

LN11

Buckling of Thin Plates and Shells

- ① Buckling of cylindrical shells subjected to uniformly-distributed axial loading

- ① Governing equation:

$$D \frac{d^4 w}{dx^4} + N_x \frac{d^2 w}{dx^2} + E h \frac{w}{a^2} = 0$$

where D = flexural rigidity of thin shells,

w = radial displacement,

N_x = axial force (positive when in compression)

E = Young's modulus

h = thickness of the shell

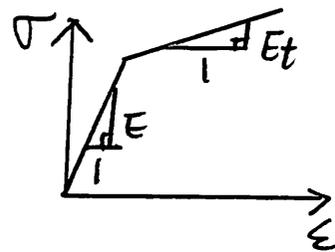
a = radius of the shell

- ② Solution:

$$\text{Elastic buckling: } \sigma_{cr} = \frac{N_{cr}}{h} = D \left(\frac{m^2 \pi^2}{h l^2} + \frac{E}{a^2 D} \frac{l^2}{m^2 \pi^2} \right)$$

where l = length of the shell

$$\text{Plastic buckling: } \sigma_{cr} = \frac{h \sqrt{E E_t}}{a \sqrt{3(1-\nu^2)}}$$



- ③ Assume slender cylindrical shells, $\Rightarrow \frac{l}{r} > 150$

$$\Rightarrow \sigma_{cr} = \frac{\pi^2 E}{(l/r)^2} = D \left(\frac{m^2 \pi^2}{h l^2} + \frac{E}{a^2 D} \frac{l^2}{m^2 \pi^2} \right)$$

global
buckling

local
buckling

When global buckling dominates,

$$\Rightarrow \frac{\pi^2 E}{(l/r)^2} < D \left(\frac{m^2 \pi^2}{h l^2} + \frac{E}{a^2 D} \times \frac{l^2}{m^2 \pi^2} \right)$$

$$\Rightarrow \left(\frac{r}{l} \right)^2 < D \left(\frac{m^2}{E h l^2} + \frac{l^2}{a^2 D m^2 \pi^4} \right)$$

$$\Rightarrow \left(\frac{r}{l} \right)^2 < D \left(\frac{1}{E h} \cdot \frac{m^2}{l^2} + \frac{1}{a^2 D \pi^4} \times \frac{l^2}{m^2} \right)$$

$$\Rightarrow \frac{r}{l} < \sqrt{D \left(\frac{1}{E h} \cdot \frac{m^2}{l^2} + \frac{1}{a^2 D \pi^4} \times \frac{l^2}{m^2} \right)} \quad \#$$

When local buckling dominates,

$$\Rightarrow \frac{r}{l} > \sqrt{D \left(\frac{1}{E h} \cdot \frac{m^2}{l^2} + \frac{1}{a^2 D \pi^4} \times \frac{l^2}{m^2} \right)} \quad \#$$

Buckling of uniformly loaded circular thin plates

① Governing equation:

$$r^2 \frac{d^2 \phi}{dr^2} + r \frac{d\phi}{dr} - \phi = 0.$$

where r = radial variable, ϕ = angular variable

② B.C.: $\phi(r=a) = 0 \Rightarrow a = \text{radius} \Rightarrow \text{Fixed-ended}$
 $\phi(r=0) = 0 \Rightarrow \text{sym.}$

③ Solution to the governing equation is:

$$\phi = A_1 J_1 \left(\sqrt{\frac{Nr}{D}} \cdot r \right) + A_2 \cdot Y_1 \left(\sqrt{\frac{Nr}{D}} \cdot r \right)$$

where A_1 & A_2 are constants.

J_1 = Bessel function of the first kind,

Y_1 = Bessel function of the second kind,

(See slide #26 for J_1 & Y_1 .)

$$\Rightarrow N_{cr} = (Nr)_{cr} = \frac{14.68 D}{a^2} \quad (\text{fixed-ended})$$

$$= (Nr)_{cr} = \frac{4.20 D}{a^2} \quad (\text{simply-supported})$$