Buckling of Beams – I

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Outline

• Buckling failure of beams
• Uniform and non-uniform torsion of thin-walled open sections
• Analysis of lateral buckling of beams
• Effect of type of cross-section on the critical moment
• Failure modes of beams when subjected to strong axis bending
• Summary
Beams – I  

• **Buckling failures of beams:**  
  – In-plane bending → Elastic buckling load  
  – Out-of-plane bending and twisting → Elastic lateral torsional buckling load  

  – Factors affecting the buckling of beams  
    • Cross-section shape → I  
    • Unbraced length (slenderness) → Lateral torsional buckling  
    • Support conditions → End warping restraint torsion  
    • Type and position of the applied load  
      – Bending vs. Bending + Torsion  
      – Stabilizing and destabilizing effects
Beams – I

• Lateral torsional buckling of a clamped beam
Beams – I

- Lateral torsional buckling of a clamped beam
Beams – I

• Lateral torsional buckling of C beams
Beams – I

• Buckling of I/H beams
Beams – I

Buckling of an I-beam member (Source: UC Berkeley)
Beams – I

Buckling of a box girder (Source: Unknown)
Beams – I

Torsional buckling of I-beam members (Source: The Steel Construction Institute)
Beams – I

Torsional buckling of I-beam members (Source: The Steel Construction Institute)
Beams – I

Torsional buckling of an I-beam members
(Source: The Steel Construction Institute)

(Source: Dave Coxon)
Beams – I

• **Uniform and non-uniform torsion of thin-walled open sections**
  - Uniform (pure) torsion, $T_{sv}$
  - Non-uniform (warping restraint) torsion, $T_w$

\[
\frac{T}{J} = \frac{\tau}{R} = \frac{G \theta}{L}
\]
Beams – I

- **Uniform and non-uniform torsion of thin-walled open sections**
  - Total $T = T_{sv} + T_w$
• **Analysis of lateral buckling of beams**
  – Assumptions:
    • Geometrically perfect
    • Non-eccentrically loaded
    • Small deflection theory
    • Plane remains plane

  – Simply-supported rectangular beam under pure bending
    • Governing equations
      – In-plane bending
      – Out-of-plane bending
      – Torsion
Beams – I

• **Analysis of lateral buckling of beams**
  – Simply-supported *rectangular beam* under pure, uniform bending
    • Characteristic equation of the system

• Boundary conditions

• Solution of the critical moment
Beams – I

- **Analysis of lateral buckling of beams**
  - Simply-supported I-beam under pure, uniform bending
    - Governing equations
      - In-plane bending
      - Out-of-plane bending
      - Torsion
    - Characteristic equation of the system
    - B.C.
    - Solution of the critical moment
Beams – I

- **Analysis of lateral buckling of beams**
  - Comparison between the solutions of *rectangular* and *I-beam* sections
    - Possibility of lateral buckling of beams

- Effects of twisting resistance and warping resistance
Beams – I

• **Analysis of lateral buckling of beams**
  – Simply-supported I-beam under pure, non-uniform bending
    • Equivalent moment factor

  • Approximate solution of the critical moment
Beams – I

- Effect of type of cross-section on the critical moment
Beams – I

- Yield moment and plastic moment
Beams – I

• Failure modes of beams when subjected to strong axis bending
  – Plastic yielding
    • $M_P < M_{cr}$
  
  – Elastic pure torsion
    • $(M_2)_{int} = GJ\phi < M_{cr}$
  
  – Elastic lateral torsional buckling
    • $M_{cr} < (M_2)_{int}$
    • $M_{cr} < M_P$
Summary

• The torsional capacity of beams consists of two parts; the twisting resistance and the warping resistance.

• Twisting and out-of-plane deformation are usually coupled when warping occurs.

• Lateral torsional buckling of beams will only occur when the moment of inertia of the weak axis equals the moment of inertia of the strong axis. \( \rightarrow \) It will never occur in circular and square box cross sections.

• Actual failures of beams could be due to plastic yielding, elastic pure torsion, or elastic torsional buckling.