CIVE.5120 Structural Stability (3-0-3)
03/28/17

Buckling of Beams – I

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Outline

- Buckling failure of beams
- Uniform and non-uniform torsion of thin-walled open sections
- Analysis of lateral buckling of beams
- Effect of type of cross-section on the critical moment
- Failure modes of beams when subjected to strong axis bending
- Summary

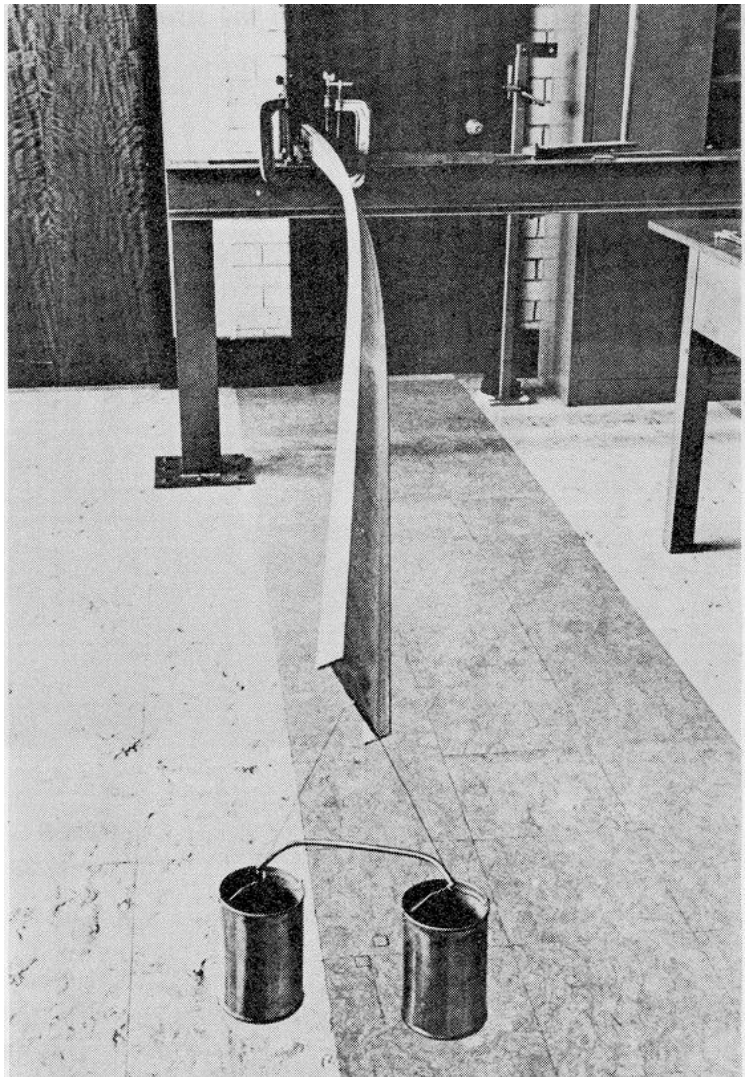
Beams – I

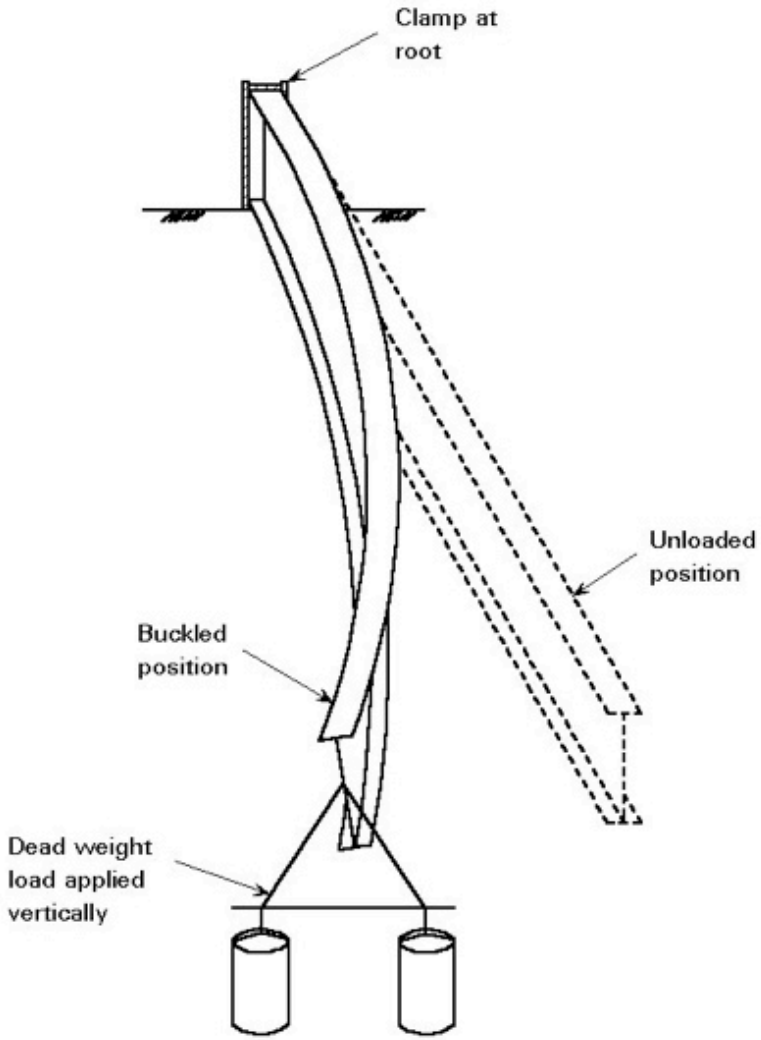
- **Buckling failures of beams:**
  - In-plane bending → Elastic buckling load
  - Out-of-plane bending and twisting → Elastic lateral torsional buckling load

  - Factors affecting the buckling of beams
    - Cross-section shape → I
    - Unbraced length (slenderness) → Lateral torsional buckling
    - Support conditions → End warping restraint torsion
    - Type and position of the applied load
      - Bending vs. Bending + Torsion
      - Stabilizing and destabilizing effects

Beams – I

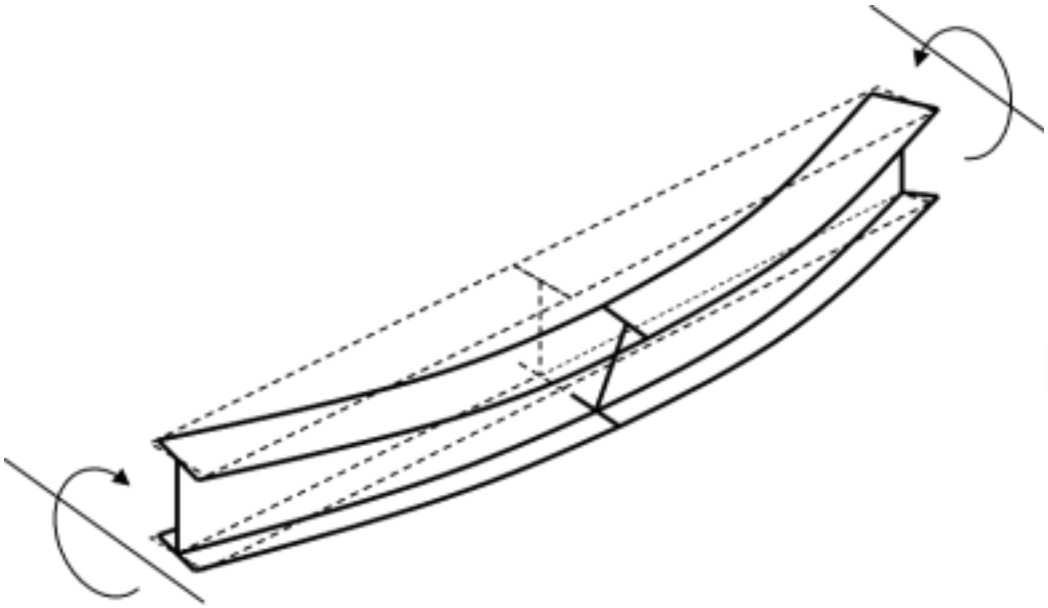
- **Lateral torsional buckling of a clamped beam**

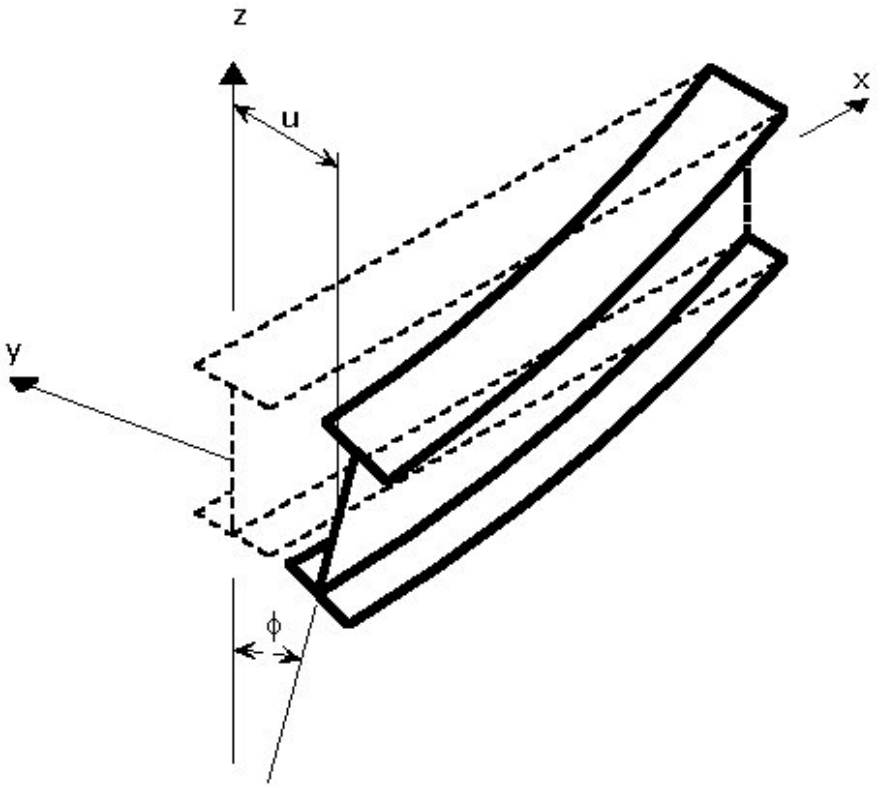




Beams – I

- **Lateral torsional buckling of a clamped beam**

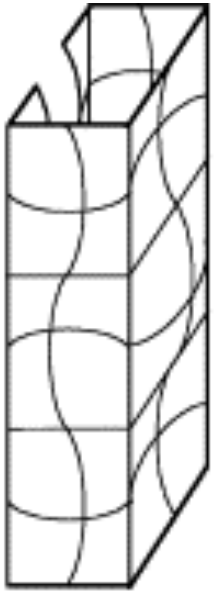




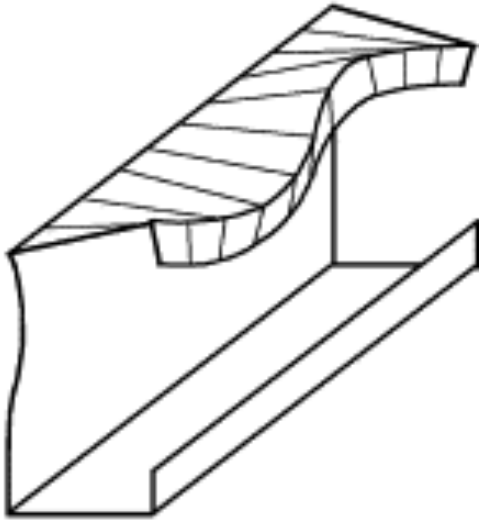
Beams – I

- **Lateral torsional buckling of C beams**

(a)



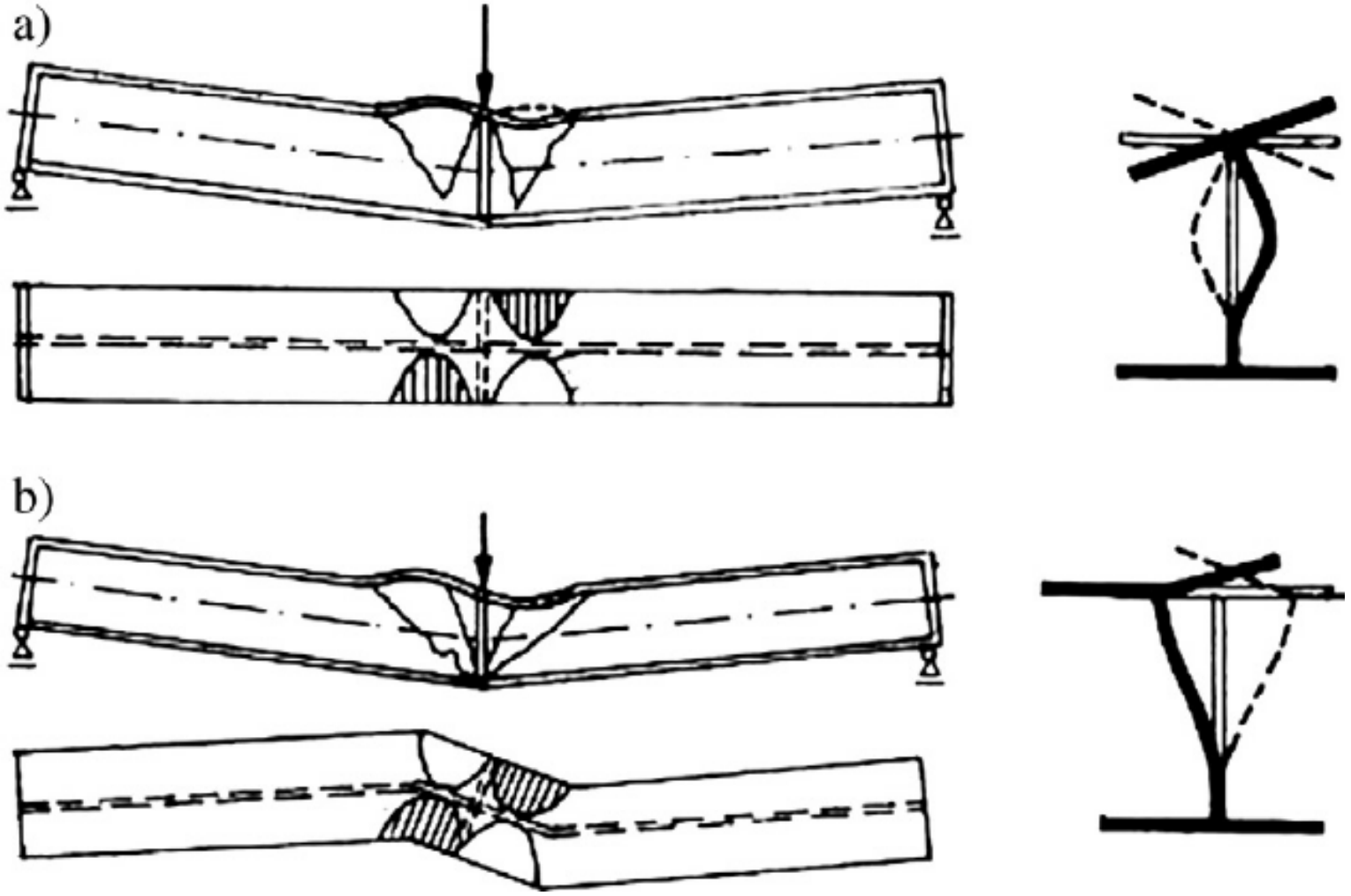
(b)



(c)

Beams – I

- **Buckling of I/H beams**



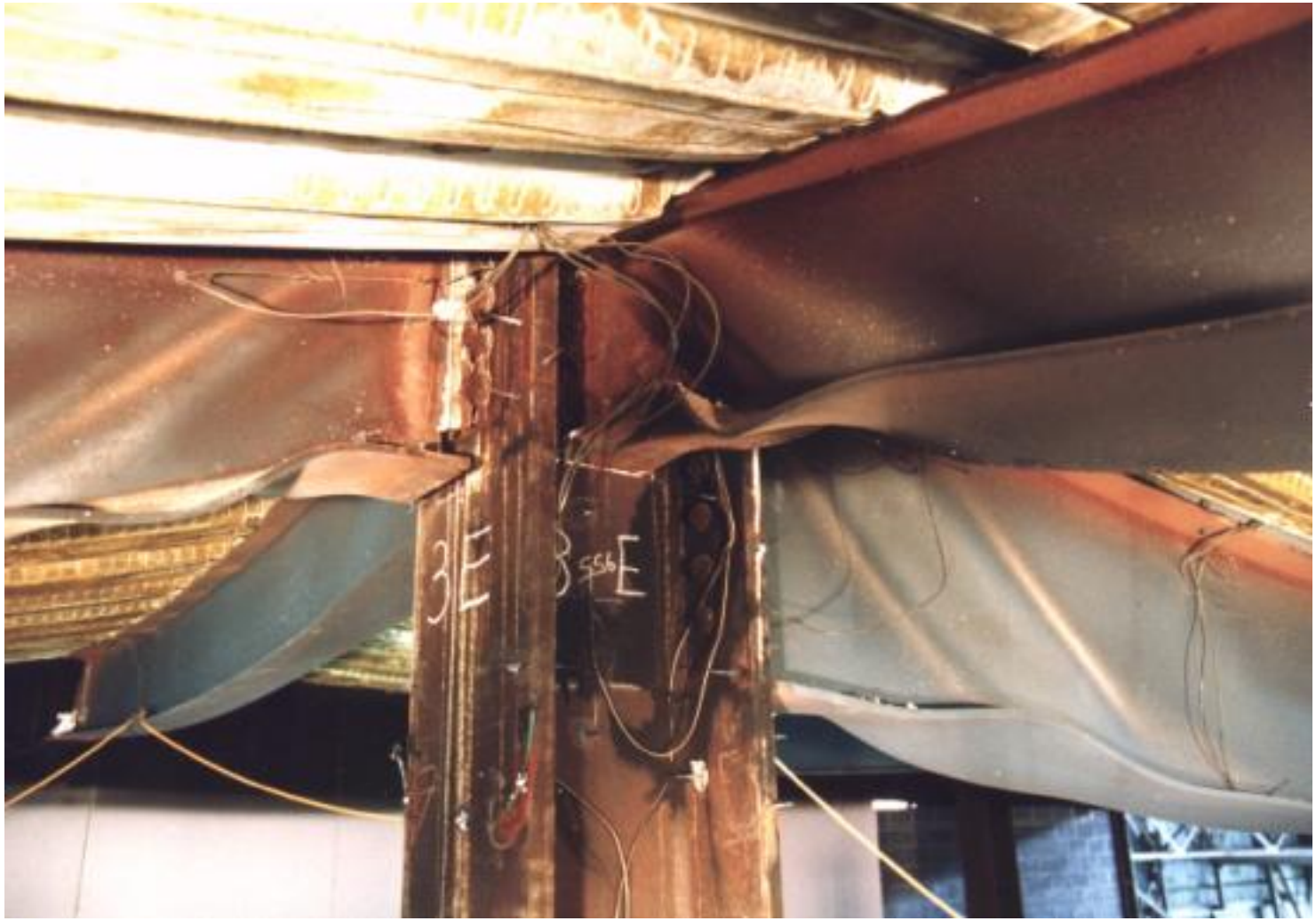
Beams – I

Buckling of an I-beam member (Source: UC Berkeley)

Beams – I

Buckling of a box girder (Source: Unknown)

Beams – I



Torsional buckling of I-beam members (Source: The Steel Construction Institute)

Beams – I

Torsional buckling of I-beam members (Source: The Steel Construction Institute)

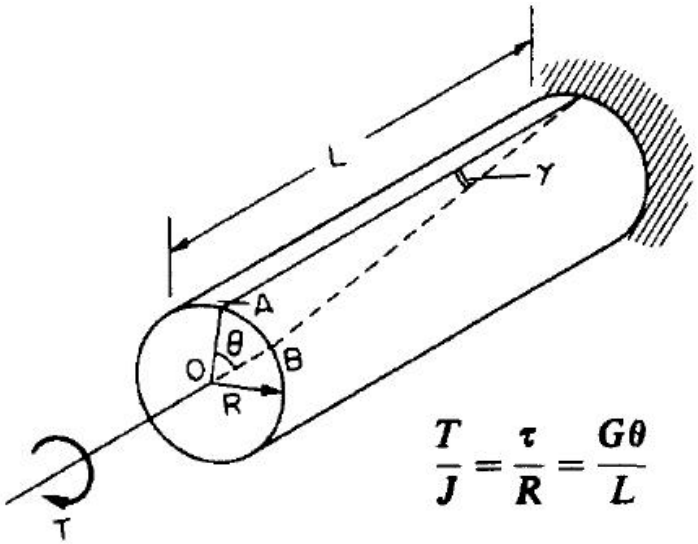
Beams – I

Torsional buckling of an I-beam members
(Source: The Steel Construction Institute)

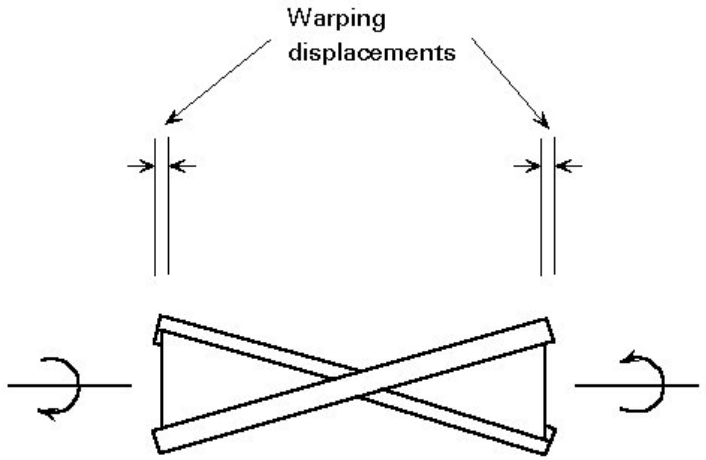
(Source: Dave Coxon)

Beams – I

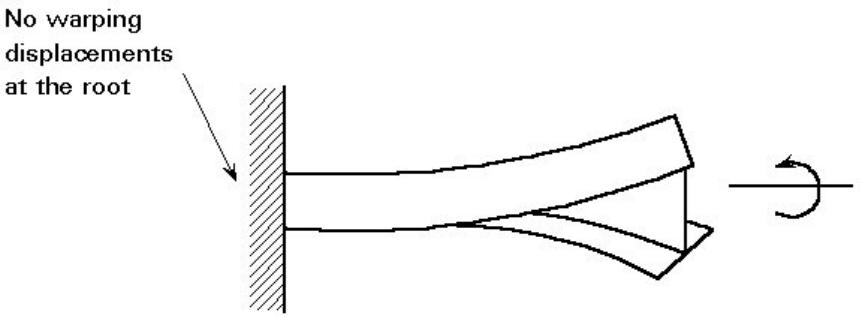
- **Uniform and non-uniform torsion of thin-walled open sections**
  - Uniform (pure) torsion, T_{sv}
  - Non-uniform (warping restraint) torsion, T_w



$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}$$



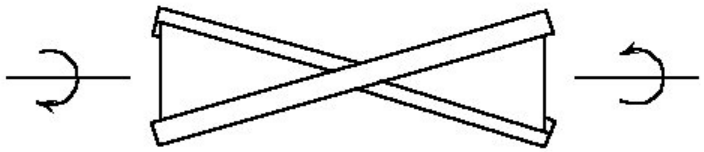
(a) Torsion with unrestrained warping



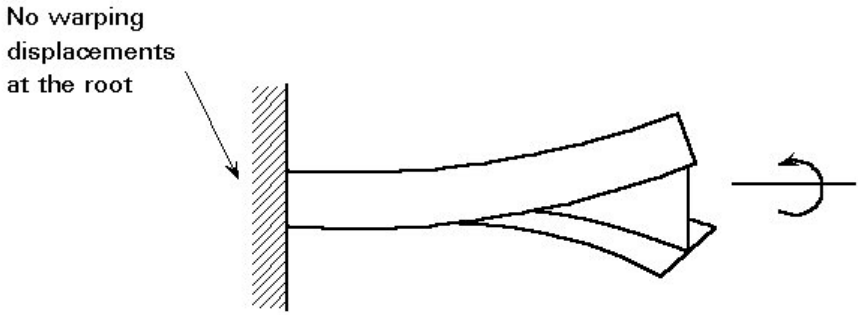
(b) Torsion with restrained warping

Beams – I

- **Uniform and non-uniform torsion of thin-walled open sections**
  - Total $T = T_{sv} + T_w$



(a) Torsion with unrestrained warping



(b) Torsion with restrained warping

Beams – I

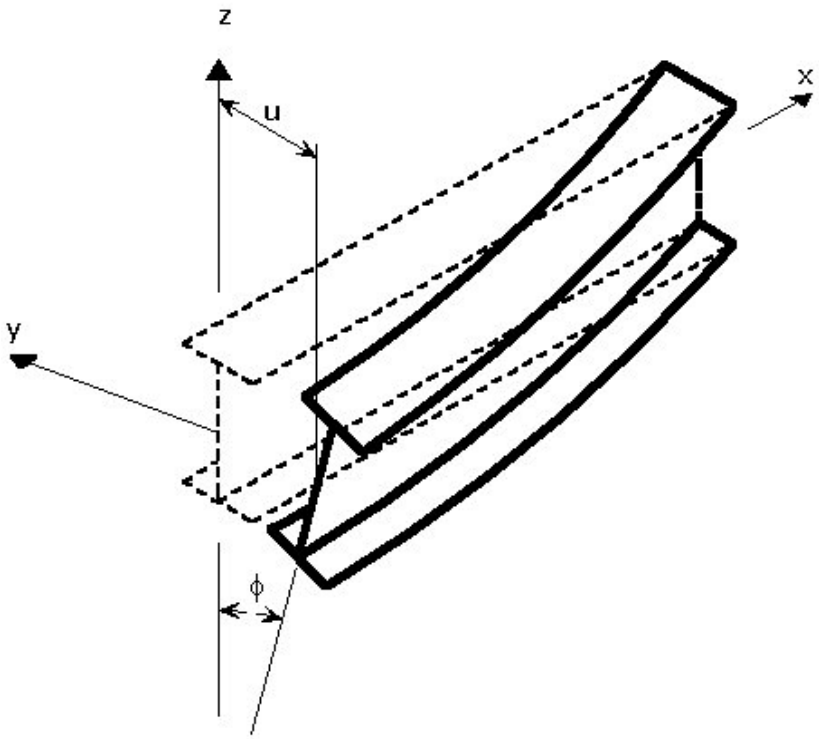
- **Analysis of lateral buckling of beams**
  - Assumptions:
    - Geometrically perfect
    - Non-eccentrically loaded
    - Small deflection theory
    - Plane remains plane
  - Simply-supported rectangular beam under pure bending
    - Governing equations
      - In-plane bending
      - Out-of-plane bending
      - Torsion

Beams – I

- **Analysis of lateral buckling of beams**
  - Simply-supported rectangular beam under pure, uniform bending
    - Characteristic equation of the system
    - Boundary conditions
    - Solution of the critical moment

Beams – I

- **Analysis of lateral buckling of beams**
  - Simply-supported I-beam under pure, uniform bending
    - Governing equations
      - In-plane bending
      - Out-of-plane bending
      - Torsion
    - Characteristic equation of the system
    - B.C.
    - Solution of the critical moment



Beams – I

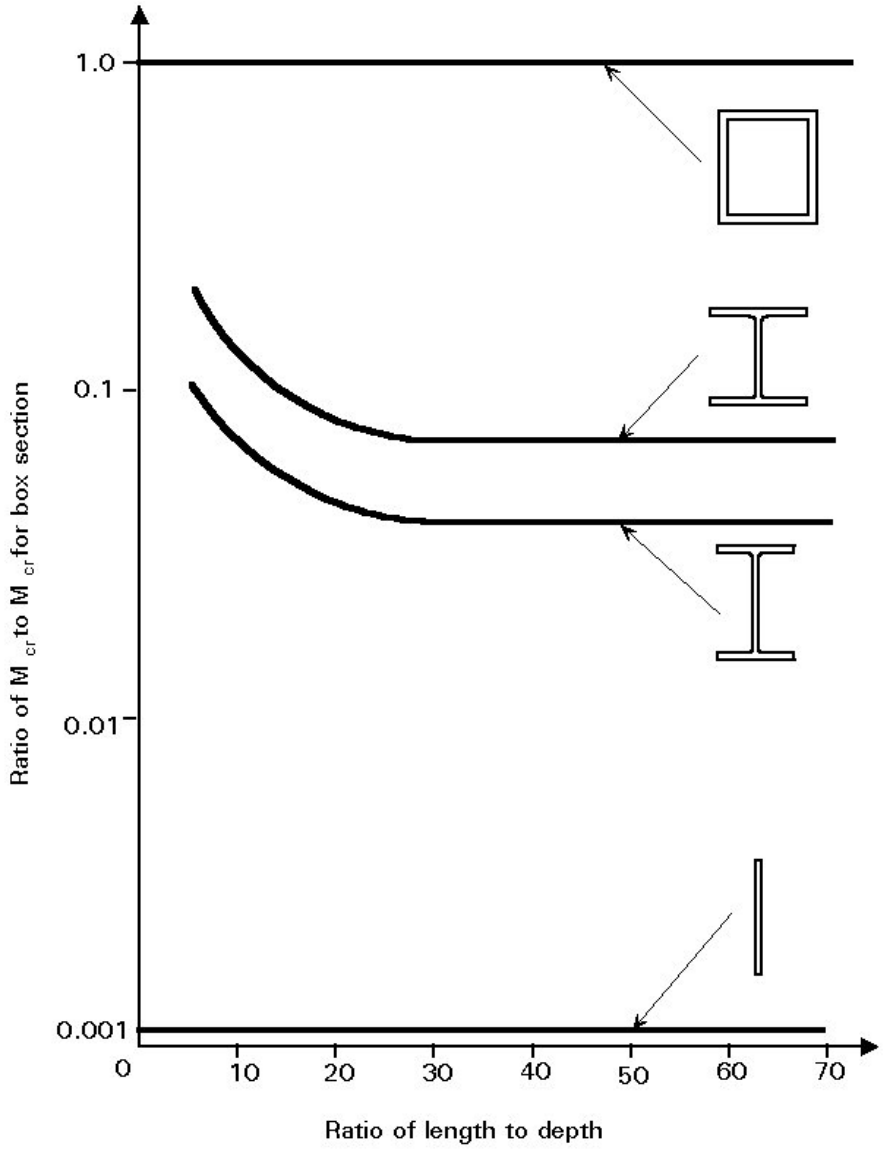
- **Analysis of lateral buckling of beams**
  - Comparison between the solutions of rectangular and I-beam sections
    - Possibility of lateral buckling of beams
    - Effects of twisting resistance and warping resistance

Beams – I

- **Analysis of lateral buckling of beams**
  - Simply-supported I-beam under pure, non-uniform bending
    - Equivalent moment factor
    - Approximate solution of the critical moment

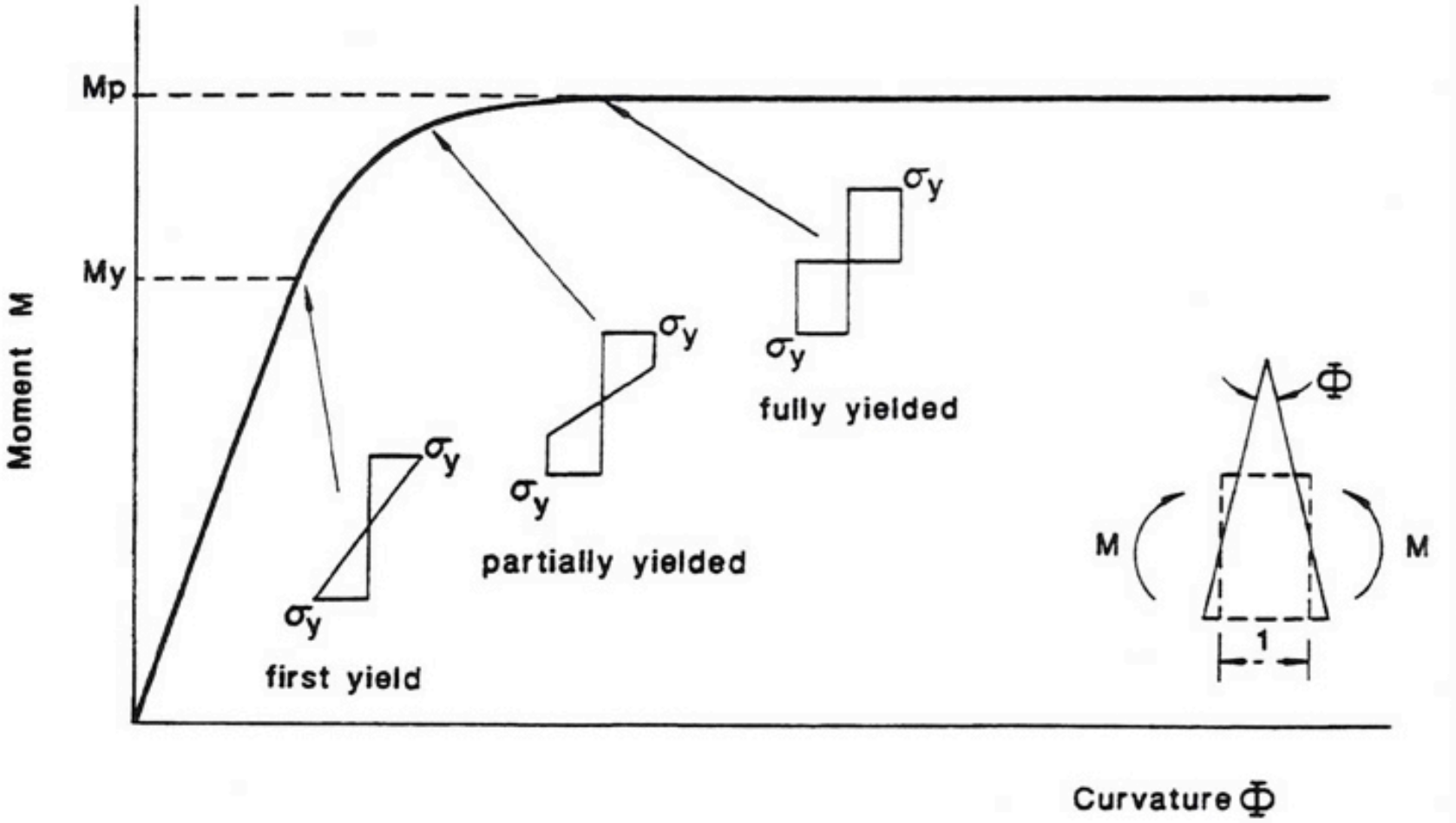
Beams – I

- **Effect of type of cross-section on the critical moment**



Beams – I

- **Yield moment and plastic moment**



Beams – I

- **Failure modes of beams when subjected to strong axis bending**
  - Plastic yielding
    - $M_P < M_{cr}$
  - Elastic pure torsion
    - $(M_z)_{int} = GJ\phi < M_{cr}$
  - Elastic lateral torsional buckling
    - $M_{cr} < (M_z)_{int}$
    - $M_{cr} < M_P$

Summary

- The torsional capacity of beams consists of two parts; the twisting resistance and the warping resistance.
- Twisting and out-of-plane deformation are usually coupled when warping occurs.
- Lateral torsional buckling of beams will only occur when the moment of inertia of the weak axis equals the moment of inertia of the strong axis. → It will never occur in circular and square box cross sections.
- Actual failures of beams could be due to plastic yielding, elastic pure torsion, or elastic torsional buckling.