CIVE.5120 Structural Stability (3-0-3)
03/07/17

Buckling of Rigid Frames – II

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Outline

- Elastic critical loads – Slope deflection method
  - Non-sway case
  - Sway case
- Second-order elastic analysis
- Plastic analysis – Plastic collapse loads
  - Hinge-by-hinge method
  - Mechanism method
- Elastic-plastic-failure interaction – Merchant-Rankine equation
- Summary

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- **Elastic critical load – Slope deflection method**
  - General procedures:
    - List the slope-deflection equation for each internal moment at the joints.
    - Apply force equilibrium at the joints.
    - Obtain the characteristic equation of the frame, which is a function of the stability functions. → Stability functions in the slope-deflection equations; $s_{ij} = (c_{ij} L) / (EI)$ → c_{ij} = stiffness coefficient → $c_{ij} = c_{ij}(kL) = c_{ij}(P_{cr})$
    - Find kL when $s_{ij} = 0$ or $\det| s_{ij} | = 0$.
    - Find $P = P_{cr}$ at the value of kL.

    Note: Graphical or trial-and-error methods are usually required since s_{ij} is a the combination of trigonometric functions.

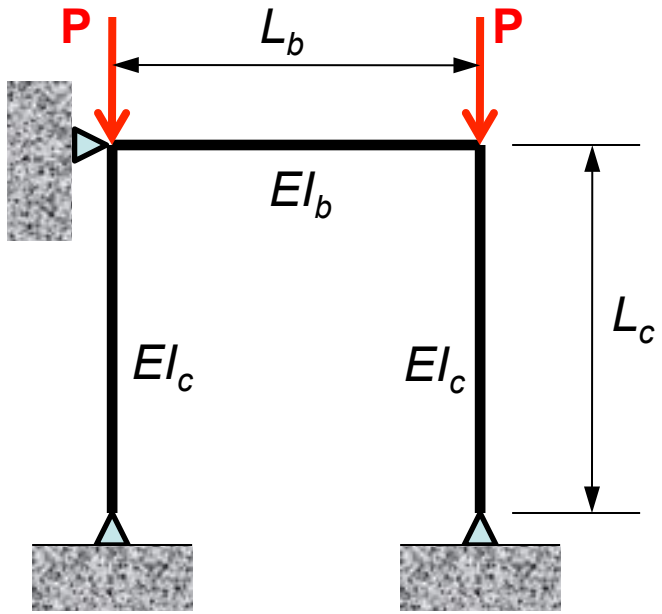
Rigid Frames – II

- **Elastic critical load – Slope deflection method**
  - Non-sway case

1. Slope-deflection equations

2. Force equilibrium at the joints

3. Characteristic equation of the frame



Rigid Frames – II

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  - Sway case

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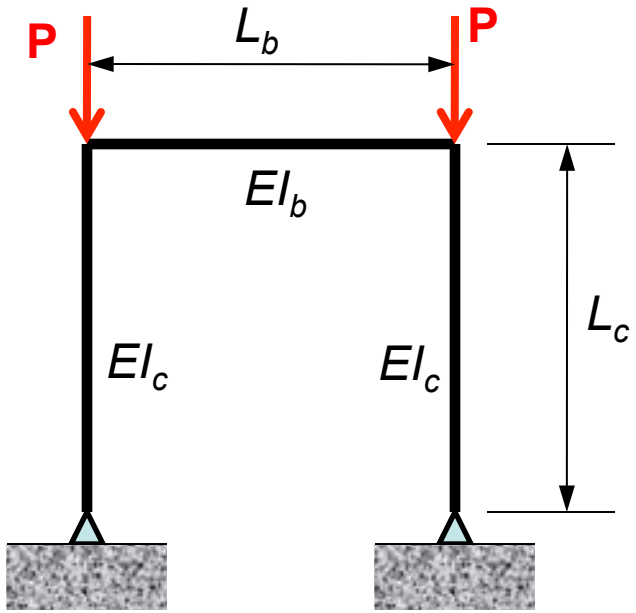

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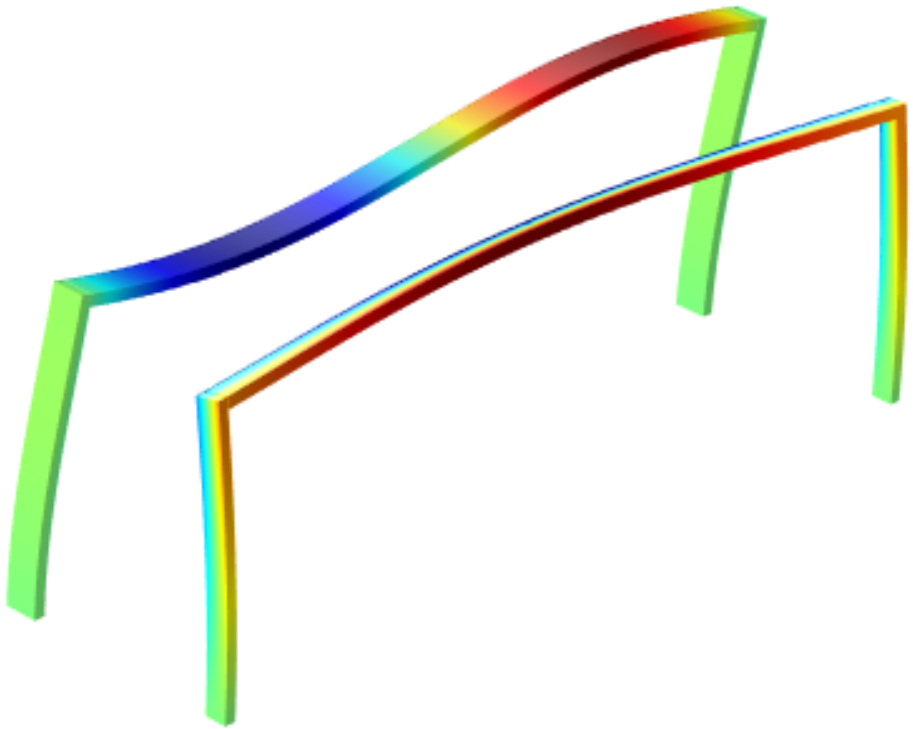
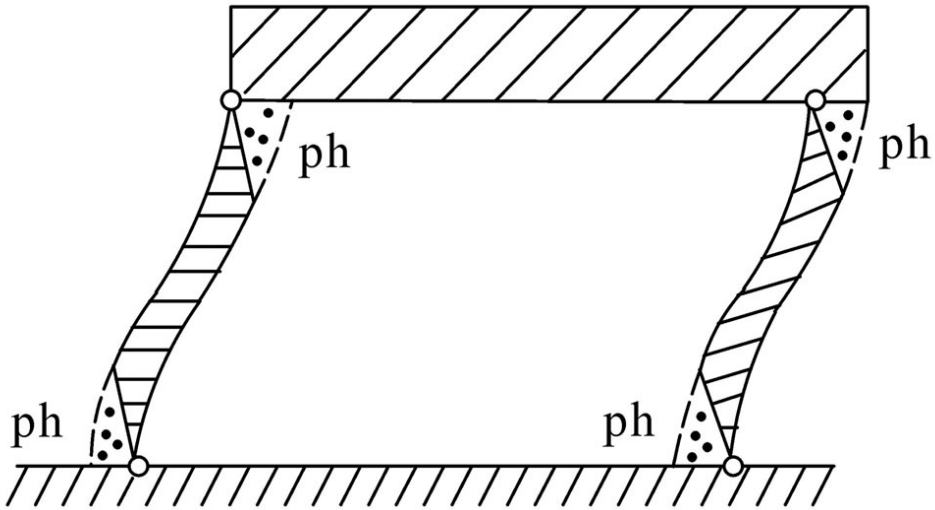
- **Elastic critical load – Slope deflection method**
  - Sway case





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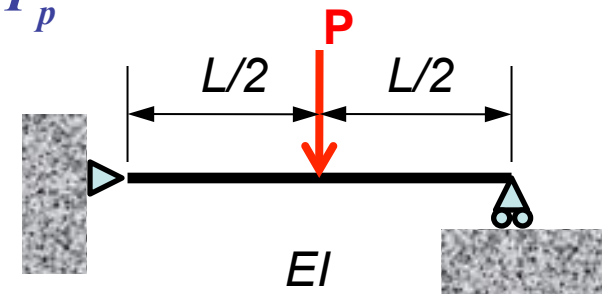
- **Second-order elastic analysis**

Rigid Frames – II

- **Plastic analysis – Plastic collapse loads, P_p**
  - Hinge-by-hinge method
    - Formation of the 1st hinge



    - Formation of the 2nd hinge

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- **Plastic analysis – Plastic collapse loads, P_p**
  - Hinge-by-hinge method
    - Possible locations for the formation of plastic hinges
      - Supports
      - Location of concentrated/point loads
      - Joints
      - Maximum internal bending moment
      - Change of cross-sectional properties (e.g., E or A→I)

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- **Plastic analysis – Plastic collapse loads, P_p**
  - Mechanism method
    - Principle of virtual work/displacement (upper bound theorem)
      - Found value may not be the true value; could be a larger one.
    - General procedures:
      - Locate the possible plastic hinges.
      - Determine **all** geometrically possible mechanisms.
      - Complete critical loads associated with each possible mechanism.
      - Select the **lowest** critical load.
      - Check to see that $|M| \le M_p$ at all points of the structure.
    - Note:
      - Theoretically, all possible mechanisms should be checked.
      - Usually, we check only several mechanisms based on the judgment, then construct the bending moment diagram to see if the plasticity condition is satisfied.
  - Another example: The yield line theory for concrete structures

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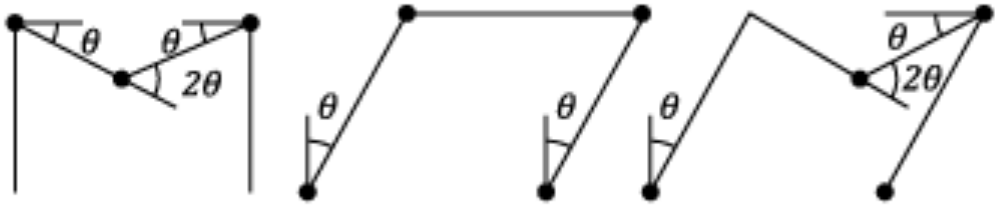
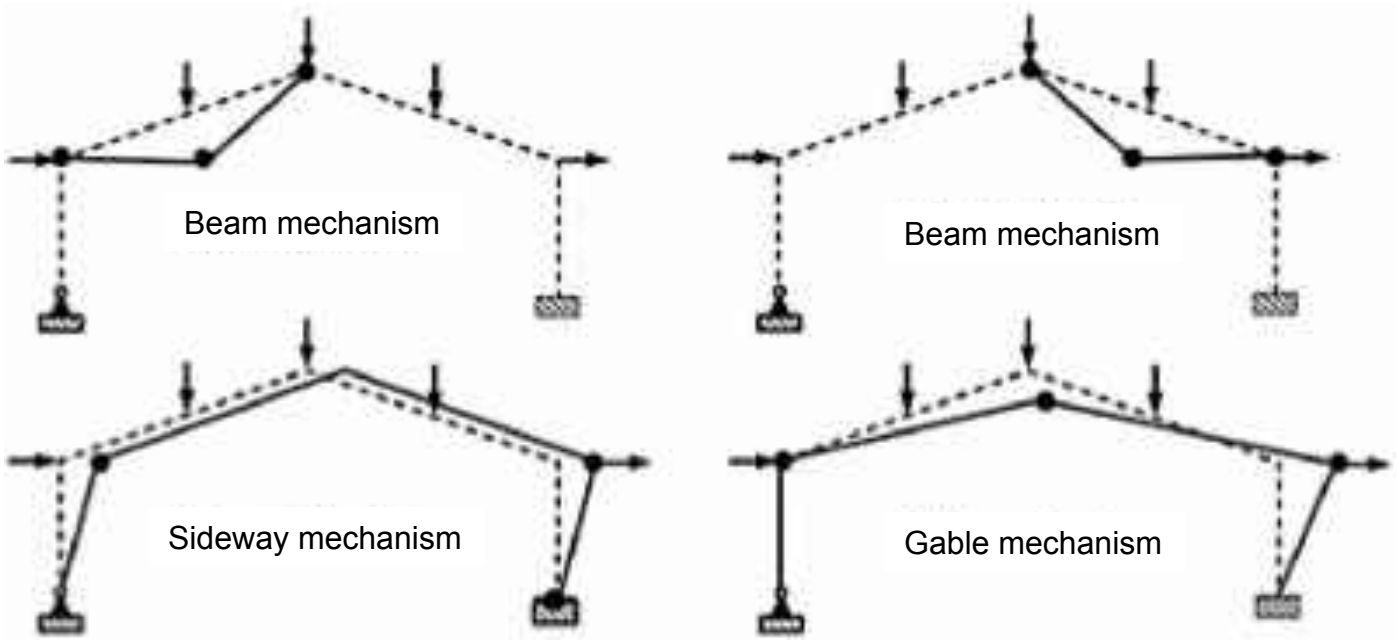
- **Plastic analysis – Plastic collapse loads, P_p**
  - Mechanism method
    - Basic mechanisms:
      - Beam mechanism
      - Sideway/sway mechanism
      - Gable mechanism
      - Joint/combined mechanism





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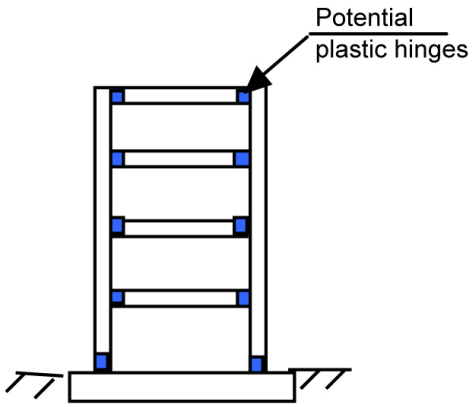
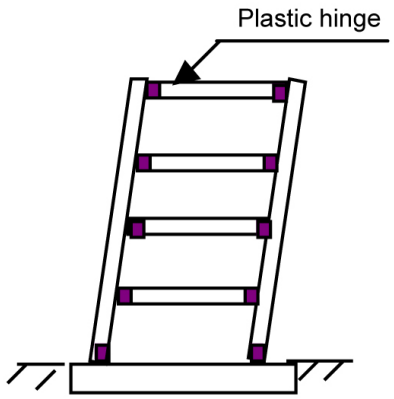
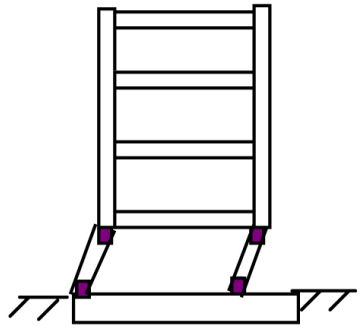
- **Plastic analysis – Plastic collapse loads, P_p**
  - Mechanism method



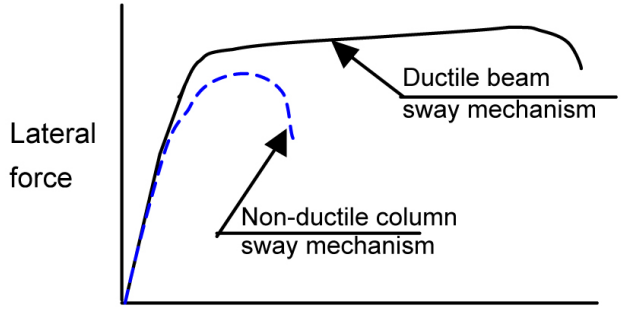
(a) Location of potential plastic hinges



(b) Ductile beam sway mechanism



(c) Non-ductile column sway mechanism



(d) Lateral force versus displacement

(Source: Canterbury Earthquakes Royal Commission, New Zealand)

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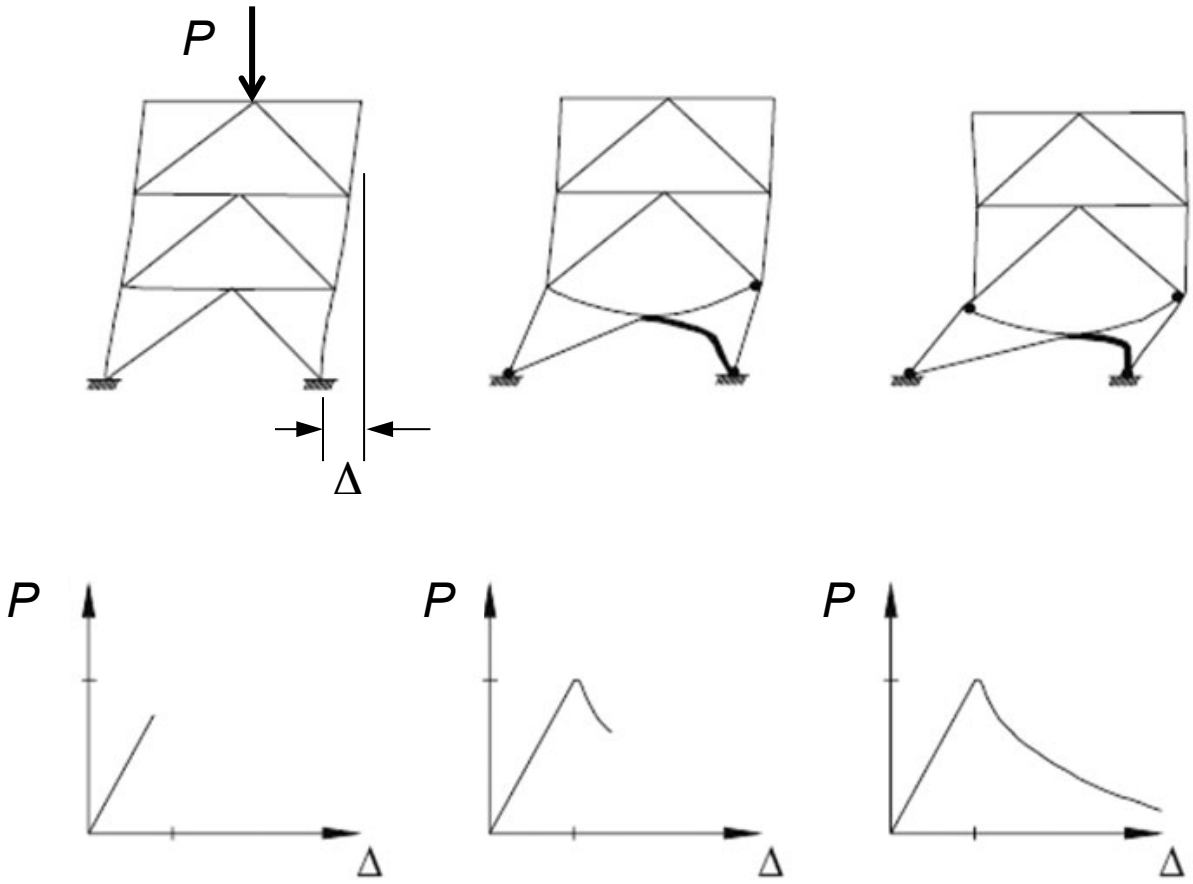
- **Failure mechanisms of chevron bracing frames**



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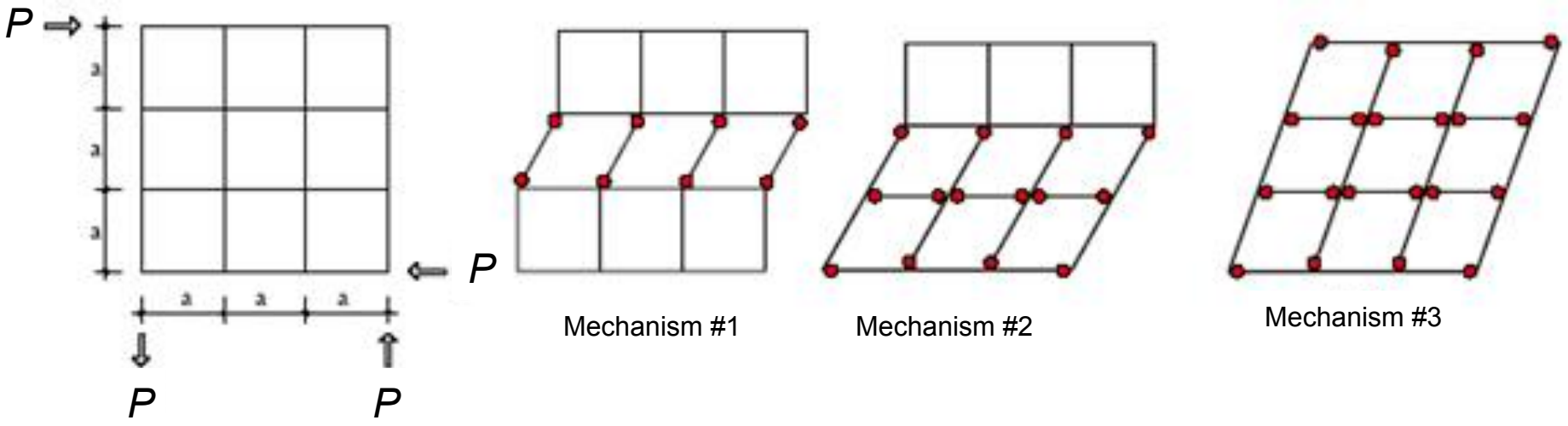
- **Failure mechanisms of a frame structure**



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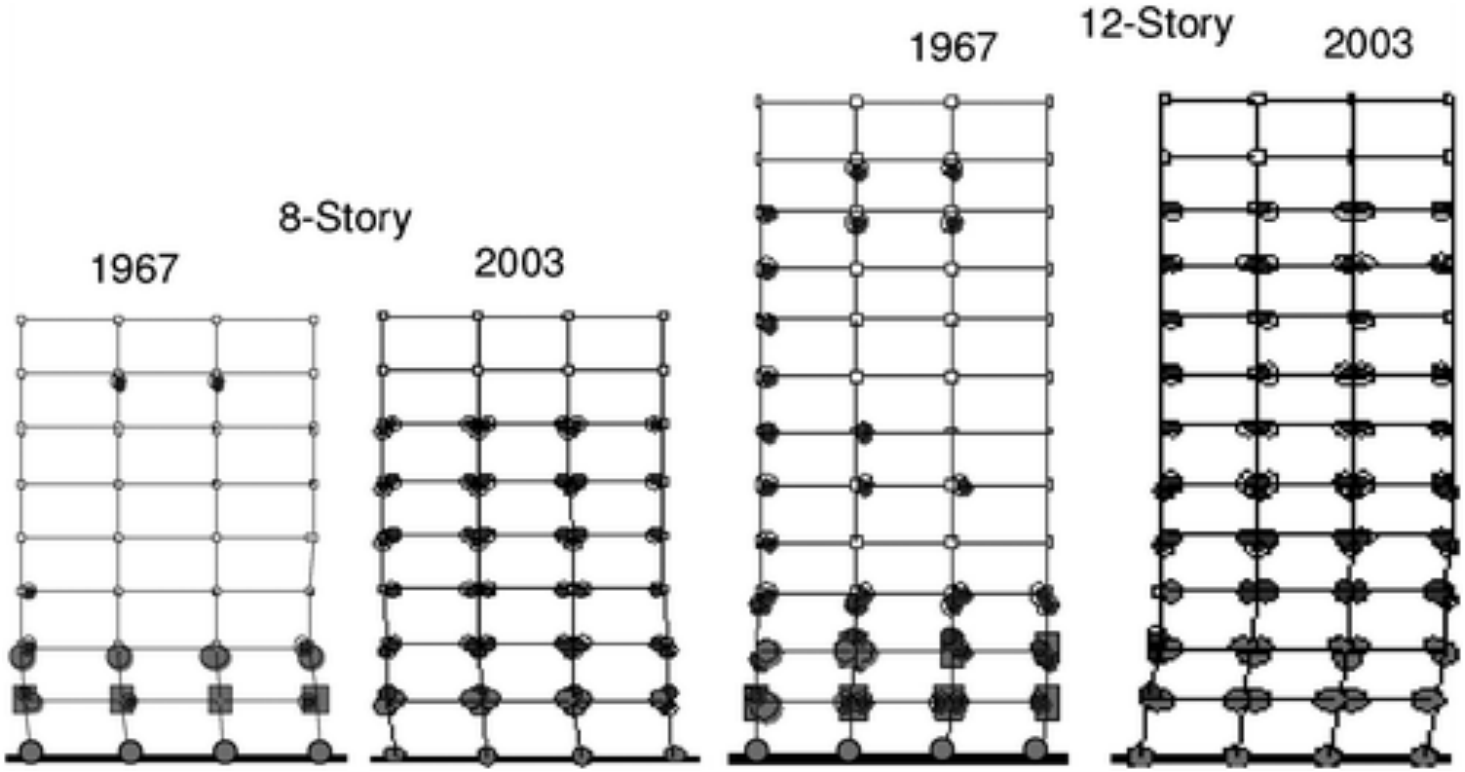
- **Formation of plastic hinges in frames**



(Liel, Heaselton, and Deierlein (2011), *J. Struct. Eng.*, ASCE, 137 (4))

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- **Collapse of a RC structure**

(Mosalam and Günay (2012) “Chapter 23: Seismic Analysis and Design of Masonry-Infilled Frames,” in *Structural and Geotechnical Engineering*, S.K. Kunnath (ed), *Encyclopedia of Life support Systems* (EOLSS) Publishers, Oxford, UK.)

Rigid Frames – II

- **Plastic analysis – Plastic collapse loads, P_p**
  - Mechanism method
    - Number of independent mechanisms, N_M:

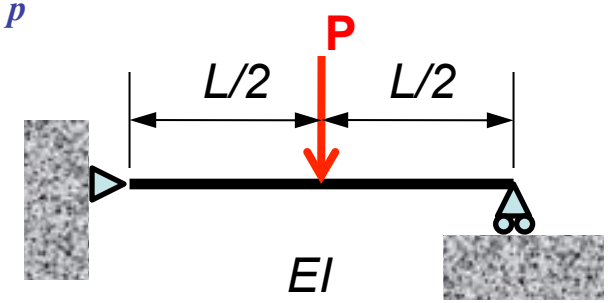
      N_{PH} = No. of possible plastic hinge locations

      N_I = No. of structural indeterminacy

      $\Rightarrow$ **$N_M = N_{PH} - N_I$**

Rigid Frames – II

- **Plastic analysis – Plastic collapse loads, P_p**
  - Mechanism method



Rigid Frames – II

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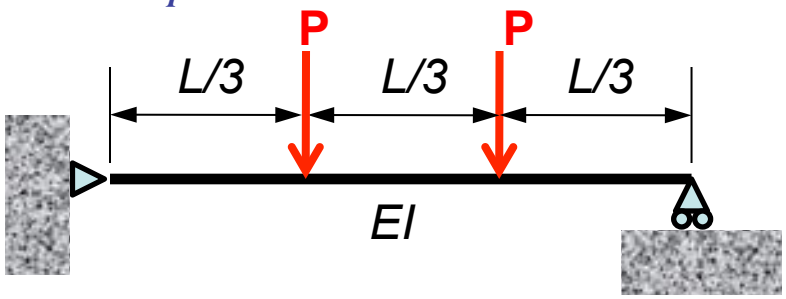

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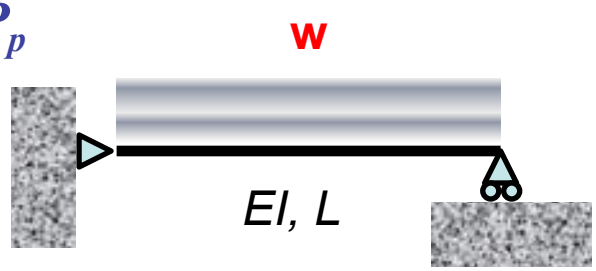

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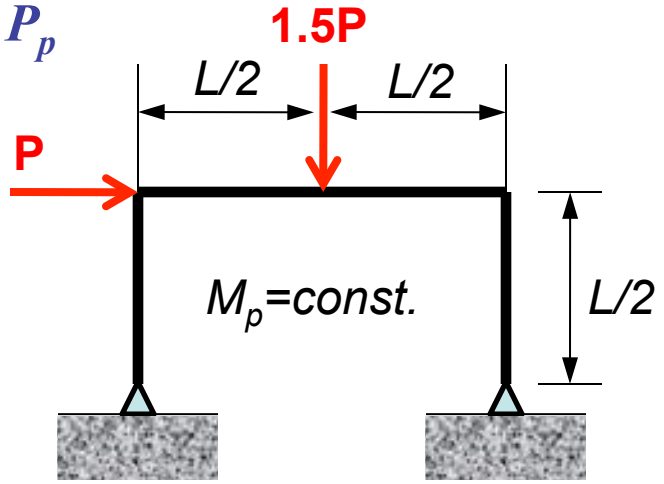

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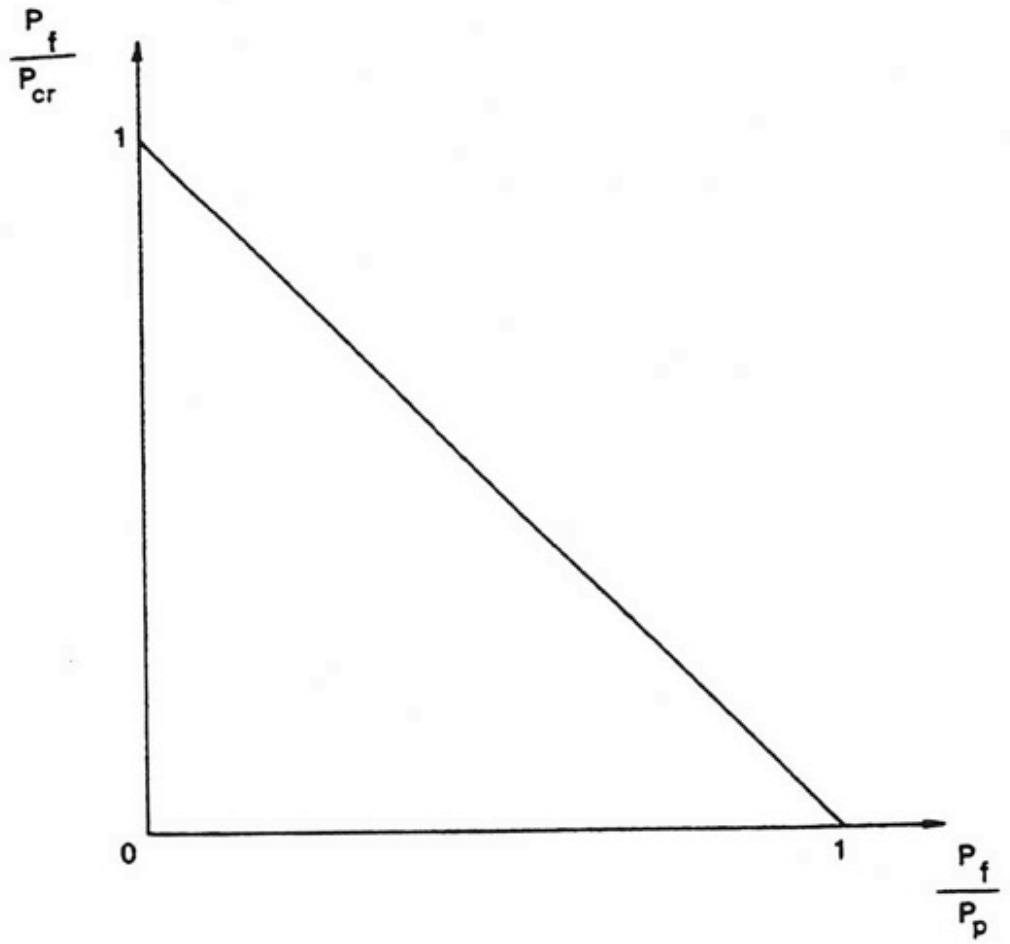
- **Elastic-plastic-failure interaction – Merchant-Rankine equation**
  - Elastic buckling load, P_{cr}
  - Plastic collapse load, P_p
  - Actual failure load, P_f

$$\frac{P_f}{P_{cr}} + \frac{P_f}{P_p} = 1 \qquad (4.7.1)$$



Summary

- In the elastic stability analysis, we can determine the elastic critical load (P_{cr}) by
  - The differential equation method
  - The slope-deflection method
  - The matrix stiffness method (not covered)

- In the plastic stability analysis, we can determine the plastic collapse load (P_p) by
  - The hinge-by-hinge method
  - The mechanism method

- Actual failure load (P_f) can be estimated by the Merchant-Rankine equation.

- In most cases, collapse of structures is a result of an interaction of the effects of instability and plasticity.