Forward and Inverse Dielectric Modeling of Oven-Dried Cement Paste Specimens in the Frequency Range of 1.02 GHz to 4.50 GHz

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ABSTRACT

The use of radar non-destructive evaluation (NDE) technique for condition assessment of deteriorated civil infrastructure systems is an effective approach for preserving the sustainability of these systems. Radar NDE utilizes the interaction between radar signals (electromagnetic waves) and construction materials for surface and subsurface sensing based on dielectric properties and geometry. In the success of radar inspection, it is imperative to develop models capable of predicting the dielectric properties of the materials under investigation. The dielectric properties (dielectric constant and loss factor) of oven-dried cement paste specimens with water-to-cement (w/c) ratios (0.35, 0.40, 0.45, 0.50, 0.55) in the frequency range of 1.02 GHz to 4.50 GHz were studied and modeled using modified Debye's models. An open-ended coaxial probe and a network analyzer were used to measure dielectric properties. Forward models are proposed and inversed for predicting the w/c ratio of a given oven-dried cement paste specimen. Modeling results agreed with the experimental data. The proposed models can be used for predicting the dielectric properties of oven-dried cement paste specimens. Also, the modeling approach can be applied to other cementitious materials (e.g., concrete) with additional modification.

Keywords: : Dielectric dispersion, microwave frequency, water-to-cement ratio, inverse modeling

1. INTRODUCTION

Measurement and modeling of dielectric properties (dielectric constant and loss factor) of construction materials are indispensable knowledge to the inspection and monitoring of critical civil infrastructure systems (e.g., reinforced and prestressed concrete bridges, tunnels, buildings) using radar and microwave non-destructive evaluation (NDE) techniques. Understanding the reflection, scattering, attenuation, and transmission of radar/microwave signals (electromagnetic or EM waves) inside dielectrics like reinforced concrete (RC) relies on the dielectric or EM properties of RC and the geometry of each element in RC. As a lossy dielectric, radar/microwave signals transmitted through and reflected from RC depend on the dielectric properties of concrete and steel reinforcing bars (rebars). Since the dielectric property of steel is well known, the key to successfully predict the dielectric property of RC or prestressed concrete (PC) is the dielectric property of concrete. Such knowledge is important in locating subsurface steel rebars,[1] detecting subsurface rebar corrosion,[2,3] and assessing concrete integrity[4] using radar/microwave NDE techniques like ground penetrating radar.

However, concrete, as a cementitious composite, is the assembly of hydrated cement (cement hydration products), fine and coarse aggregates, moisture (liquid water), and air. These components form a multi-phase dielectric system whose overall/effective dielectric properties depend on i) dielectric properties and geometry of individual components, ii) volumetric fractions of individual components, and iii) spatial distributions of individual components. To fully understand and precisely predict the dielectric property of concrete, one must have all the information in order to derive a material/dielectric model with high fidelity. This demands the knowledge of dielectric property of all components in concrete, their volumetric fractions, and their spatial distributions. The paper presents a systematic approach to investigate how the dielectric property of concrete changes with different concrete compositions (concrete mix design) and measurement conditions (frequency) by

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presenting our work on the measurement and modeling of oven-dried cement paste specimens (w/c ratio = 0.35, 0.40, 0.45, 0.50, 0.55) in the frequency range of 1.02 GHz to 4.50 GHz.

In the past, research work on the dielectric measurement and modeling of oven-dried cement paste in the microwave frequency range was reported in literature.[5–7] Hasted and Shah[8] measured the dielectric constant and loss factor of hardened cement paste specimens (oven-dried and moist) at 3, 9, 24 GHz, using guided waves. Gu and Beaudoin[9] also measured the dielectric properties of cement paste specimens saturated in lime solution for one year in the frequency range of 1MHz to 1GHz. De Loor et al.[5] conducted dielectric measurements of moist and oven-dried cement paste specimens at 3, 3,75, 7.45, and 9.375 GHz using a coaxial waveguide system. They found that the increase of moisture content in cement paste specimens is responsible for the increase of their dielectric constant and loss factor measurements, while the increase of measurement frequency leads to the decrease of dielectric constant. Similar finding was reported by other researchers in different frequency bands and on different hardened cement paste specimens.[6,10,11]

While the dielectric constant and loss factor measurements of oven-dried cement paste were reported by various research teams, theoretical/dielectric modeling effort is behind dielectric measurement effort as found from our literature review. One recent modeling effort,[12] based on the Cole-Cole model,[13] on oven-dried cement paste (heated up to 75°C) assumed single relaxation time for the material in the frequency range of 100 kHz to 10 MHz. However, material design parameters (e.g., w/c ratio) were not considered in their model. Keddam et al.[14] used the capacitor method to measure the dielectric properties of oven-dried cement paste in the frequency range of 100 kHz to 40 MHz . They used a single relaxation time capacitor model to model the complex electric capacitance (related to complex electric permittivity or dielectric properties of an oven-dried cement paste specimens). However, material design parameters such as the water-to-cement (w/c) ratio was not considered in their model. Assuming single relaxation time, Yu[15] also proposed dielectric model for oven-dried cement paste as a function of frequency and w/c ratio using experimental data from literature in the frequency range of 3 GHz to 24 GHz, based on an assumption of constant product of w/c ratio and infinite dielectric permittivity.

2. EXPERIMENTAL WORK

2.1 Specimen Description

Oven-dried cement paste panel specimens were manufactured by mixing Portland cement Type I/II and water based on five different water-to-cement (w/c) ratios (0.35, 0.40, 0.45, 0.50, and 0.55; by weight), producing a total of five specimens with the same dimensions (1ft-by-1ft-by-1in) as shown in Figure 1. The specimens were moist cured for seven days, room-conditioned at an average temperature and RH (relative humidity) of 24°C and 27%, respectively, for three months before oven-drying at a temperature of 105°C. Detailed description of the specimens are provided in Table 1.

Table 1: Details of specimen

Specimen	w/c	Mass before OD (lbs)	Mass after OD (lbs)	Mass loss (%)
CP35	0.35	10.035	9.650	3.84
CP40	0.40	9.225	8.870	3.85
CP45	0.45	8.910	8.520	4.38
CP50	0.50	8.790	8.385	4.61
CP55	0.55	8.100	7.725	4.63

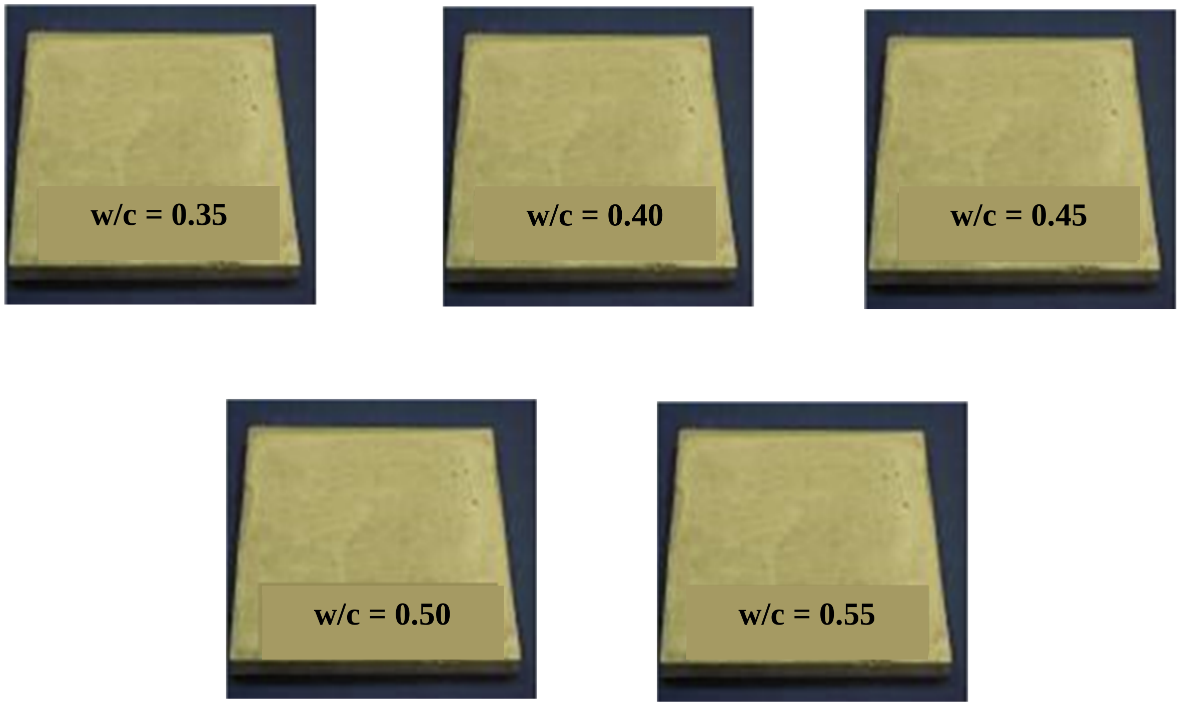

Figure 1: Oven-dried cement paste specimen panels

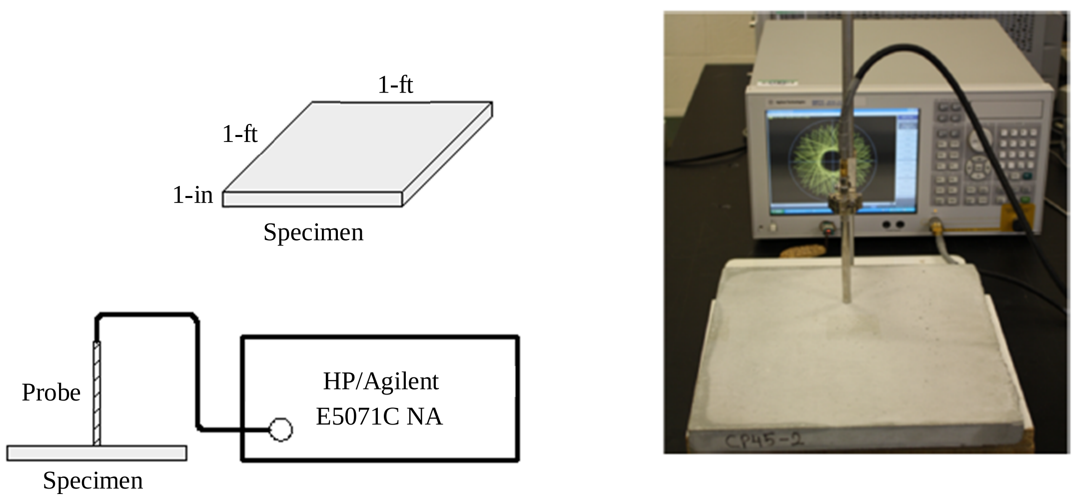

Figure 2: Setup of contact dielectric measurement system and dimensions of specimen

2.2 Dielectric Measurement

Dielectric properties of the oven-dried cement paste panels were measured using an open-ended coaxial probe and an HP/Agilent E5071C Network Analyzer (NA) in the frequency range of 1.02 GHz to 4.50 GHz. The setup of the measurement system and the dimensions of the panels are shown in Figure 2. Measurements were randomly taken at sixty points within the 1ft-by-1 ft surface of the panels. The dielectric constant (ε_r') and loss factor (ε_r'') for each panel were calculated by finding the average of the sixty data points to take into account the spatial variation in the measurement. The results of dielectric measurement are shown in Figure 3.

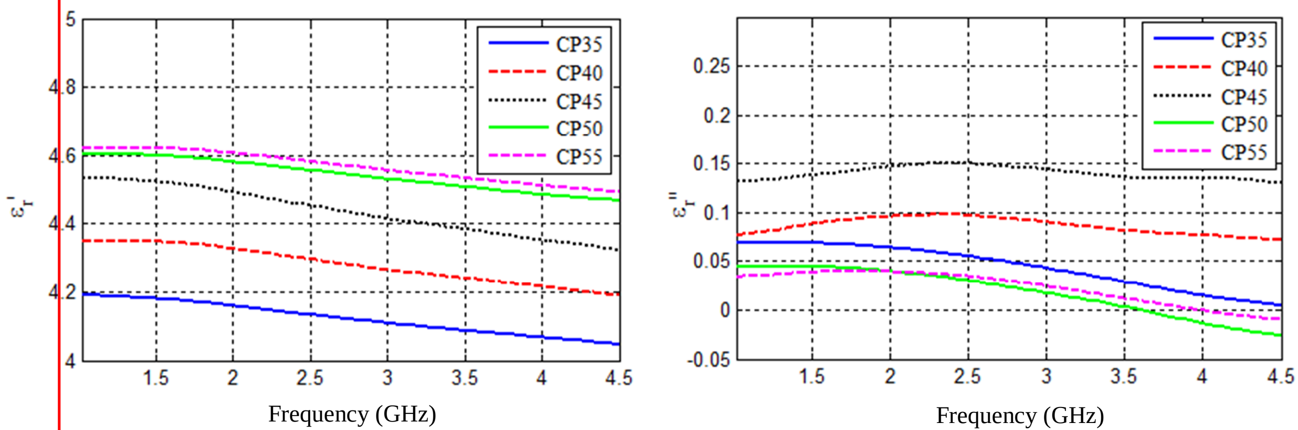

Figure 3: Dielectric measurements of oven-dried cement paste specimens

3. MODELING APPROACH

3.1 Forward Modeling

In the forward modeling of this research, modified Debye's models were developed for predicting the dielectric constant and loss factor of oven-dried cement paste specimens in the considered frequency range. The objective of forward modeling in this research is to predict the dielectric constant (real part of the relative complex electric permittivity) and loss factor (imaginary part of the relative complex electric permittivity) of oven-dried cement paste in the frequency range of 1.02 GHz to 4.50 GHz. In this modeling approach, oven-dried cement paste was assumed to be homogenous, suggesting a single relaxation time for the material. Three parameters (w/c ratio, C_1, C_2) were introduced in the developed models to account for the dielectric dispersion as a function of frequency and the w/c ratio. Eqs. (1) and (2) describe the forward models for dielectric constant and loss factor of oven-dried cement paste specimens in the frequency range of 1.02 GHz to 4.50 GHz.

$$\varepsilon_r'(\omega) = \varepsilon_\infty + \frac{(\varepsilon_s - \varepsilon_\infty)}{1 + (\omega\tau)^2} - \frac{\psi}{10} \tag{1}$$

$$\varepsilon_r''(\omega) = \frac{\omega\tau(\varepsilon_s - \varepsilon_\infty)}{1 + (\omega\tau)^2 - C_2} - C_1 \tag{2}$$

where ε_r' = dielectric constant, ε_r''= loss factor, ω = frequency (GHz), ε_s = dielectric constant at $\omega = 0$, ε_∞ = dielectric constant at $\omega = \infty, \tau$ =relaxation time (ns) , ψ = water-to-cement-ratio. Values of parameters C_1 and C_2 were determined by nonlinear best fitting. The parameters of the model are shown in Table 2. A procedure summarizing the steps in the forward modeling is provided in the following:

1. Collect dielectric constant and loss factor measurements of oven-dried cement paste specimens with known water-to-cement ratio (ψ) in the frequency range of 1.02 GHz to 4.50 GHz.
2. Initial guesses of ε_∞, τ and ε_s are obtained using a procedure previously developed by our group.[16]
3. Use nonlinear curve fitting (sum of squared errors, SSE, criterion) to obtain the final values of ε_∞, τ, ε_s, C_1 and C_2 for Eqs. (1) and (2).

Table 2: Model parameters

Specimen	ε_∞	ε_s	τ	$\frac{\psi}{10}$	C_1	C_2
CP35	4.0200	4.2648	0.3367	0.035	0.6738	0.1420
CP40	4.1522	4.4395	0.3172	0.040	0.4945	0.0991
CP45	4.2823	4.6423	0.3434	0.045	0.3465	0.0650
CP50	4.4485	4.7002	0.3274	0.050	0.6500	0.1710
CP55	4.4758	4.7182	0.3044	0.055	0.6726	0.1660

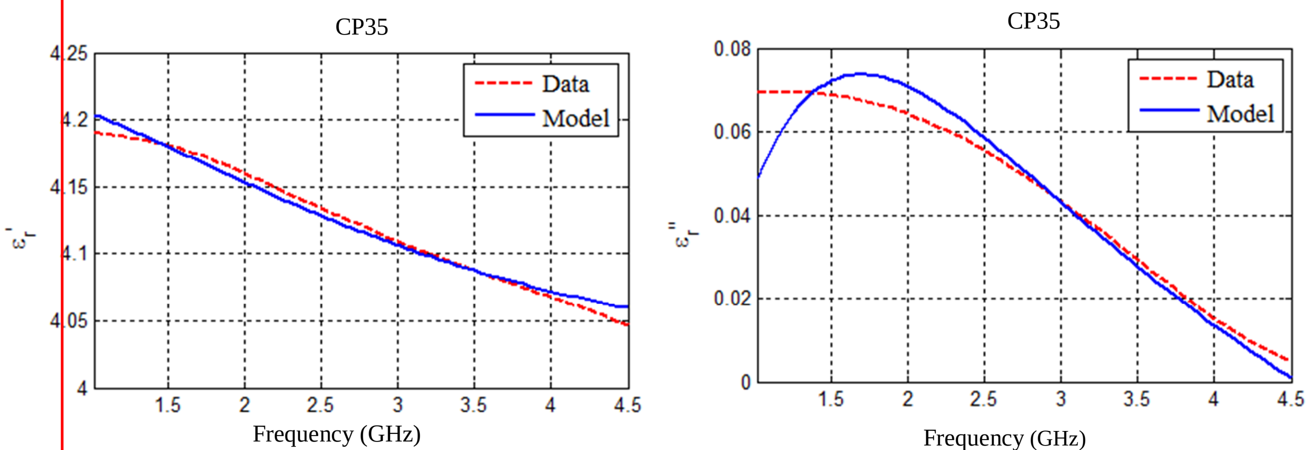

Figure 4: Comparison between the data of the dielectric constant and loss factor and the values obtained by modeling for CP35

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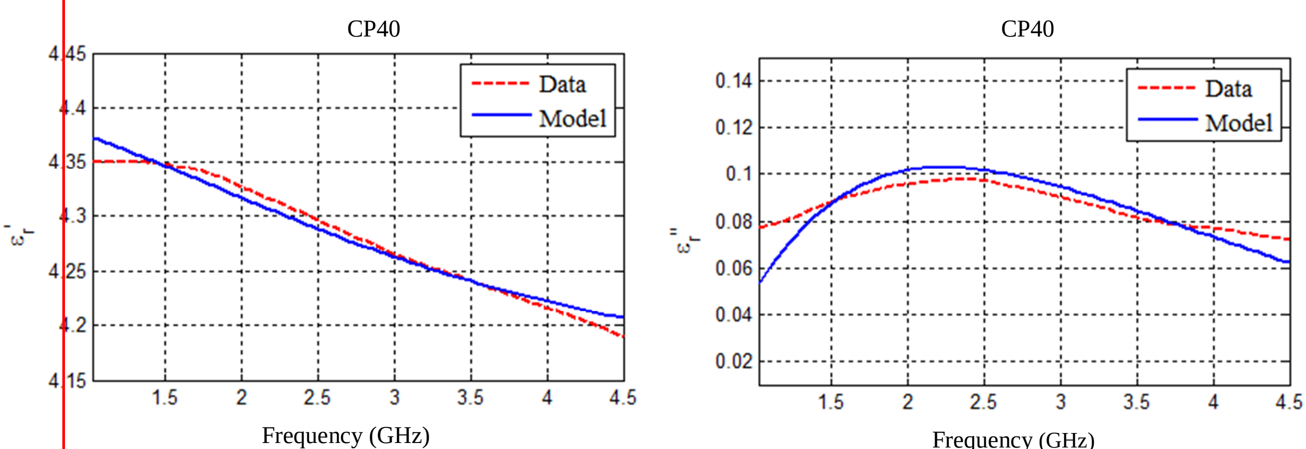

Figure 5: Comparison between the data of the dielectric constant and loss factor and the values obtained by modeling for CP40

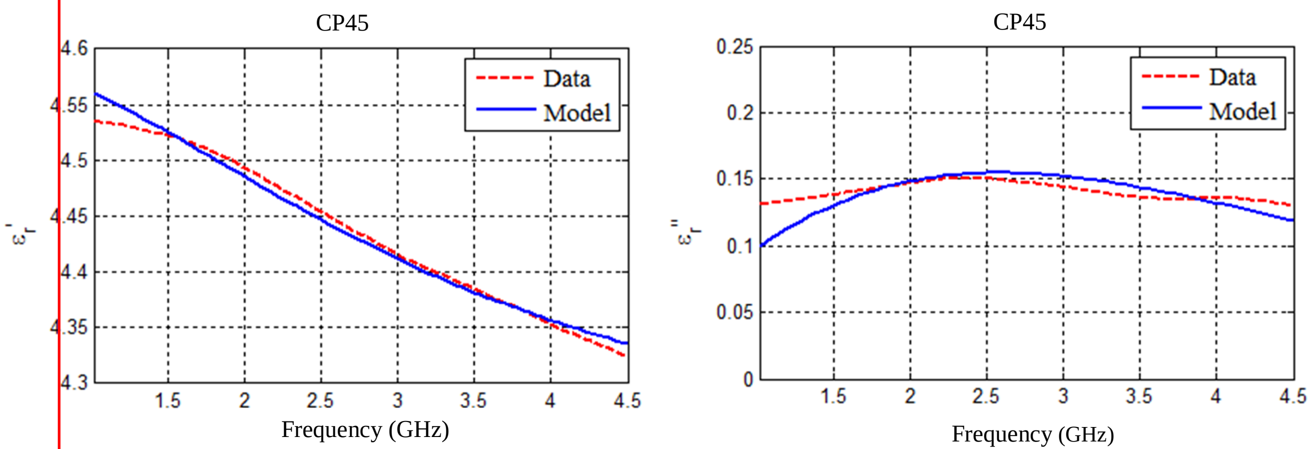

Figure 6: Comparison between the data of the dielectric constant and loss factor and the values obtained by modeling for CP45

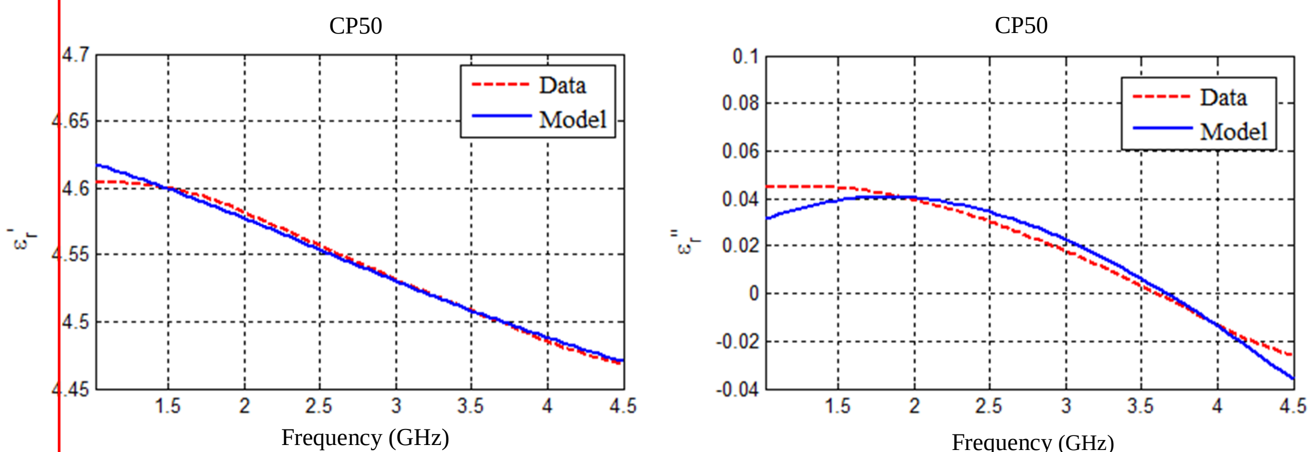

Figure 7: Comparison between the data of the dielectric constant and loss factor and the values obtained by modeling for CP50

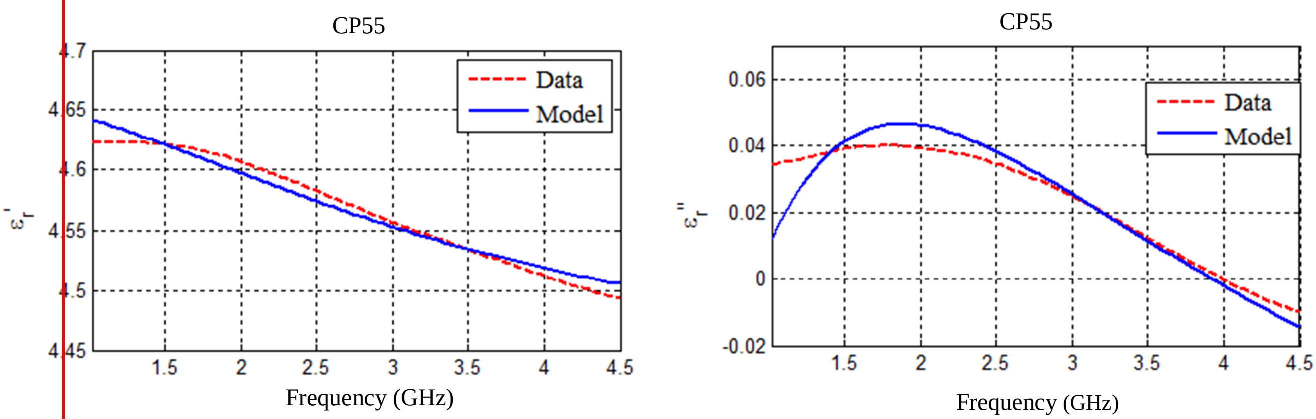

Figure 8: Comparison between the data of the dielectric constant and loss factor and the values obtained by modeling for CP55

3.2 Inverse Modeling

The objective of inverse modeling in this research is to predict the w/c ratio of oven-dried cement paste in the frequency range of 1.02 GHz to 4.50 GHz. With mathematical manipulation using linear algebra, Eq. (1) and (2) can be converted to the following form (Eq. (3)) in which three measurements of dielectric constant loss factor are needed.

$$\begin{bmatrix} (\varepsilon_\infty - \frac{\psi}{10})\tau^2 \\ -\tau^2 \end{bmatrix} = \begin{pmatrix} \omega_1^2 - \omega_2^2 & \varepsilon_{r1}^{\prime(m)}\omega_1^2 - \varepsilon_{r2}^{\prime(m)}\omega_2^2 \\ \omega_1^2 - \omega_3^2 & \varepsilon_{r1}^{\prime(m)}\omega_1^2 - \varepsilon_{r3}^{\prime(m)}\omega_3^2 \end{pmatrix}^{-1} \begin{bmatrix} \varepsilon_{r1}^{\prime(m)} - \varepsilon_{r2}^{\prime(m)} \\ \varepsilon_{r1}^{\prime(m)} - \varepsilon_{r3}^{\prime(m)} \end{bmatrix} \tag{3}$$

where $\varepsilon_{r1}^{\prime(m)}, \varepsilon_{r2}^{\prime(m)}$ and $\varepsilon_{r3}^{\prime(m)}$ are three experimentally measured dielectric constants at frequencies ω_1, ω_2 and ω_3, respectively. Once the parameters (ε_∞ and τ^2) are determined, the w/c ratio (ψ) of oven-dried cement paste can be estimated using Eq. (4) which represents the inverse model of oven-dried cement paste.

$$\psi = 10\left\{\varepsilon_\infty + \frac{[\varepsilon_{r1}^{\prime(m)}\varepsilon_{r2}^{\prime(m)}(\omega_1^2-\omega_2^2) - \varepsilon_{r1}^{\prime(m)}\varepsilon_{r3}^{\prime(m)}(\omega_1^2-\omega_3^2) + \varepsilon_{r2}^{\prime(m)}\varepsilon_{r3}^{\prime(m)}(\omega_2^2-\omega_3^2)]}{[\varepsilon_{r1}^{\prime(m)}(\omega_2^2-\omega_3^2) - \varepsilon_{r2}^{\prime(m)}(\omega_1^2-\omega_3^2) + \varepsilon_{r3}^{\prime(m)}(\omega_1^2-\omega_2^2)]}\right\} \tag{4}$$

A procedure summarizing the steps in inverse modeling is provided in the following:

1. Select three experimentally measured dielectric constants and loss factors in the frequency of 1.02 GHz to 4.5 GHz.
2. Develop an algebraic equation of the form shown in Eq. (3).
3. Solve for ε_∞ and τ^2 .
4. Substitute ε_∞ into Eq. (4) to predict the w/c ratio (ψ).

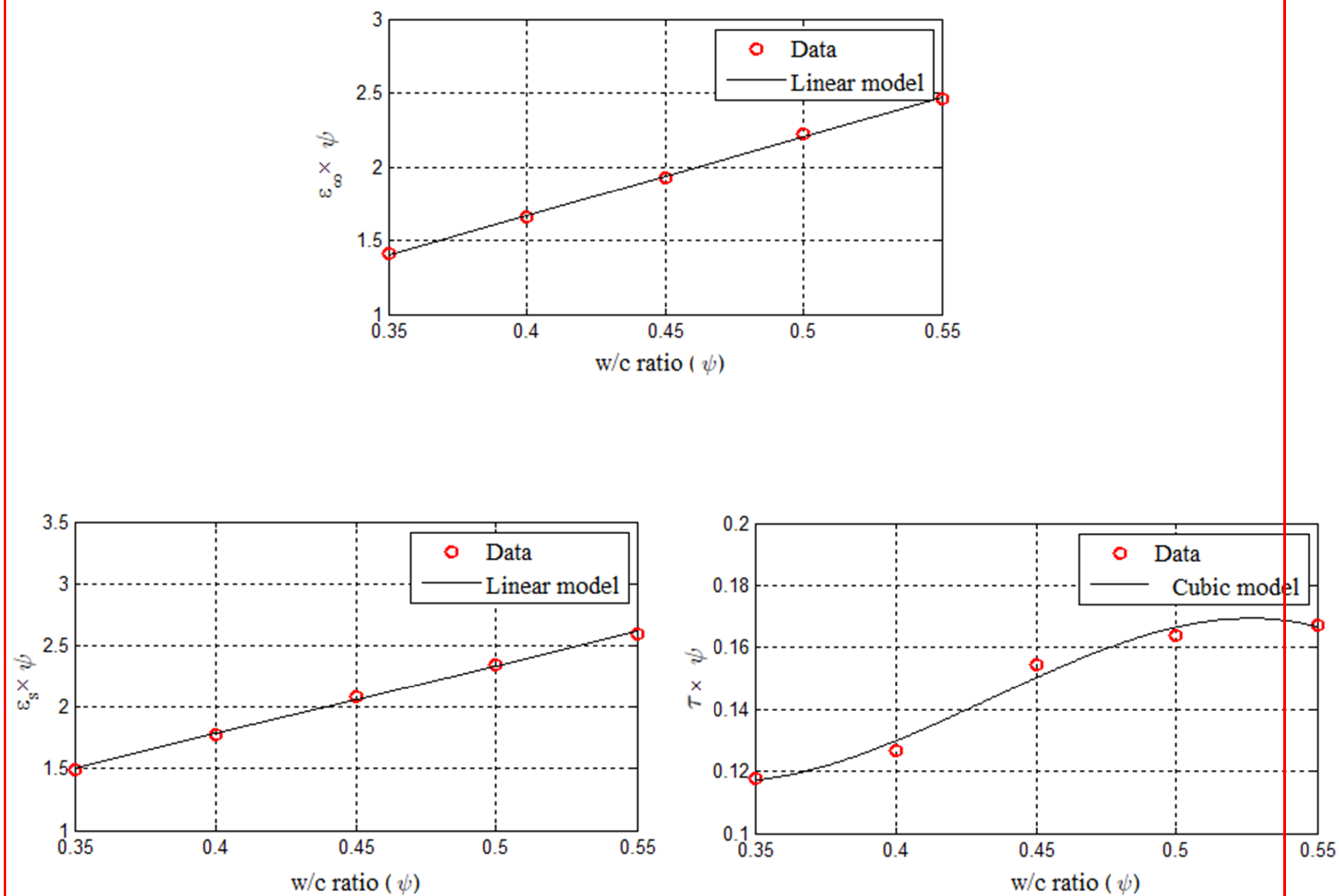

Figure 9: Relationships between ε_∞, ε_s,τ and w/c ratio

4. RESULTS

Modeling results are discussed in the following sections. Performance of developed forward and inverse models is evaluated and modeling issues are addressed.

4.1 Forward modeling

To evaluate the performance of developed forward models, dielectric measurements and model predictions are compared in Figures 4 to 8, for dielectric constant and loss factor.

In the performance of predicting dielectric constant, the forward models generally show very good agreement with experimental data in the frequency range of 1.5 GHz to 4.0 GHz. When outside this frequency range, the models tend to overestimate the dielectric constants of oven-dried cement paste specimens with w/c ratios 0.35, 0.40, 0.45, 0.55 (except for 0.50 w/c ratio).

In the loss factor part, more prediction errors are observed in the frequency range of 1.20 GHz to 2.50 GHz. The models either underestimate or overestimate the loss factors of oven-dried cement paste specimens with w/c ratios 0.35 and 0.55, but tend to underestimate the loss factors of oven-driend cement paste specimens with 0.40, 0.45, and 0.50 in the frequency range of 1.20 GHz to 1.50 GHz. Unlike the dielectric constant models, the loss factor models tend to underestimate the measured loss factor of oven-dried cement paste in the frequencies higher than 4 GHz.

4.2 Inverse modeling

In the inverse modeling of this research, key model parameters ($\epsilon_\infty, \epsilon_s, \tau$) must be estimated from measurement data in order to obtain systematic predictions of the w/c ratio. From the relationships between key model parameters and the w/c ratio, consistency of model predictions can be assessed. In Figure 9, relationships between $\epsilon_\infty, \epsilon_s, \tau$ and the w/c ratio are illustrated.

In Figure 9, the relationship between the w/c ratio ψ and the product of the infinite relative electric permittivity (dielectric constant at infinite frequency) and the w/c ratio $\epsilon_\infty \times \psi$ was found to be linear from the curve fitting using experimentally measured dielectric constants and loss factors. It can be described by

$$\begin{aligned} \varepsilon_\infty \times \psi &= 5.3455\psi - 0.4693 \\ \Rightarrow \varepsilon_\infty &= 5.3455 - \frac{0.4693}{\psi} \end{aligned} \tag{5}$$

Similar relationship was found between the w/c ratio and the product of the static relative electric permittivity (dielectric constant at zero frequency) and the w/c ratio $\epsilon_s \times \psi$, described by

$$\begin{aligned} \varepsilon_s \times \psi &= 5.5579\psi - 0.4405 \\ \Rightarrow \varepsilon_s &= 5.5579 - \frac{0.4405}{\psi} \end{aligned} \tag{6}$$

However, the relationship between the w/c ratio and the product of the relaxation time and the w/c ratio $\tau \times \psi$ was found to be nonlinear, as formulated by

$$\begin{aligned} \tau \times \psi &= -16.043\psi^3 + 20.827\psi^2 - 8.589\psi + 1.26 \\ \Rightarrow \tau &= -16.043\psi^2 + 20.827\psi - 8.589 + \frac{1.26}{\psi} \end{aligned} \tag{7}$$

5. SUMMARY

In this paper, the forward and inverse modeling of over-dried cement paste specimens in the microwave frequency range of 1.02 GHz to 4.50 GHz is reported. Modified Debye's models are proposed for predicting the dielectric constant and loss factor of oven-dried cement paste in the considered frequency range, as our forward modeling result. These models are further converted into inverse models for predicting the water-to-cement ratio of oven-dried cement paste in the considered frequency range. In general, proposed forward models provide very good predictions of dielectric constant and loss factor in the frequency range of 1.50 GHz to 4.00 GHz. Better prediction performance of the forward models is observed on dielectric constant over the ones on loss factor. Compared to other models for oven-dried cement paste,[14,17] the proposed forward models (single relaxation time or single polarization) incorporate the w/c ratio and demonstrate consistent patterns between model parameters (e.g., ε_s,ε_∞, and τ) and the w/c ratio. The proposed inverse model provides a systematic approach to determine the w/c ratio of oven-dried cement paste specimens. Other mechanical and durability properties of cementitious composites can be estimated by the w/c ratio. This approach is of great potential to be applied to other cementitious composites such as cement mortar and concrete.

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