CIVE.5120 Structural Stability (3-0-3)
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Buckling of Beams – II

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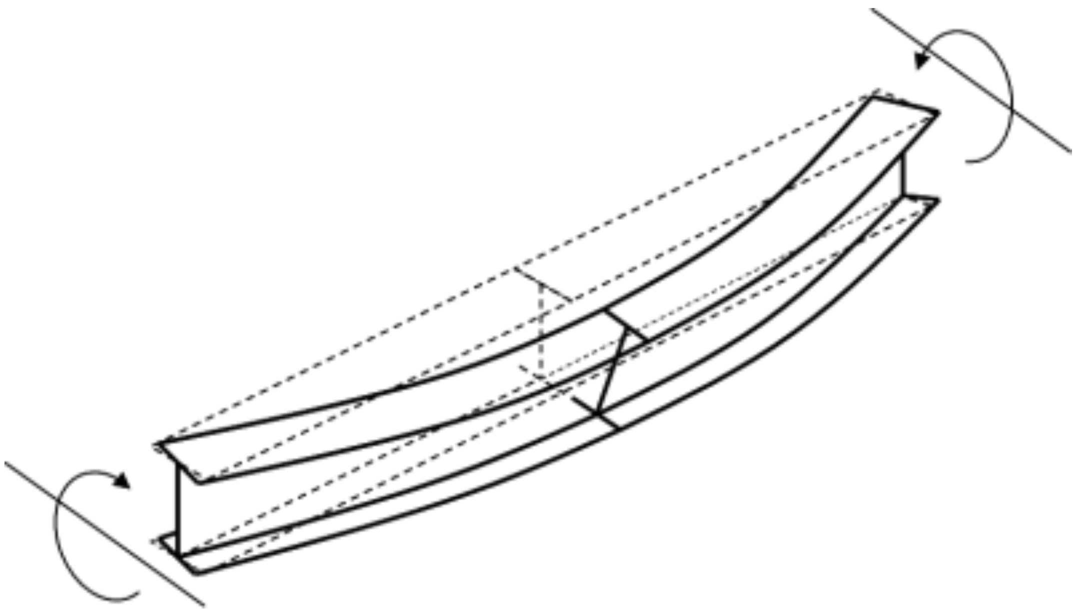
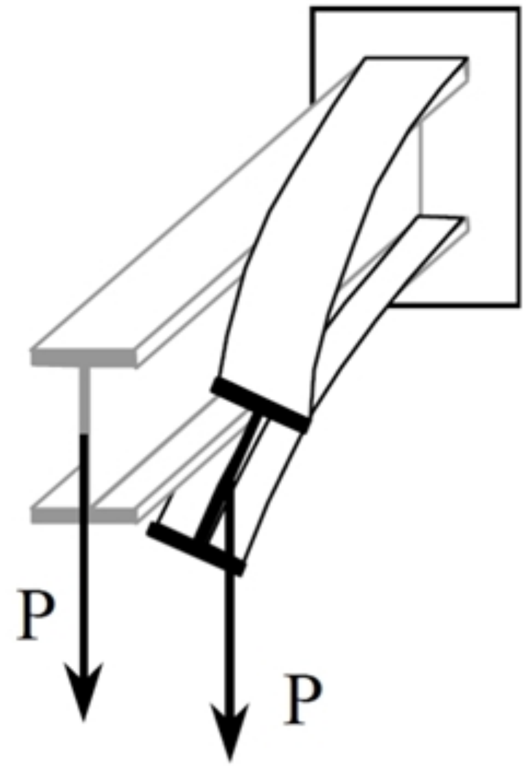
Outline

- Analysis of lateral buckling of beams
  - Simply-supported I-beam under a central concentrated load
  - Simply-supported I-beam under a uniformly distributed load
  - Out-of-plane bending and torsional buckling of doubly symmetric sections
  - Continuous beams
- Effect of the location of loading
- Review on the determination of the shear center
- Effect of boundary condition
- Summary

Beams – II

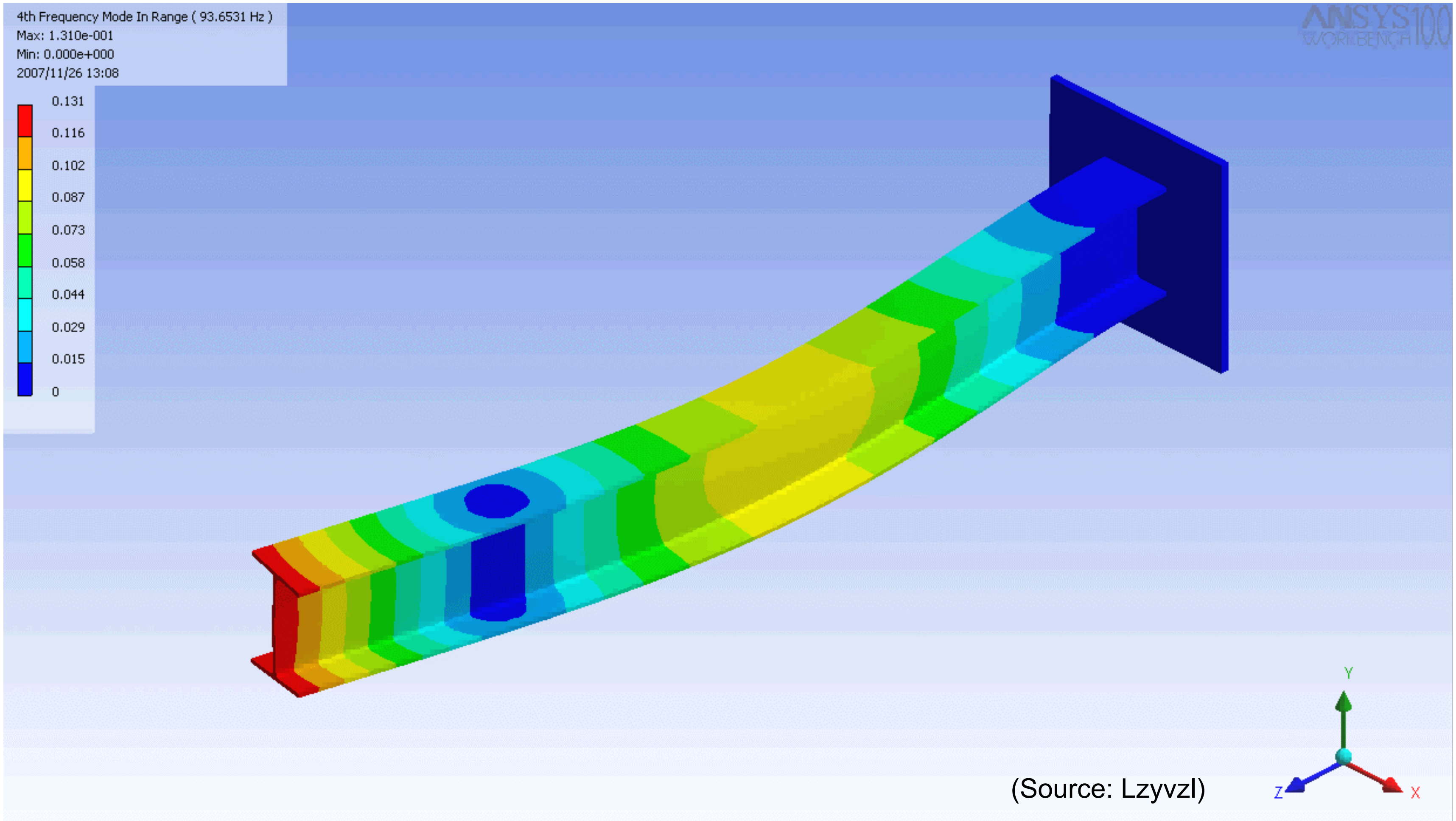
- **Analysis of lateral buckling of beams**





Beams – II

- **Analysis of lateral buckling of beams** – Stress-distribution



(Source: Lzyvzl)

Beams – II

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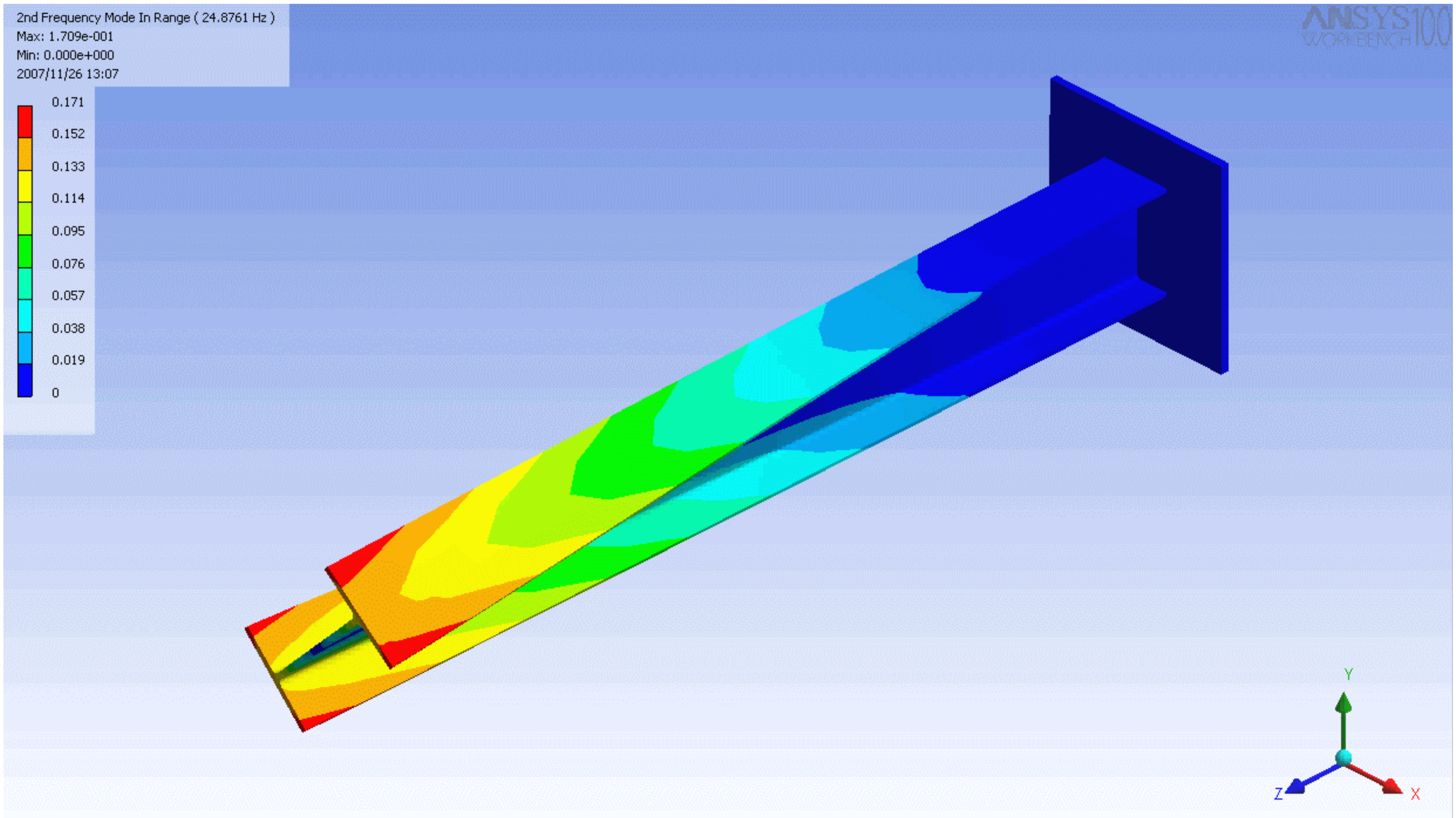
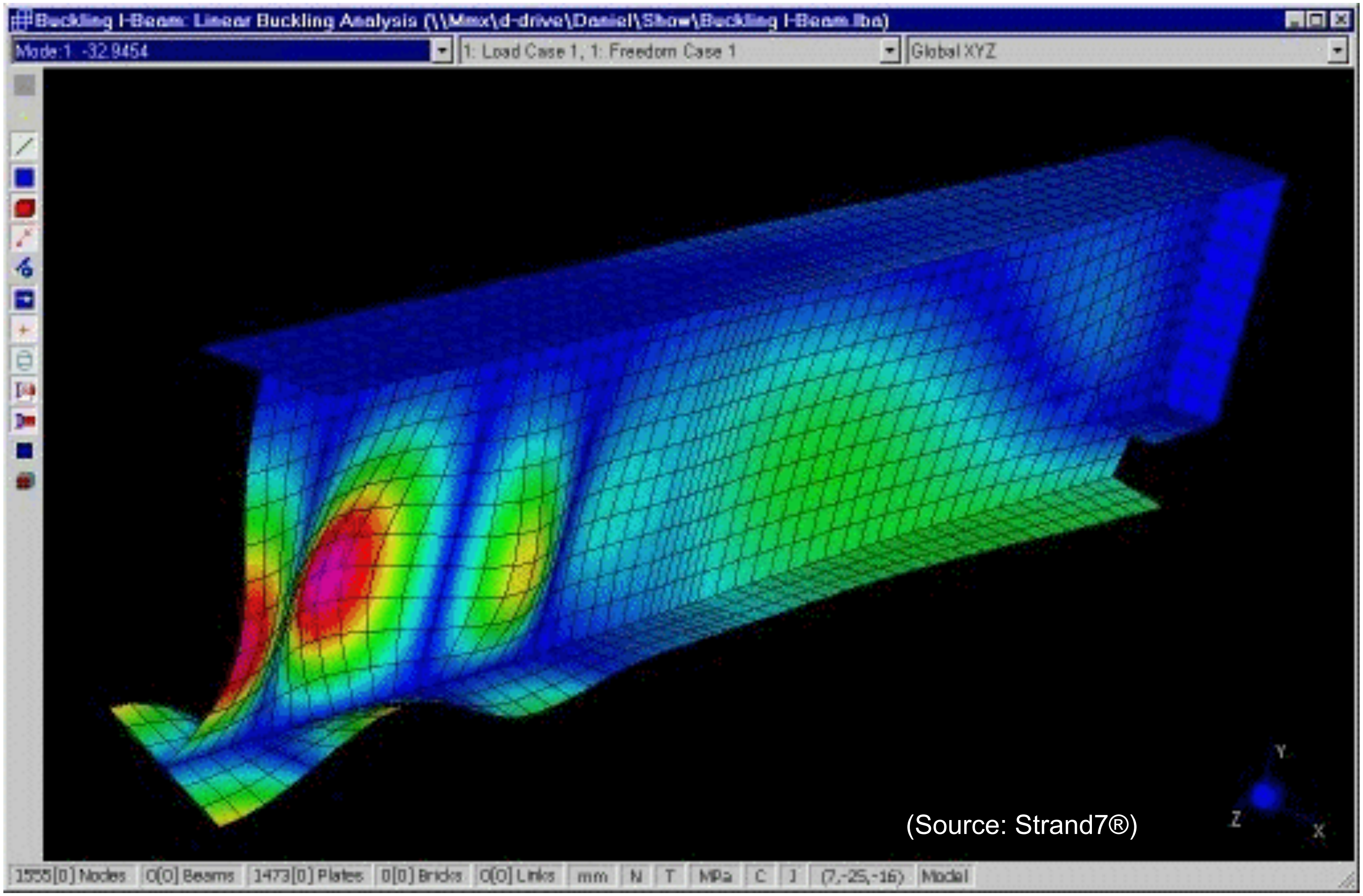


(Source: Lzyvzl)

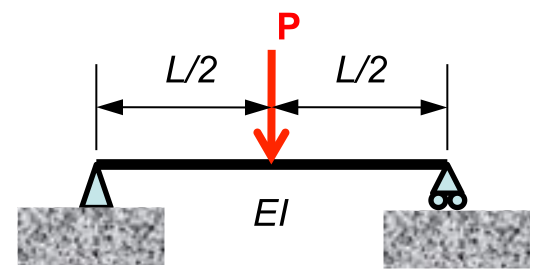
Beams – II

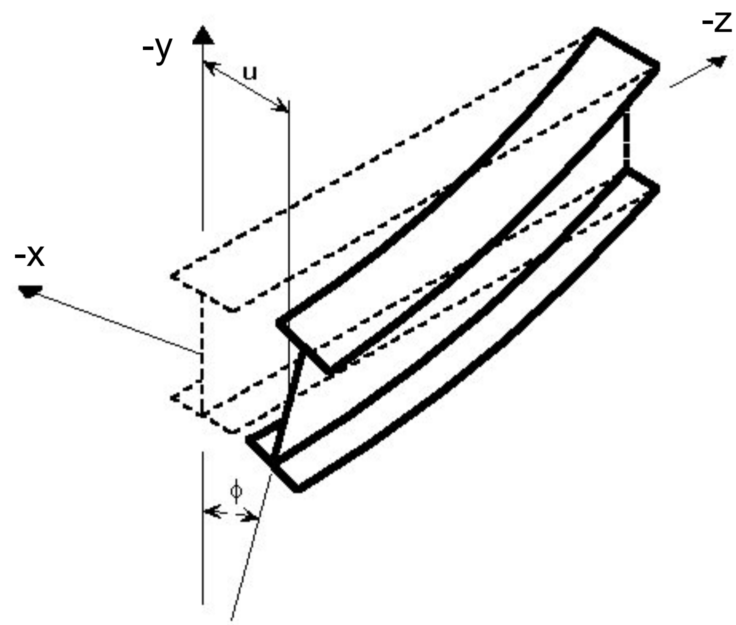
- **Analysis of lateral buckling of beams** – Stress-distribution



Beams – II

- **Analysis of lateral buckling of beams**
  - Simply-supported I-beam under a central concentrated load
    - Governing equations
      - In-plane bending
      - Out-of-plane bending
      - Torsion
    - Characteristic equation of the system

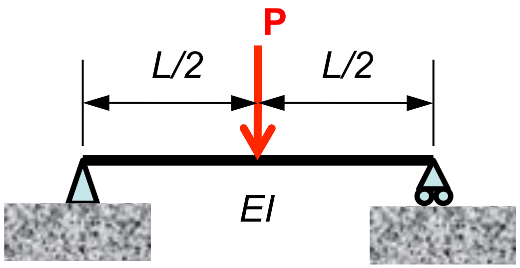




Beams – II

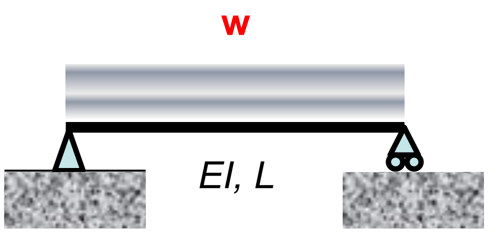
- **Analysis of lateral buckling of beams**
  - Simply-supported I-beam under a central concentrated load
    - B.C.

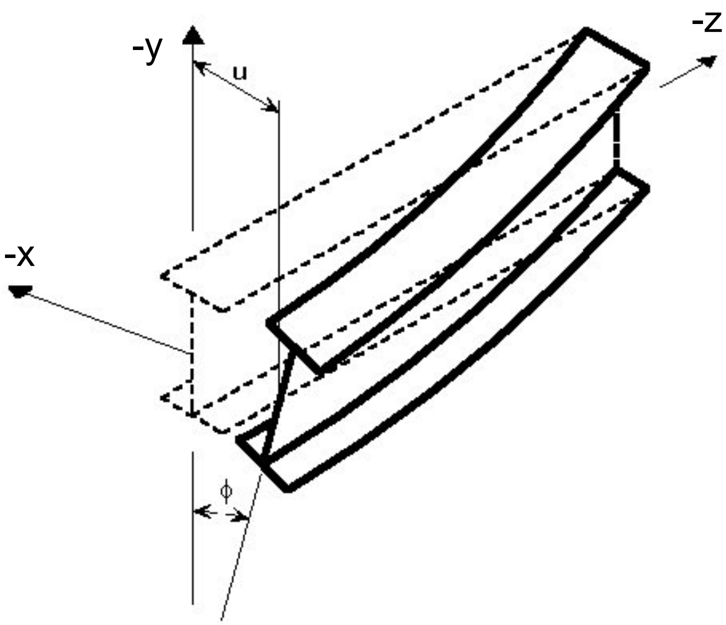
    - Solution of the critical moment



Beams – II

- **Analysis of lateral buckling of beams**
  - Simply-supported I-beam under a uniformly distributed load
    - Governing equations
      - In-plane bending
      - Out-of-plane bending
      - Torsion
    - Characteristic equation of the system





Beams – II

- **Analysis of lateral buckling of beams**
  - Simply-supported I-beam under a uniformly distributed load
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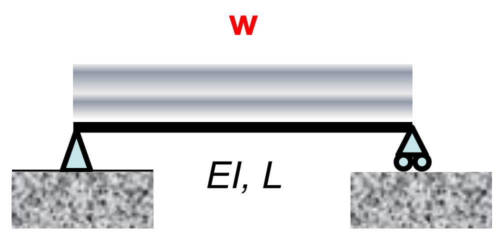


Beams – II

- **Analysis of lateral buckling of beams**
  - Out-of-plane bending and torsional buckling of doubly symmetric sections (Clark and Hill 1962)

Beams – II

- **Analysis of lateral buckling of beams**
  - Continuous beams

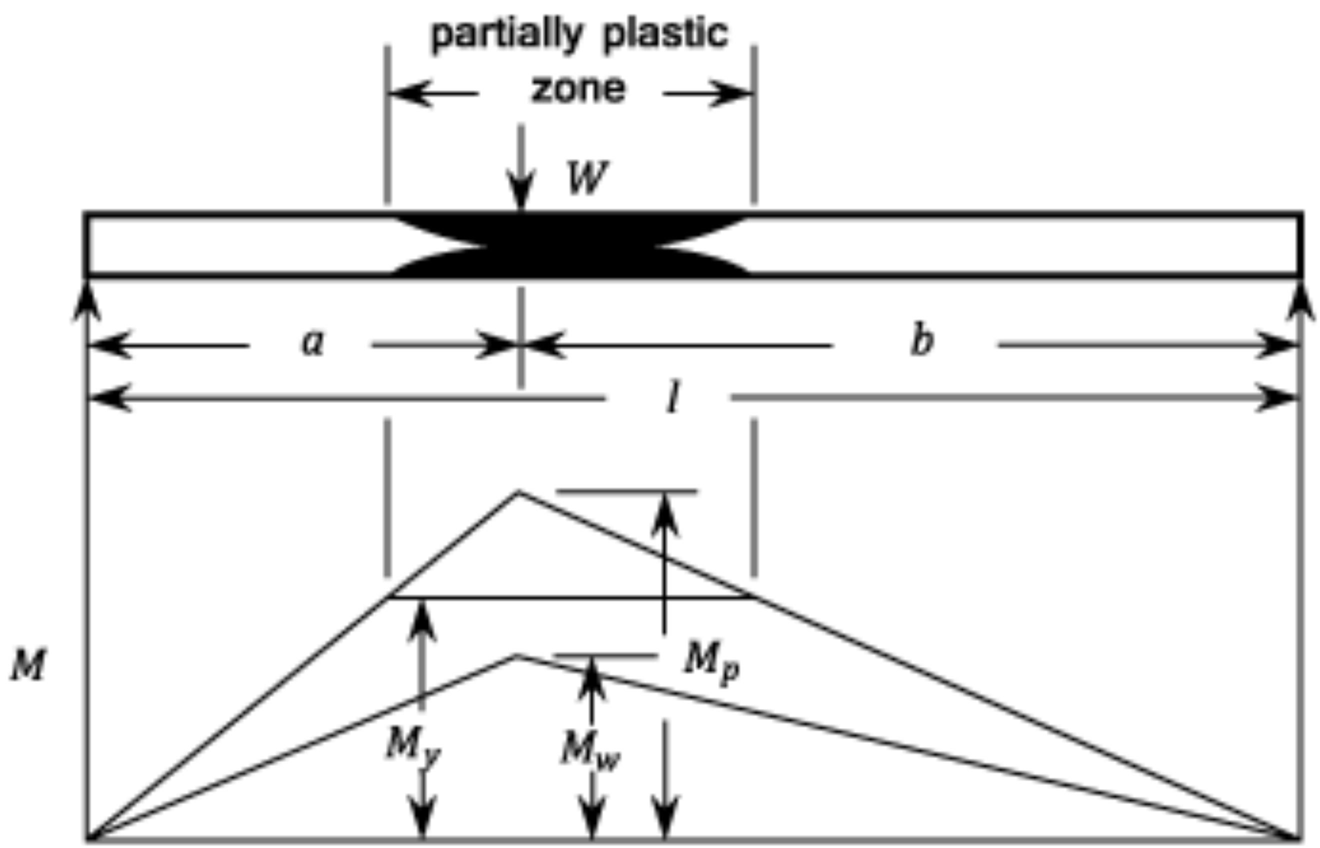
Beams – II

- **Effect of the location of loading**
  - Above the shear center
  - At the shear center
  - Under the shear center

Q: How does this phenomenon affect the design of beams for lateral torsional buckling?

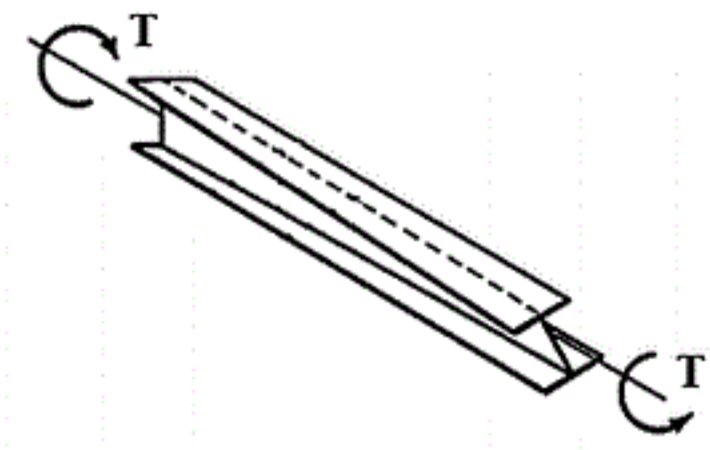
Beams – II

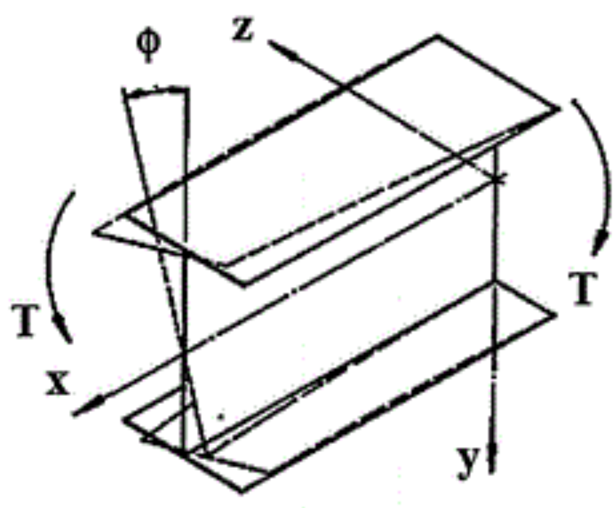
- **Formation of plastic hinge on a steel I-beam cross section**

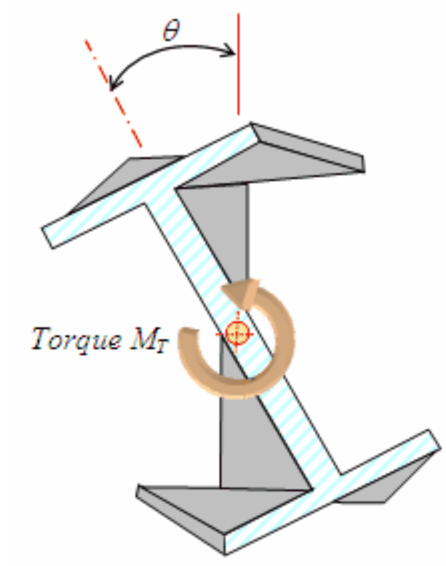


Beams – II

- **Uniform torsion of thin-walled open sections**

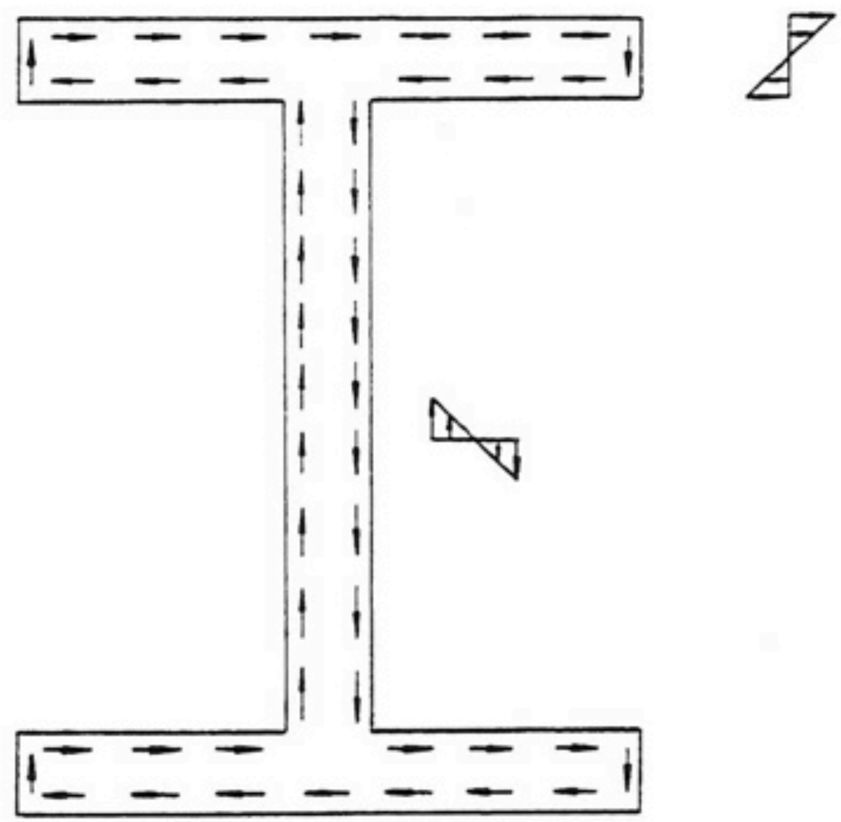


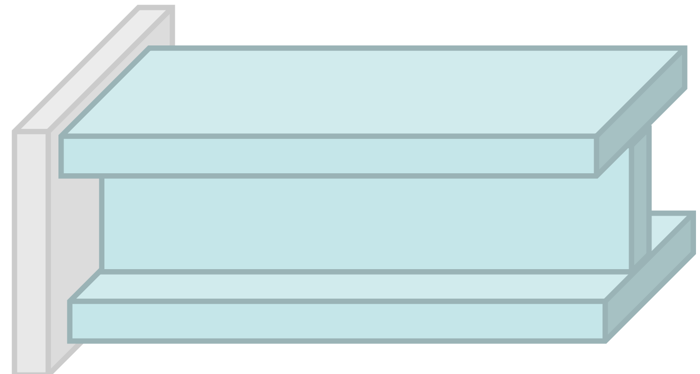




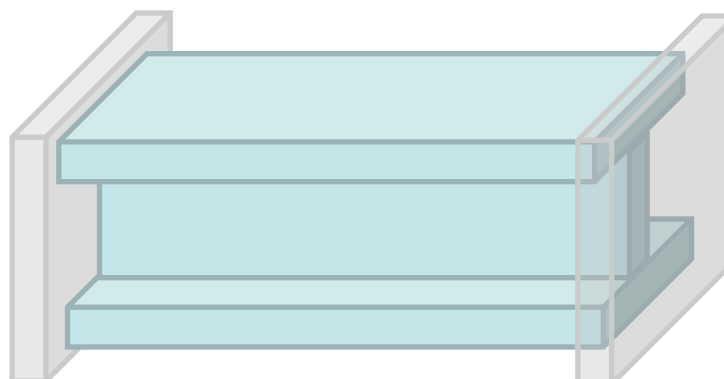
Beams – II

- **Venant shear stress distribution in an I-section**





Warping permitted

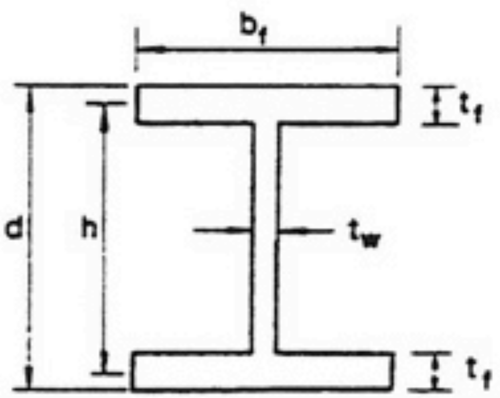


Warping restrained

If warping restrained at the support, Venant shear will be developed.

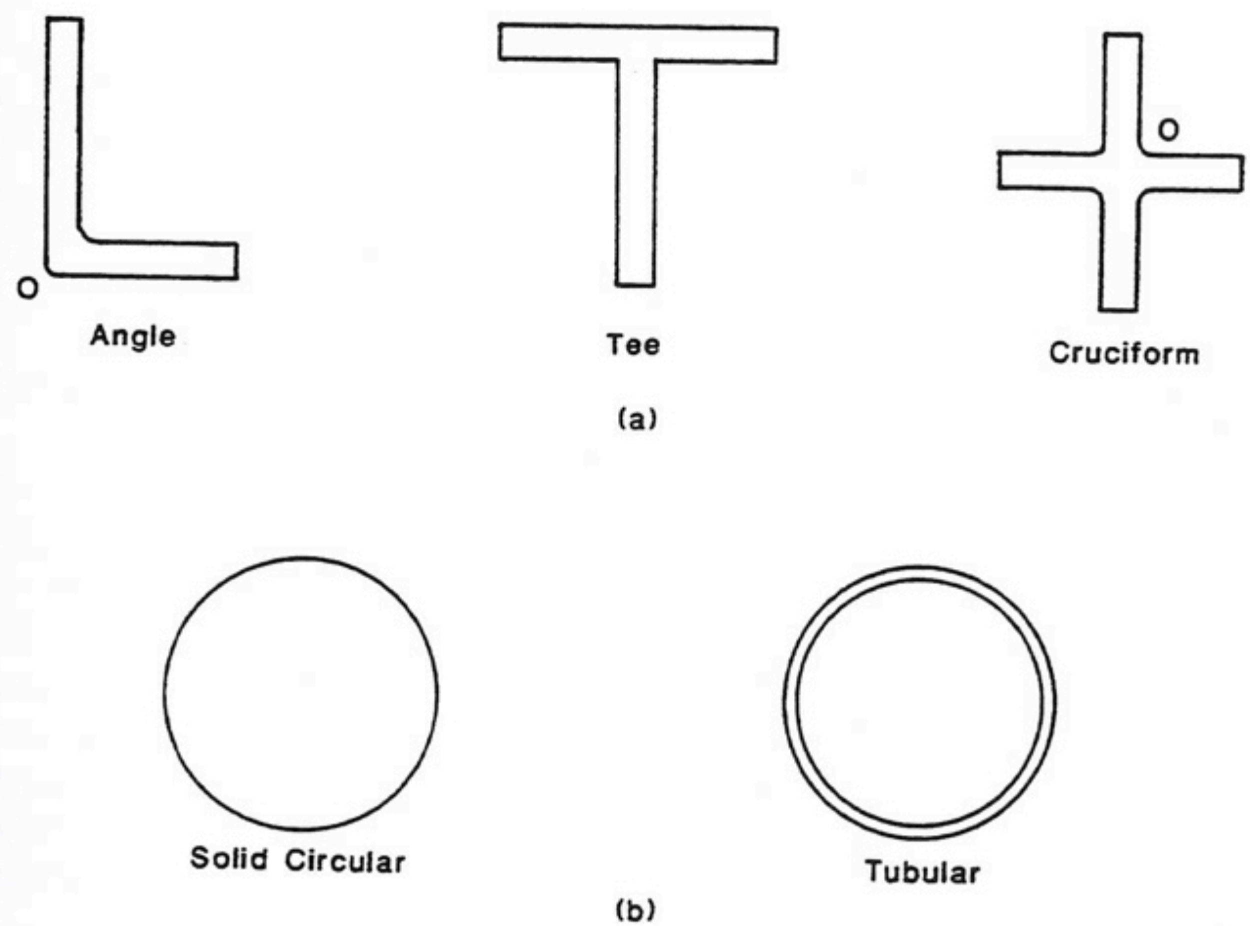
- **Warping constant, C_w**

Table 5.1 Torsional Constant and Warping Constant for a Doubly Symmetric I-Section

	Torsional Constant	Warping Constant
	$J = \frac{2b_f t_f^3 + (d-2t_f)\, t_w^3}{3}$	$C_w = \frac{t_f\, b_f^3\, h^2}{24}$

- **Cross sections without any Venant shear stress (C_w = *0*)**

FIGURE 5.9 Cross sections with $C_w = 0$



- **Equivalent moment factor, C_b**

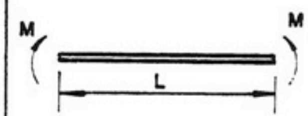
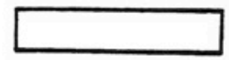
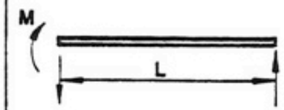
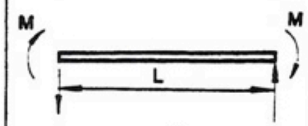
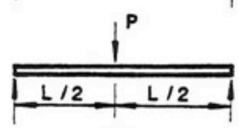
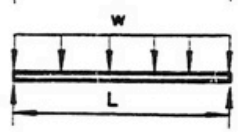
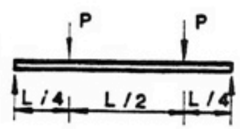
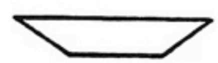
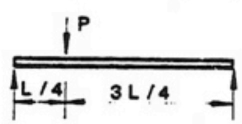
Loadings	Bending Moment Diagrams	M_{cr}	C_b
		M_{cr}	1.00
		M_{cr}	1.75
		M_{cr}	2.30
		$\frac{P_{cr} L}{4}$	1.35
		$\frac{W_{cr} L^2}{8}$	1.13
		$\frac{P_{cr} L}{4}$	1.04
		$\frac{3 P_{cr} L}{16}$	1.44

Table 5.2 Values of C_b for different loading cases (all loads are applied at shear center of the cross section)

- **Effect of the location of a concentrated load on an I-beam**

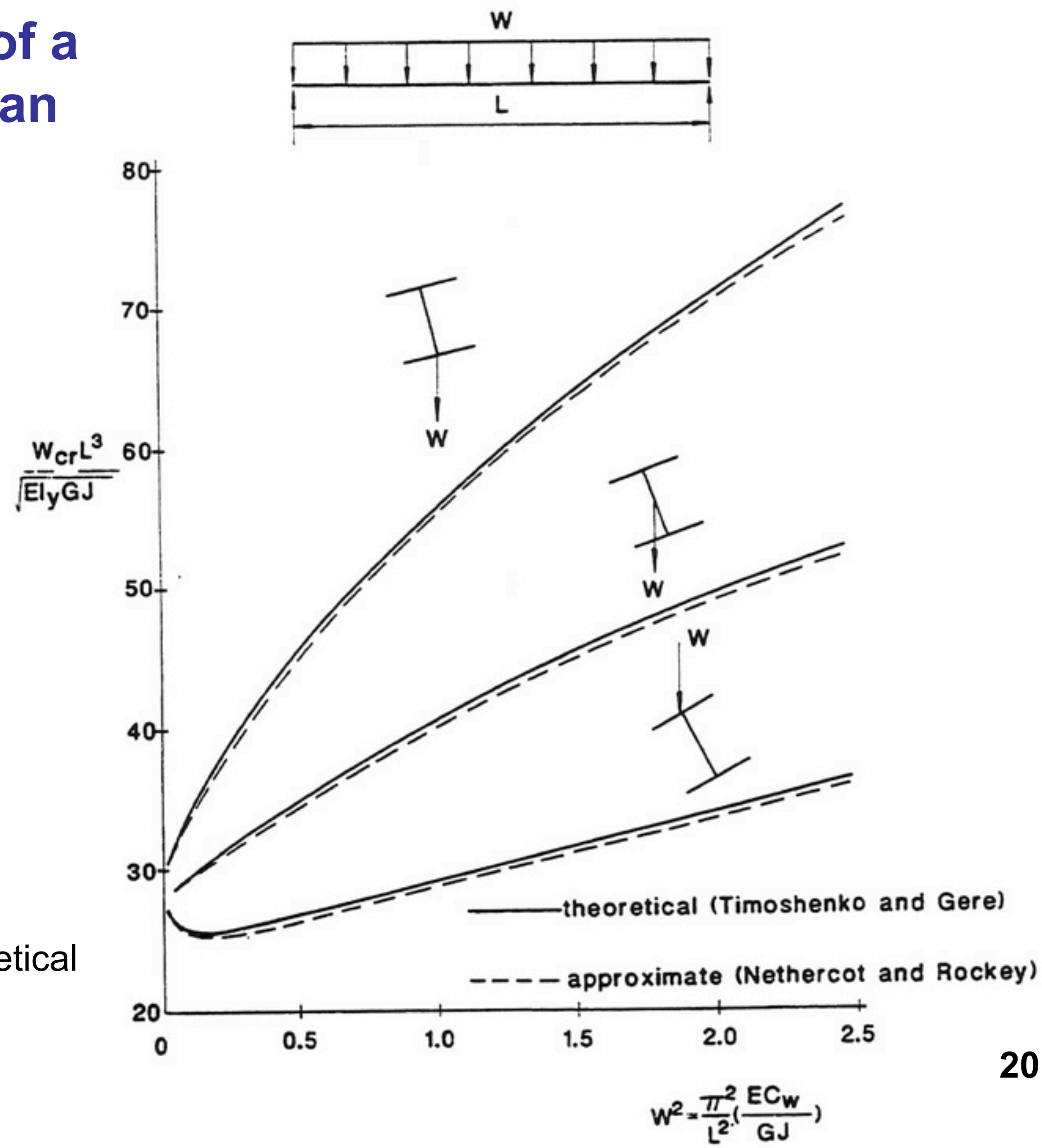
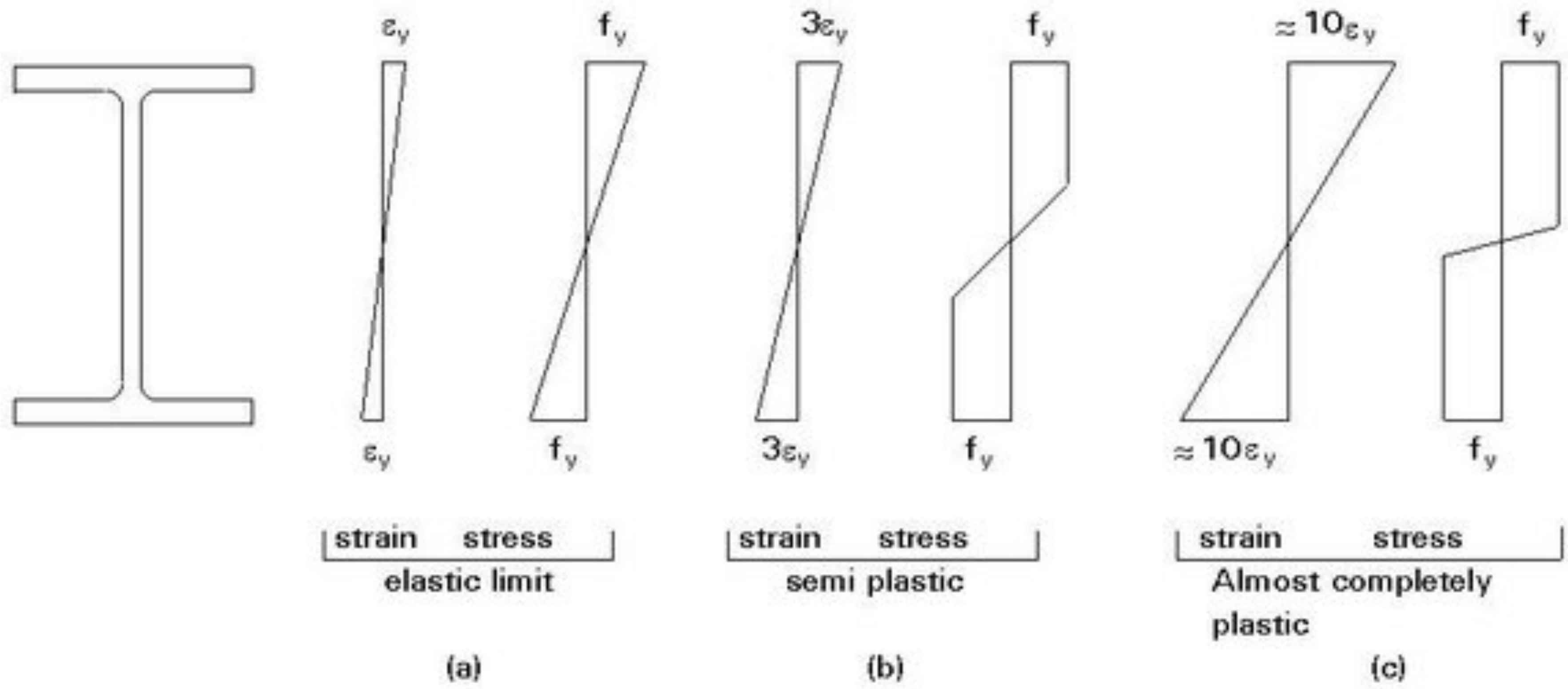


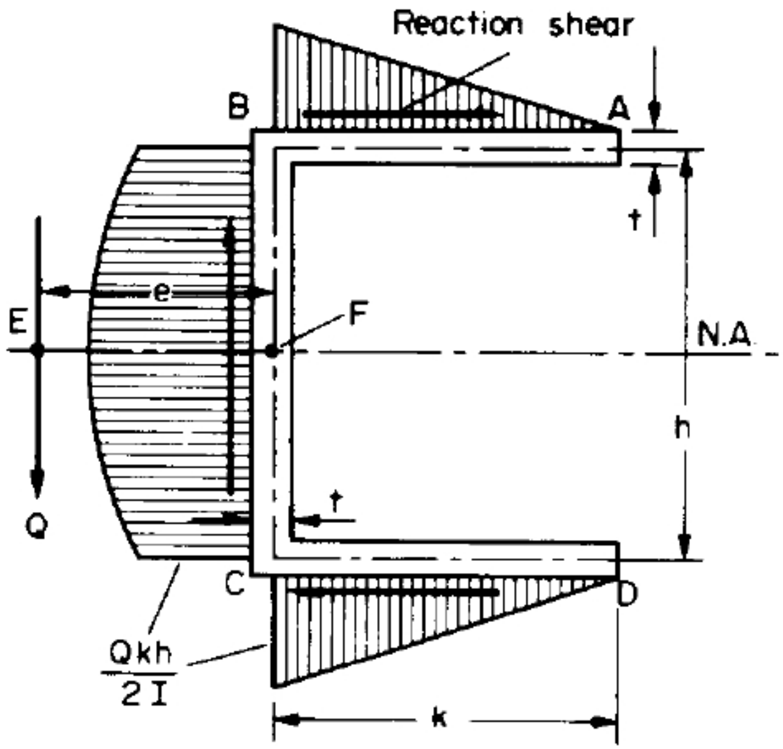
FIGURE 5.19 Comparison of theoretical and approximate solutions

Beams – II

- **Formation of plastic hinge on a steel I-beam cross section**



- **Review – Determination of the shear center**
  - Definition: The shear center of a section is the location at which the application of a concentrated load will result in zero twist of the section.
  - Example: Channel section beams



$$\tau_B = \frac{QA\bar{y}}{It} = \frac{Q \times kt}{It} \times \frac{h}{2} = \frac{Qkh}{2I}$$

$$\tau_A = 0$$

Average stress X area $= \frac{1}{2} \times \frac{Qkh}{2I} \times kt = \frac{Qk^2ht}{4I}$

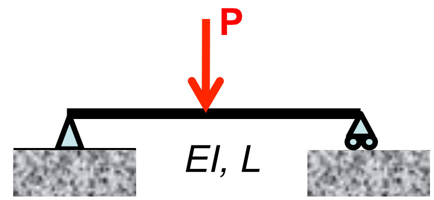
$$Q \times e = \frac{Qk^2ht}{4I} \times h$$

$$e = \frac{k^2h^2t}{4I}$$ → **Shear center**

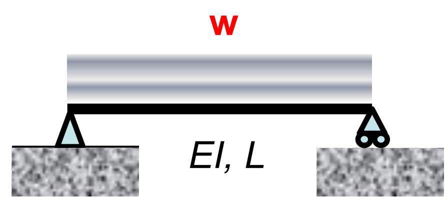
(Source: E.J. Hearn (1997))

Beams – II

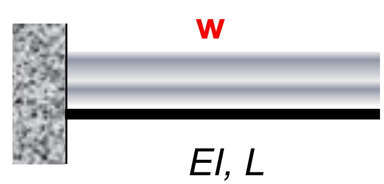
- **Effect of boundary conditions**
  - Simply-supported, concentrated load



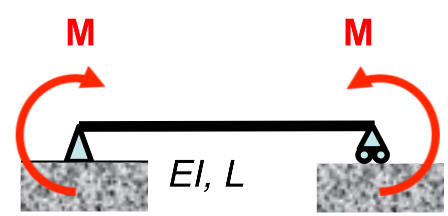
  - Simply-supported, uniformly distributed load



  - Cantilever (one end fixed and the other free), uniformly distributed load



  - Fixed-ended, equal end moments



Beams – II

- **Effect of boundary condition**
  - Asymmetric boundary conditions
    - Warping prevented in one plane and bending permitted in another plane

Bending permitted

Twisting permitted but warping prevented

  - Concept of the effective length *KL*

Summary

- The boundary condition of beams can be considered by different values of the effective length, KL, which depends on
  - Unbraced length of the beam
  - Material properties E and G,
  - Cross-section geometry c_w and J,
  - Types of loadings
  - Location of the load w.r.t. the shear center of the cross section.

- For design purpose, it is conservative to evaluate the critical load for each span in a continuous beam by assuming the ends of the span are simply supported.