CIVE.5120 Structural Stability (3-0-3)
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Buckling of Rigid Frames – I

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Outline

- Effect of geometrical imperfection
  - P-δ and P-Δ effects
- Load-deflection behavior of frames
- Analysis approaches
  - D.E. method
  - Slope-deflection method
  - Matrix stiffness method
- Slope-deflection method
- Elastic critical loads – D.E. method
  - Non-sway case
  - Sway case
- Summary
- References

Buckling of Rigid Frame – I

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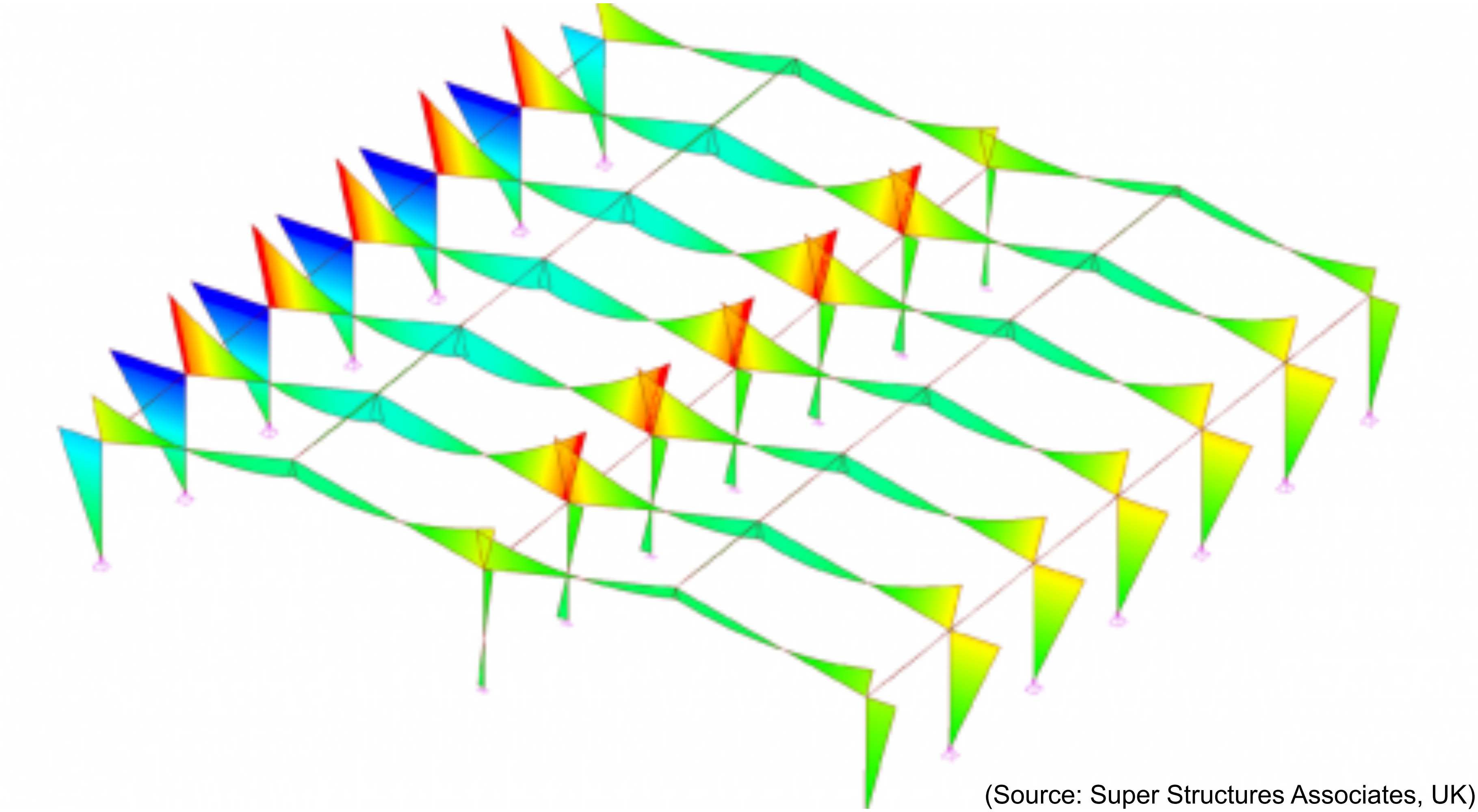
- Moment distribution

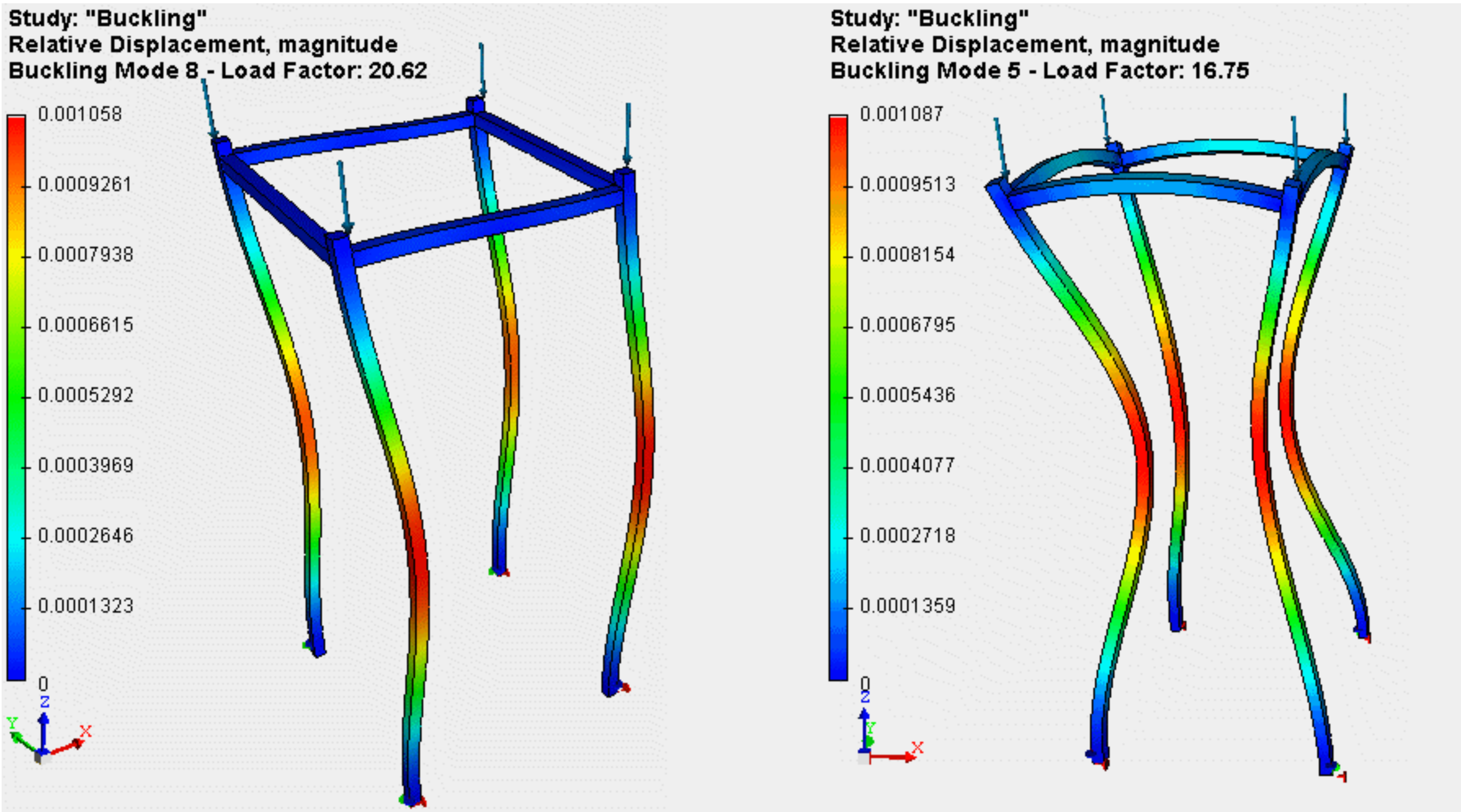


(Source: Super Structures Associates, UK)

Buckling of Rigid Frame – I

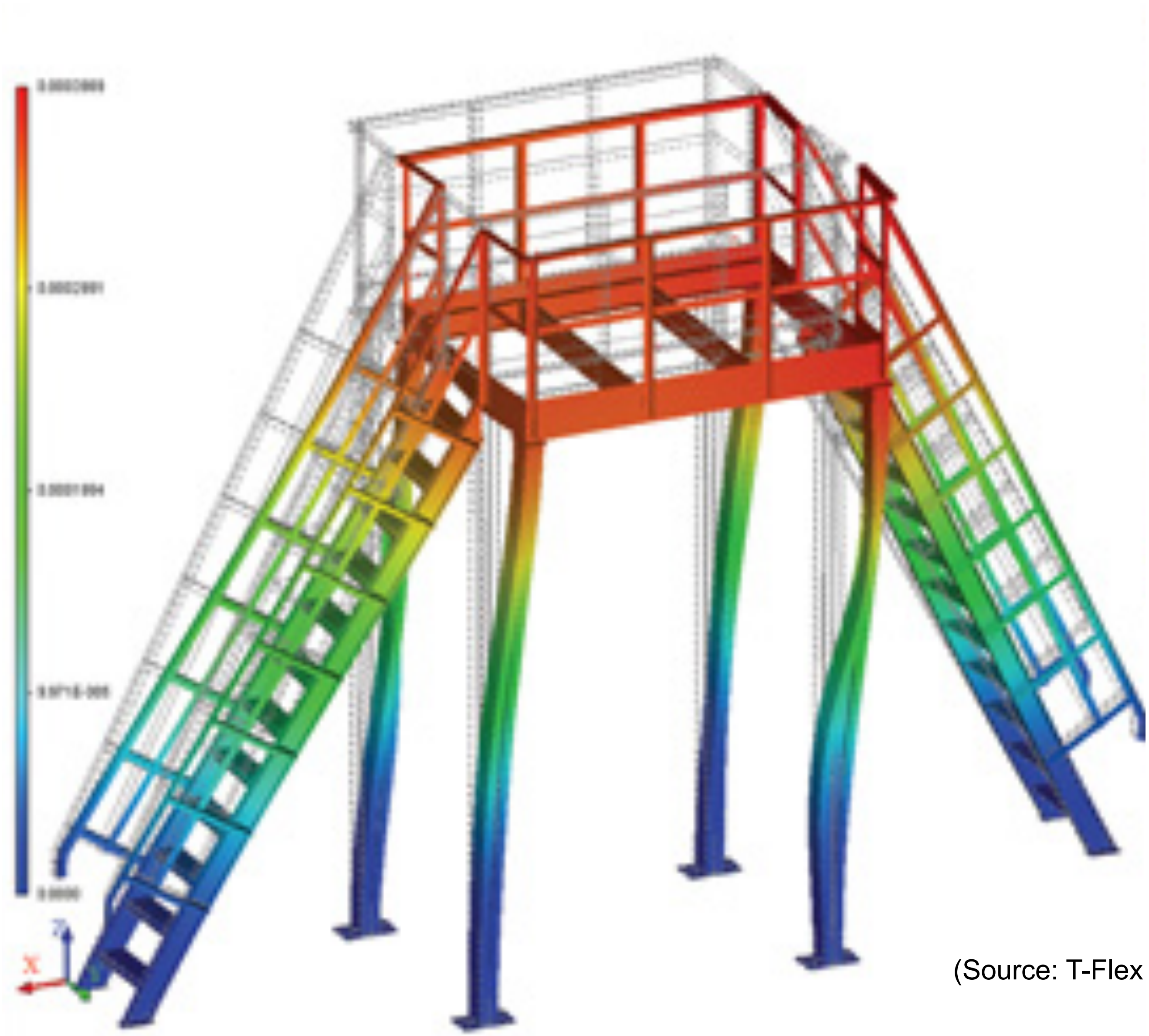
(Source: J.E. Steel, AZ)

Buckling of Rigid Frame – I



(Source: AutoFEM Analysis)

Buckling of Rigid Frame – I



(Source: T-Flex Analysis)

Rigid Frames – I

- **Effects of geometric imperfection**
  - P-δ and P-Δ effects

Rigid Frames – I

- **Load-deflection behavior of frames**
  - Elastic buckling load, P_{cr}
  - Plastic collapse load, P_p
  - Actual failure load, P_f

Rigid Frames – I

- **Analysis approaches**
  - D.E. method
    - Force based
    - Second-order and fourth-order
  - Slope-deflection method
    - Displacement based
    - Matrix form
  - Matrix stiffness method
    - Force based
    - Matrix form

Slope Deflection Method

- Slope deflection method is a displacement-based analysis for indeterminate structures
- Unknown displacements are first written in terms of the loads by using load-displacement relationships; then these equations are solved for the displacements.
- Once the displacements are obtained, unknown loads are determined from the compatibility equations using load-displacement relationships.

– Nodes: Specified points on the structure that undergo displacements (and rotations).

– Degrees of Freedom (DOF): These displacements (and rotations) are referred to as degrees of freedom.

Slope Deflection Method

To clarify these concepts we will consider some examples, beginning with the **beam in Fig. 1(a).** Here any load P applied to the beam will cause node A only to rotate (neglecting axial deformation), while node B is completely restricted from moving. **Hence the beam has only one degree of freedom, θ_A.**

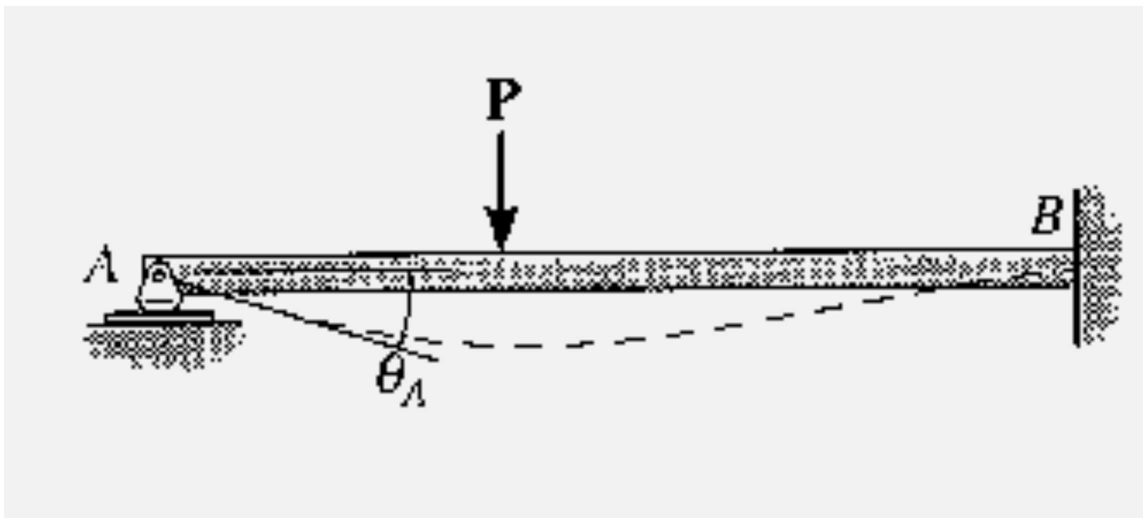


Fig. 1 (a)

Slope Deflection Method

The **beam in Fig. 1(b) has** node at A, B, and C, and so has **four degrees of freedom, designed by the rotational displacements θ_A, θ_B, θ_C, and the vertical displacement Δ_C.**

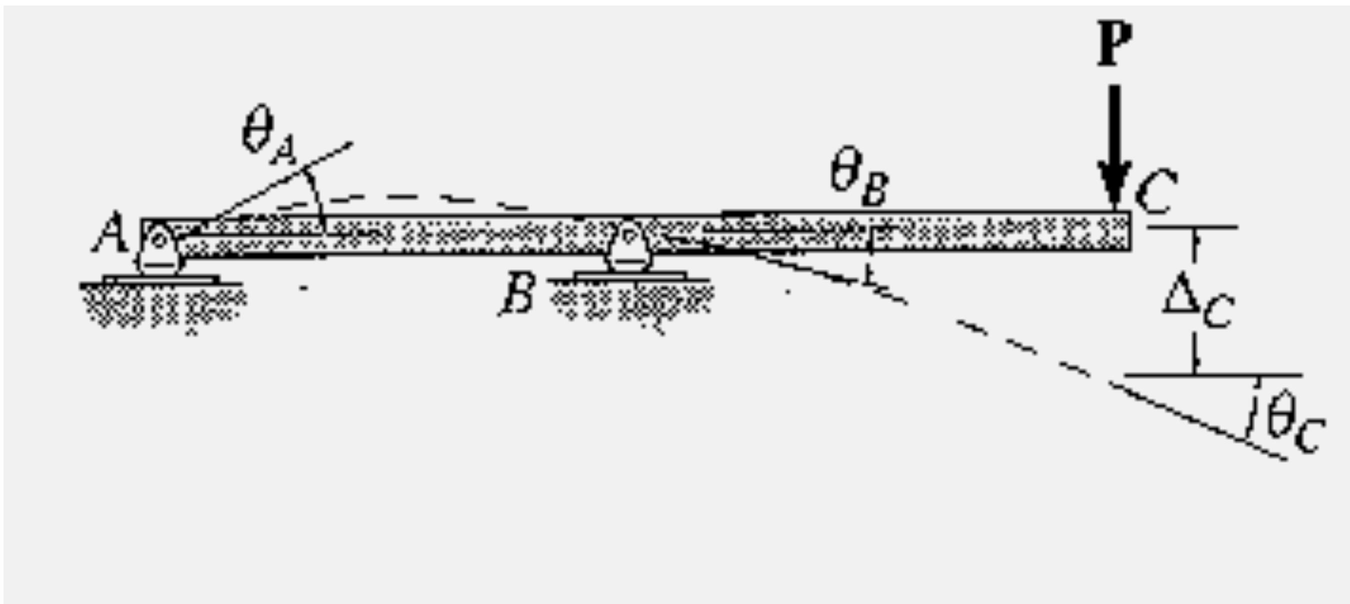


Fig. 1 (b)

Slope Deflection Method

Consider now the **frame in Fig. 1(c).** Again, if we neglect axial deformation of the members, an arbitrary loading P applied to the frame can cause nodes B and C to rotate nodes can be displaced horizontally by an equal amount. The frame therefore **has three degrees of freedom, θ_A, θ_B, Δ_B.**

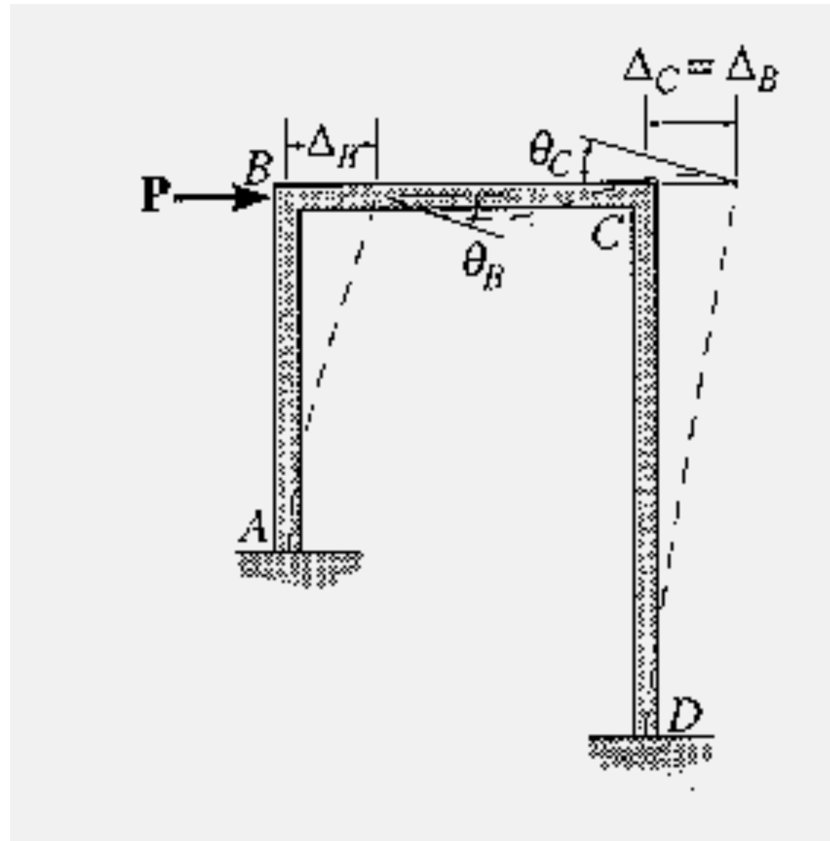
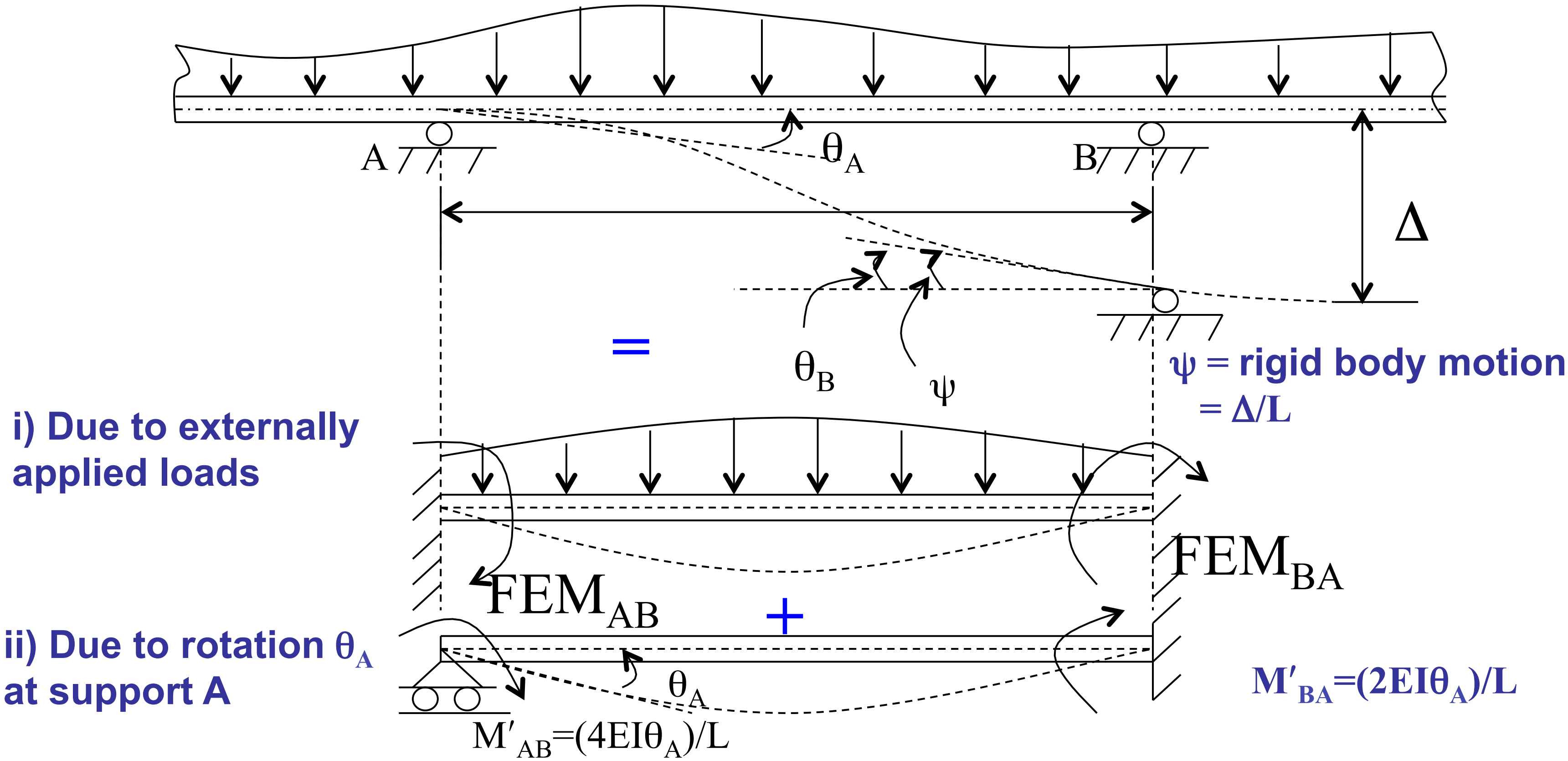


Fig. 1 (c)

Slope Deflection Method

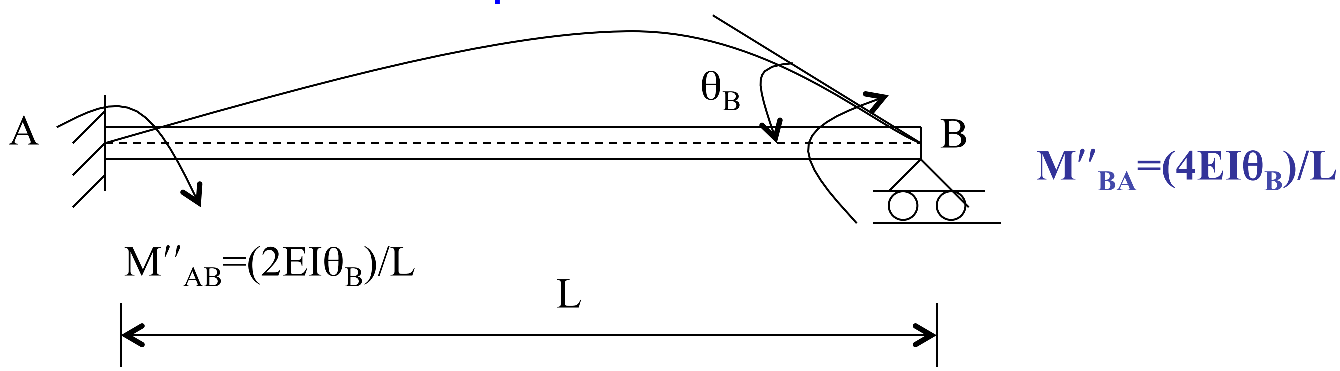
Consider portion AB of a continuous beam, shown below, subjected to a distributed load w(x) per unit length and a support settlement of Δ at B; EI of the beam is constant.



Slope Deflection Method

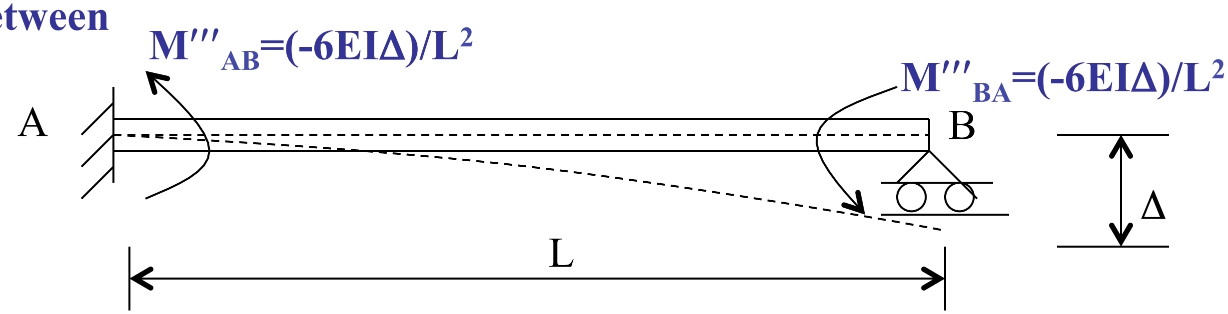
iii) Due to rotation θ_B at support B

+



iv) Due to differential settlement of Δ (between A and B)

+



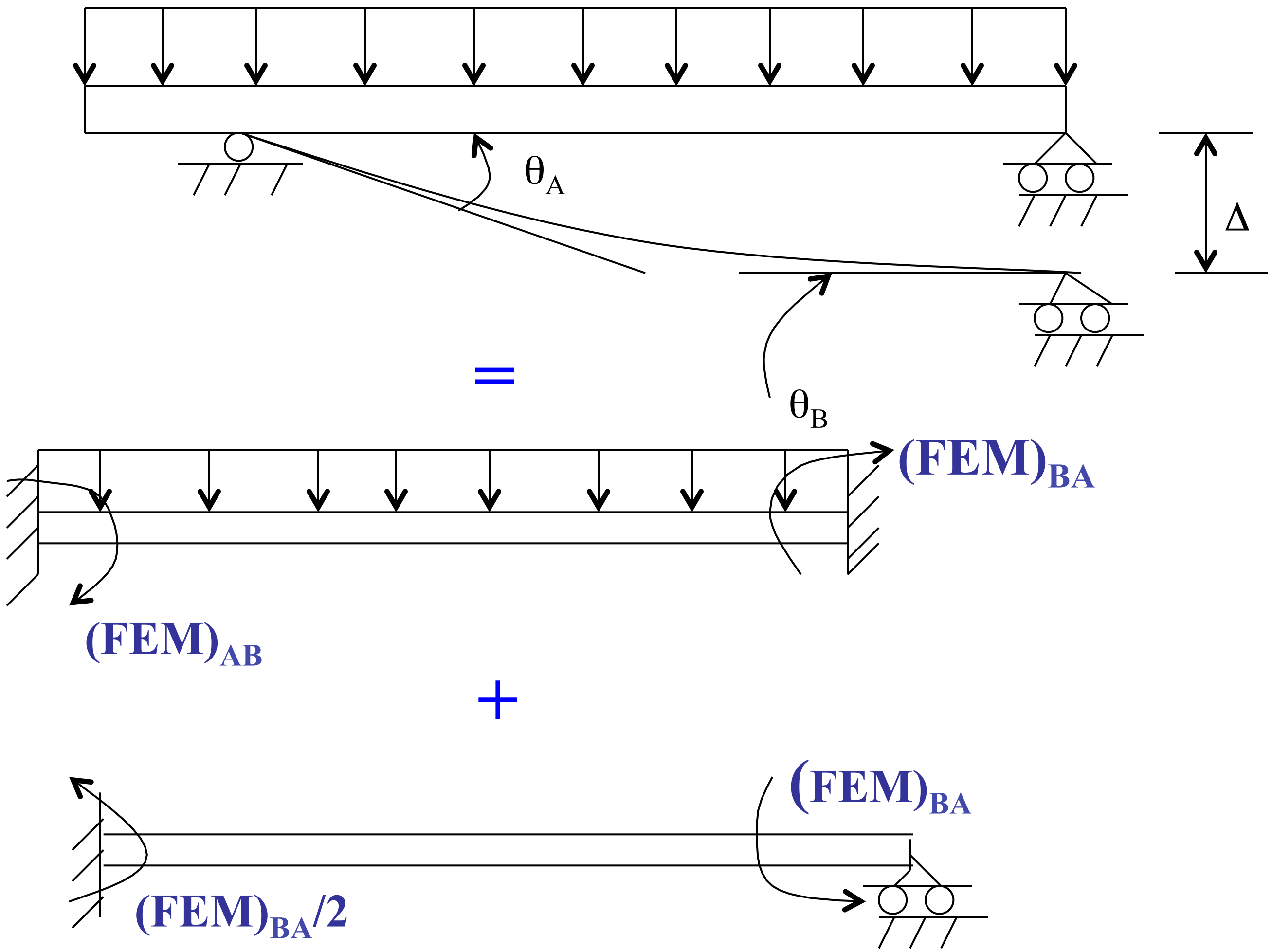
Slope Deflection Method

- **Governing equation –**

$$
\begin{aligned}
M_{AB} &= \mathrm{FEM}_{AB} + M'_{AB} + M''_{AB} + M'''_{AB} \\
&= \mathrm{FEM}_{AB} + \frac{4EI\theta_A}{L} + \frac{2EI\theta_B}{L} - \frac{6EI\Delta}{L^2} \\
&= \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right) + \mathrm{FEM}_{AB}
\end{aligned}
$$

$$
\begin{aligned}
M_{BA} &= \mathrm{FEM}_{BA} + M'_{BA} + M''_{BA} + M'''_{BA} \\
&= \mathrm{FEM}_{BA} + \frac{2EI\theta_A}{L} + \frac{4EI\theta_B}{L} - \frac{6EI\Delta}{L^2} \\
&= \frac{2EI}{L}\left(\theta_A + 2\theta_B - \frac{3\Delta}{L}\right) + \mathrm{FEM}_{BA}
\end{aligned}
$$

Slope Deflection Method



Slope Deflection Method

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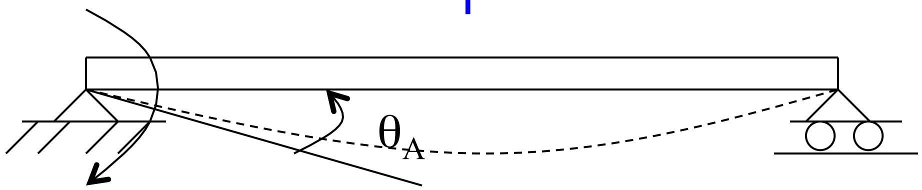
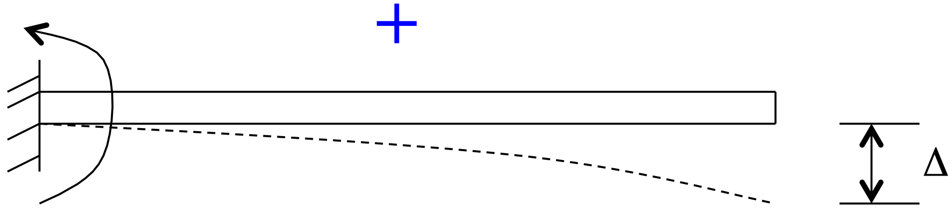


$M''_{AB}=(3EI\theta_A)/L$

+



$M'''_{AB}=(3EI\Delta)/L^2$

$\Delta = PL^3/(3EI)$,
$M = PL = (3EI\Delta/L^3)(L) = 3EI\Delta/L^2$

$$M_{AB} = \underline{[(FEM)_{AB} - (FEM)_{BA}/2]} + (3EI\theta_A)/L - (3EI\Delta)/L^2$$

Modified FEM at end A

Rigid Frames – I

- **Elastic critical load – Differential equation approach**
  - Non-sway case

1. Governing equations

   1.1 For the beam

   1.2 For the column

2. Displacement solution & boundary conditions

Rigid Frames – I

- **Elastic critical load – Differential equation approach**
  - Non-sway case

3. Compatibility condition

4. Characteristic equation → Critical load, P_{cr}

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Summary

- Real structures, such as buildings, behave like frames. Boundary conditions and joint conditions become critical in determining the critical load of the structures.

- Geometric imperfection plays a key role in the critical load.

- Secondary effects (P-δ and P-Δ effects) will make the elastic load-deflection behavior become nonlinear (geometric nonlinearity)

References

- Y-Y Hsieh, S.T. Mau (1995), *Elementary Theory of Structures*, Prentice Hall, Upper Saddle River, NJ. → *Chapter 8*
- E.C. Rossow (1998), *Analysis and Behavior of Structures*, Prentice Hall, Upper Saddle River, NJ. → *Chapter 11*
- K.M. Leet, C-M Uang, A.M. Gilbert (2011), *Fundamentals of Structural Analysis*, 4th ed., McGraw-Hill, Ne York, NY. → *Chapter 12*
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