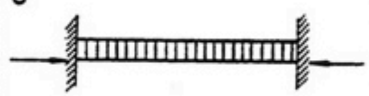
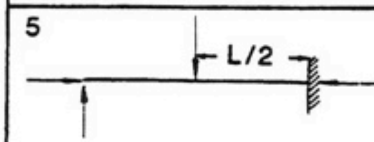
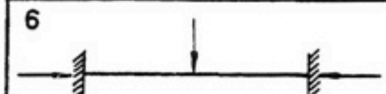
Beam-Columns – I

- Theoretical and design values:
  - Maximum deflection: y_{max}
  - Maximum internal bending moment: M_{max}

Beam-Columns – I

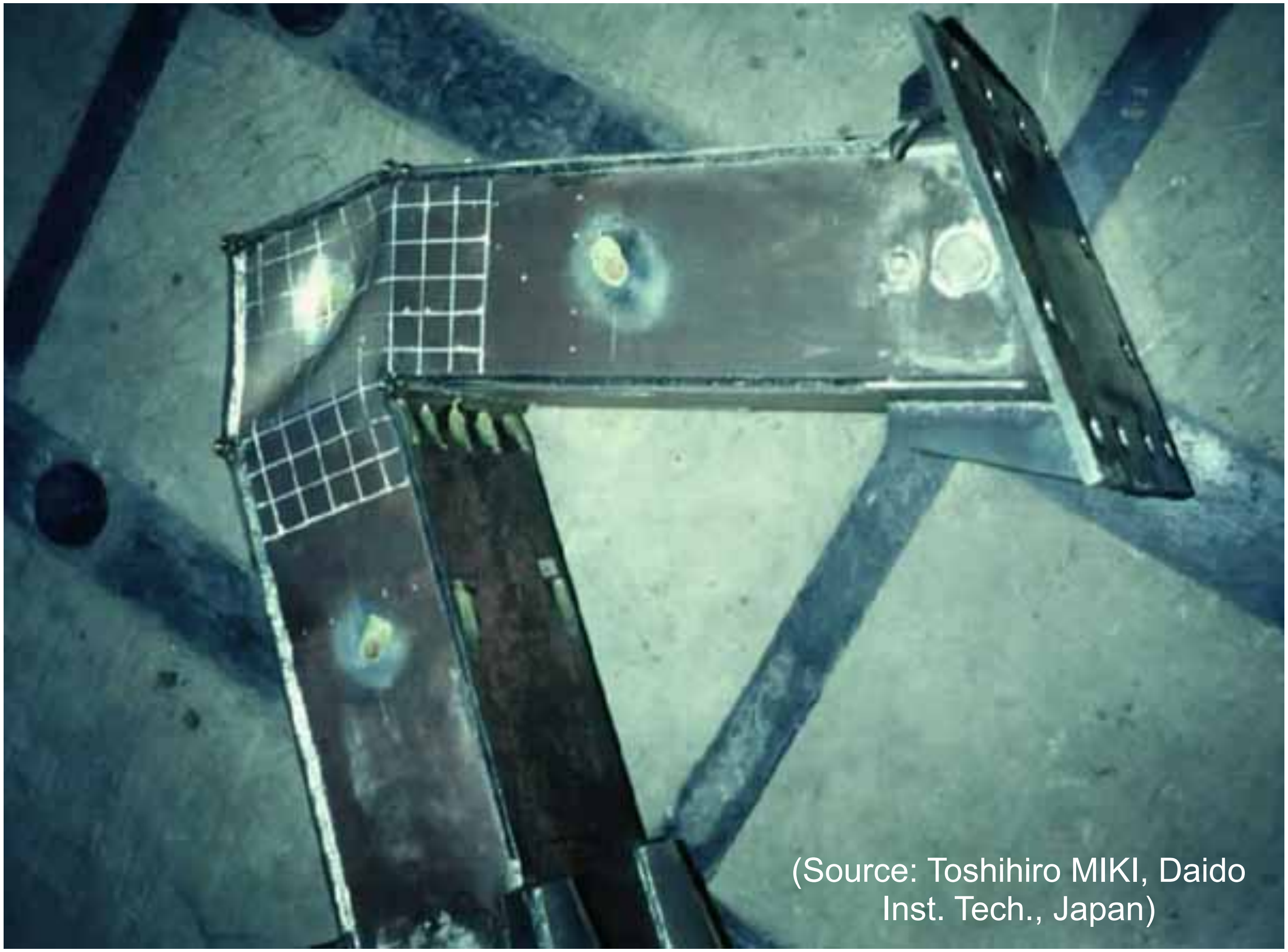
- Concept of equivalent moment (Equivalent moment factor, C_m)

Table 3.9 Values for ψ and C_m

Case	ψ	C_m
1 	0	1.0
2 	−0.4	$1-0.4\ P/P_{ek}$
3 	−0.4	$1-0.4\ P/P_{ek}$
4 	−0.2	$1-0.2\ P/P_{ek}$
5 	−0.3	$1-0.3\ P/P_{ek}$
6 	−0.2	$1-0.2\ P/P_{ek}$

Beam-Columns – I

- Failure modes – Beam-column joint failed in shear



(Source: Toshihiro MIKI, Daido Inst. Tech., Japan)

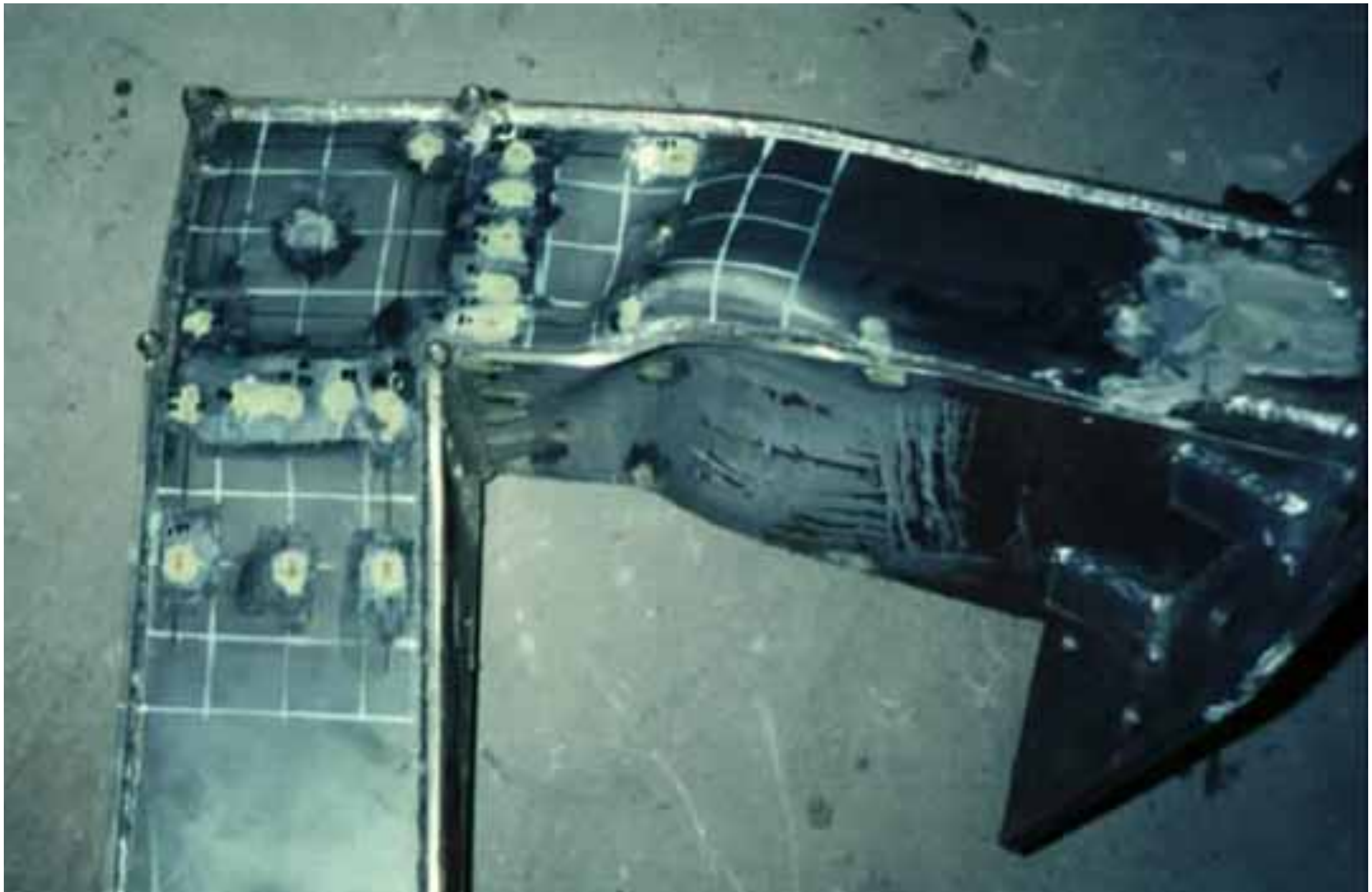
Beam-Columns – I

- Failure modes – Beam-column joint failed in bending

(Source: Toshihiro MIKI, Daido Inst. Tech., Japan)

Beam-Columns – I

- Failure modes – Beam-column joint failed in torsion



(Source: Toshihiro MIKI, Daido Inst. Tech., Japan)

Summary

- In reality, all members in a frame are beam-columns.

- When analyzing beam-columns, the primary effect (bending moment and deflection) is due to bending moment and the secondary effect (bending moment and deflection) is to due to axial force; however, the secondary bending moment and deflection can be more significant than the primary ones.

- At constant P, $w(x)$ and y_{max} are proportional to each other; however, at constant $w(x)$, P and y_{max} are **NOT** proportional to each other.

- For beam-columns subjected to end moments, $\frac{M_A}{M_B} > 0$: Double curvature; $\frac{M_A}{M_B} < 0$: Single curvature