Buckling of Beams – II

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Outline

• Analysis of lateral buckling of beams
  – Simply-supported I-beam under a central concentrated load
  – Simply-supported I-beam under a uniformly distributed load
  – Out-of-plane bending and torsional buckling of doubly symmetric sections
  – Continuous beams

• Effect of the location of loading

• Review on the determination of the shear center

• Effect of boundary condition

• Summary
Beams – II

• Analysis of lateral buckling of beams
Beams – II

• **Analysis of lateral buckling of beams** – Stress-distribution

(Source: Lzyvzl)
Beams – II

- Analysis of lateral buckling of beams – Stress-distribution

(Source: Lzyvzl)
Beams – II

• Analysis of lateral buckling of beams – Stress-distribution

(Source: Strand7®)
Beams – II

• Analysis of lateral buckling of beams
  – Simply-supported I-beam under a central concentrated load
    • Governing equations
      – In-plane bending
      – Out-of-plane bending
      – Torsion

• Characteristic equation of the system
Beams – II

• **Analysis of lateral buckling of beams**
  – Simply-supported *I*-beam under a central concentrated load
    • B.C.
    • Solution of the critical moment
Beams – II

**Analysis of lateral buckling of beams**
- Simply-supported I-beam under a uniformly distributed load
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Beams – II

• Analysis of lateral buckling of beams
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• **Analysis of lateral buckling of beams**
  – Out-of-plane bending and torsional buckling of doubly symmetric sections *(Clark and Hill 1962)*
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• Analysis of lateral buckling of beams
  – Continuous beams
Beams – II

• Effect of the location of loading
  – Above the shear center
  
  – At the shear center
  
  – Under the shear center

Q: How does this phenomenon affect the design of beams for lateral torsional buckling?
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- Formation of plastic hinge on a steel I-beam cross section
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• Uniform torsion of thin-walled open sections
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- Venant shear stress distribution in an I-section

If warping restrained at the support, Venant shear will be developed.
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- Warping constant, $C_w$

**Table 5.1 Torsional Constant and Warping Constant for a Doubly Symmetric I-Section**

<table>
<thead>
<tr>
<th>Torsional Constant</th>
<th>Warping Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J = \frac{2b_t t_f^3 + (d-2t_f) t_w^3}{3}$</td>
<td>$C_w = \frac{t_f b_t^3 h^2}{24}$</td>
</tr>
</tbody>
</table>
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- Cross sections without any Venant shear stress \( (C_w = 0) \)

**FIGURE 5.9** Cross sections with \( C_w = 0 \)

- Angle
- Tee
- Cruciform
- Solid Circular
- Tubular
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- **Equivalent moment factor, \( C_b \)**

<table>
<thead>
<tr>
<th>Loadings</th>
<th>Bending Moment Diagrams</th>
<th>( M_{cr} )</th>
<th>( C_b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M ) ( \frac{L}{2} ) ( \frac{L}{2} )</td>
<td>( M )</td>
<td>( M_{cr} )</td>
<td>1.00</td>
</tr>
<tr>
<td>( M ) ( L )</td>
<td>( M )</td>
<td>( M_{cr} )</td>
<td>1.75</td>
</tr>
<tr>
<td>( M ) ( L )</td>
<td>( M )</td>
<td>( M_{cr} )</td>
<td>2.30</td>
</tr>
<tr>
<td>( P ) ( \frac{L}{4} ) ( \frac{L}{2} ) ( \frac{L}{4} )</td>
<td>( P ) ( \frac{L}{4} ) ( \frac{L}{2} ) ( \frac{L}{4} )</td>
<td>( \frac{P_{cr} L}{4} )</td>
<td>1.35</td>
</tr>
<tr>
<td>( W ) ( L )</td>
<td>( W ) ( L )</td>
<td>( \frac{W_{cr} L^2}{8} )</td>
<td>1.13</td>
</tr>
<tr>
<td>( P ) ( \frac{L}{4} ) ( \frac{L}{2} ) ( \frac{3L}{4} )</td>
<td>( P ) ( \frac{L}{4} ) ( \frac{L}{2} ) ( \frac{3L}{4} )</td>
<td>( \frac{3P_{cr} L}{16} )</td>
<td>1.44</td>
</tr>
</tbody>
</table>

*Table 5.2 Values of \( C_b \) for different loading cases (all loads are applied at shear center of the cross section)*
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- Effect of the location of a concentrated load on an I-beam

**FIGURE 5.19** Comparison of theoretical and approximate solutions
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- Formation of plastic hinge on a steel I-beam cross section
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• **Review – Determination of the shear center**
  - Definition: The shear center of a section is the location at which the application of a concentrated load will result in zero twist of the section.
  - Example: Channel section beams

\[
\tau_B = \frac{Q A \bar{y}}{I t} = \frac{Q \times k t}{I t} \times \frac{h}{2} = \frac{Q k h}{2 I}
\]

\[
\tau_A = 0
\]

Average stress \( \times \) area = \( \frac{1}{2} \times \frac{Q k h}{2 I} \times k t = \frac{Q k^2 h t}{4 I} \)

\[
Q \times e = \frac{Q k^2 h t}{4 I} \times h
\]

\[
e = \frac{k^2 h^2 t}{4 I} \rightarrow \text{Shear center}
\]

(Source: E.J. Hearn (1997))
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• **Effect of boundary conditions**
  - Simply-supported, concentrated load
  - Simply-supported, uniformly distributed load
  - Cantilever (one end fixed and the other free), uniformly distributed load
  - Fixed-ended, equal end moments
• **Effect of boundary condition**
  – Asymmetric boundary conditions
    • Warping prevented in one plane and bending permitted in another plane
  – Concept of the effective length $KL$
Summary

• The boundary condition of beams can be considered by different values of the effective length, $KL$, which depends on
  – Unbraced length of the beam
  – Material properties $E$ and $G$,
  – Cross-section geometry $c_w$ and $J$,
  – Types of loadings
  – Location of the load w.r.t. the shear center of the cross section.

• For design purpose, it is conservative to evaluate the critical load for each span in a continuous beam by assuming the ends of the span are simply supported.