Energy-Efficient Target Monitoring in Wireless Sensor Networks

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***Abstract*—One of the fundamental purpose of sensing information is to immediately respond to any anomalies. Wireless sensor network (WSN) is a network of inexpensive, low-power nodes with embedded processors, radios, sensors, and actuators, often integrated on a single chip, to communicate with the physical world in applications, such as security and surveillance, smart classroom, monitoring of natural habitats, and medical monitoring. WSNs differ considerably from current networked and embedded systems and due to its extreme energy constraints its design requires a proper understanding of the interplay between network protocols, energy-aware design, signal-processing algorithms, and distributed programming. Though the small form-factor of sensor nodes makes them attractable for use in monitoring applications, at the same time their small size affects resources such as the energy, computational power, and storage. Therefore, improvising on the energy constraints of wireless sensor networks is crucial. We propose two base-station relocation policies that aim to minimize the energy consumed for transmitting the data to base station. Both the policies involve a mobile base station, and focus on moving the base station closer to the active sensors that detect the target. Our first policy involves having a mobile base station and relocating it to the geometric centroid of all the sensors detecting the target. This approach significantly reduces the energy overhead required for transmitting data from the sensors to the base station. Our second policy for performing network lifetime optimization is to move the base station to geometric centroid of the base station locations obtained over several time periods. However, in each case, moving the base station at each time period involves a considerable overhead and therefore we observe the effects of moving the base station after a specific number of time periods as opposed to moving after every time period. We evaluate the network lifetime performance of these two proposed policies over different network scenarios.**

Keywords: Wireless sensor networks and target monitoring.

I. INTRODUCTION

A wireless sensor network (WSN) consists of a large number of sensor nodes deployed over an area, integrated to collaborate over a wireless medium. These sensors are small in size and are able to sense, process data, and communicate with each other, typically over a radio channel. There are several applications of WSN including general engineering, agriculture and environmental monitoring, civil engineering, military applications, and health monitoring. In a typical application, a WSN is scattered in a region and is meant to collect data through its sensor nodes. The characteristics of a wireless sensor network [1] are self-organization, multi-hop cooperative relay, and large-scale dense deployment. The fundamental limitations in WSNs are node energy, transmission power, memory, and computing power.

Target monitoring is one of the important applications of WSNs. In target monitoring, the WSN is deployed over a region where the target is to be monitored. Target monitoring is concerned with approximating the trajectory of one or more moving objects based on some partial information, usually provided by sensors [2]. Target monitoring is necessary in various domains, such as computer vision [3], sensor networks [4], tactical battlefield surveillance, air traffic control, perimeter security, and first response to emergencies. A typical example is the problem of finding the trajectory of a vehicle by bearings measurements, which is a technique used by radars. Work in robotics has also considered tracking targets from moving platforms [5]. In target monitoring, when the sensors detect a target, the event is reported to the base station, which can take appropriate action (e.g., send a message on the Internet or to a satellite).

Wireless sensor networks represent a significant advance over traditional methods of monitoring. As an example, for habitat monitoring sensors can be deployed prior to the onset of the breeding season. Sensors can also be deployed on areas where it is unsafe to attempt field studies. The results of wireless sensor-based monitoring efforts are comparable with the traditional methods of monitoring. Sensor network deployment represents a substantially more economical method for conducting long-term studies than traditional methods. A “deploy ’em and leave ’em” strategy of wireless sensor usage limits logistical needs to initial placement and occasional servicing. It also greatly increases access to a wider array of study sites, often limited by concerns about frequent access and habitability [6].

Another example is structural health monitoring (SHM), a technology that estimates the structural state and detects structural change that affects the performance of a structure. Compared to the wired network, installation and maintenance are easy and inexpensive in a WSN, and disruption of the operation of the structure is minimal. The system also becomes scalable to a large number of nodes to allow dense sensor coverage of real-world structures [7].

The rest of the paper is organized in the following manner: Section II outlines the problem description and Section III describes the related background work on target monitoring. Section IV proposes energy-efficient target monitoring policies. Section V presents the simulation results and Section VI concludes the paper.

II. PROBLEM DESCRIPTION

Current research literature focuses on target detection, accurate estimation of the target’s path over a period of time, and sending the collected information to the base station. However, sensor networks are limited in terms of energy and for them to be useful in any application ensuring prolonged network lifetime is extremely important. We therefore focus on creating an approach that minimizes the energy consumed in monitoring the target and thereby ensuring prolonged network lifetime. To achieve this, we propose two policies that try to reduce the energy overhead required to transmit the data to the base station. Both the policies involve a mobile base station and focus on moving it closer to the sensors that detect the target and thereby result in reduced detection time by the base station. We also look at an approach to perform real-time target monitoring. Our approach not only tracks the target accurately, but also keeps the response time to the base-station minimal.

III. RELATED WORK

In [8], a simple distributed co-operative tracking algorithm that records the time instances when each sensor detects the object and then performs line-fitting on the resulting set of points. Instead of looking at a single position measurement, the algorithm considers the path of a moving object composed of a sequence of positions over a period of time. The only requirement for this protocol is that the density of sensor nodes be high enough for the sensing ranges of several sensors to overlap. The outline of this cooperative tracking algorithm is as follows:

1. Each node records the duration for which the object is in its range.
2. Neighboring nodes exchange these durations and their locations.
3. For each point in time, the object's estimated position is computed as a weighted average of the detecting nodes locations.
4. A line-fitting algorithm is run on the resulting set of points.

In [9], the authors investigate the potential of gateway repositioning for enhanced network performance in terms of energy, delay, and throughput. The paper addresses issues related to when the gateway should be relocated, where it would be moved to, and how to handle its motion without negative effect on data traffic. The paper presents two approaches that factor in the traffic pattern for determining a new location of the gateway for optimized communication energy and timeliness, respectively. The gateway movement is carefully managed in order to avoid packet losses.

The authors in [10] propose deploying multiple, mobile base stations to prolong the lifetime of the sensor network. The lifetime of the sensor network is split into equal periods of time known as rounds. Base stations are relocated at the start of a round. The method uses an integer linear program to determine new locations for the base stations and a flow-based routing protocol to ensure energy-efficient routing during each round. The paper proposes four metrics and evaluates the solution using those metrics. Based on the simulation results the paper shows that employing multiple, mobile base stations in accordance with the solution given by the schemes would significantly increase the lifetime of the sensor network.

IV. ENERGY-EFFICIENT TARGET MONITORING

As observed in our background survey, target tracking algorithms generally emphasize on optimizing accuracy of the target positions by reducing the difference between the actual paths taken by the target and the estimated positions obtained by applying different algorithms. However, when using an energy constrained sensor network, ensuring increased network lifetime is very important. Thus, the motivation behind our research is to propose an approach that minimizes the energy overhead in transmitting the data to the base station. We propose two policies to achieve energy-efficient target monitoring. Our first policy involves having a mobile base station and moving it to the centroid of the sensors detecting the target. This significantly reduces the energy consumed in transmitting the data to the base station. However, moving the base station frequently involves considerable energy overhead and therefore we observe the effects of moving the base station after several time period values as opposed to each time period. Our second policy for energy-efficient target monitoring is to move the base station to the centroid of the base-station locations obtained over several time period values. The traffic pattern of target arrival is crucial in both the cases and therefore we evaluate the performance of the above policies under two different traffic patterns, uniform and bursty. We simulate both the energy-efficient target monitoring policies and observe the effects of traffic pattern on network lifetime in each case.

We develop a framework to model energy-efficient target monitoring policies. The following are the important assumptions and parameters.

Assumptions

- Uniformly distributed random network topology with N nodes and a single base station.
- Nodes are static and the base station is mobile.
- t: denotes the time period in units. The lifetime of the sensor network is divided into equal periods of time known as time periods. The sensor network is considered to be alive as long as the sensors detecting the target have sufficient energy to transmit the data to the base station.

Parameters

- $P_o(x, y)$: denotes the initial location of the base station given by the centroid of all the nodes in the network.
- $P_i(x, y)$: denotes the *centroid location* (in CTS) and the *centroid of centroid locations* (in CBS) of the base station for time period t_i.
- E_d^i: represents the total energy dissipated to transmit the data to base station location $P_o(x, y)$ for time period t_i.
- E_c^i: represents the total energy dissipated to transmit the data to the *centroid location* (in CTS) and the *centroid of centroid locations* (in CBS) of the base station, $P_i(x, y)$ for time period t_i.
- E_N: denotes the *energy dissipated/time period* and is the energy required to transmit data to base station location $P_o(x, y)$ from target detecting Node N for time period t_i.
- E_N^c: denotes the *centroid energy* and is the energy required to transmit the data to the *centroid location* (in CTS) and the *centroid of centroid locations* (in CBS) of the base station, $P_i(x, y)$ from target detecting Node N for time period t_i.
- Δ: denotes the *relocation energy threshold (RET)*. RET

is the energy required to relocate the base station to the optimal location.

- The routing table is setup for all the nodes in the network using Dijkstra's shortest path routing algorithm [11].

To ensure a fully connected network we develop a Hamiltonian path connecting all the nodes in the network. A Hamiltonian path is a path in an undirected graph that visits every vertex exactly once [12]. The initial location of the base station is then calculated as the centroid of the polygon, where the nodes act as the vertices of the polygon. The centroid (c_x, c_y) is also known as the center of gravity or the center of mass and is calculated using the following formula [13],

$$c_x = \frac{1}{6A} \sum_{i=0}^{N-1} (x_i + x_{i+1})(x_i y_{i+1} - x_{i+1} y_i) and \tag{1}$$

$$c_y = \frac{1}{6A} \sum_{i=0}^{N-1} (y_i + y_{i+1})(x_i y_{i+1} - x_{i+1} y_i), \tag{2}$$

where A denotes the area of the polygon with N vertices. The area of the polygon is calculated using the formula,

$$A = 1/2 \sum_{i=0}^{N-1} (x_i y_{i+1} - x_{i+1} y_i). \tag{3}$$

Our first policy for network lifetime optimization involves relocating the base station to the centroid of the sensors detecting the target. At each time period, the *energy dissipated/time period* is calculated as the energy required to transmit the data to the base station from each of the nodes detecting the targets. Then, the centroid of the nodes detecting the targets is calculated, and base station is assumed to be positioned at that location. This location is termed as the *centroid location*. Now, the energy required for transmitting the data from the same set of nodes to the centroid location is calculated. We term this energy as the *centroid energy*. If the *energy dissipated/time period* is greater then the sum of *centroid energy* and *RET* required to move the base station, the base station is relocated, otherwise it is not. But the task of actually relocating the base station is done at the start of the next time period. This policy is termed as *centroid of target detecting sensors (CTS)*. CTS gives an increased network lifetime as at each time period we decide on relocating the base station. However, relocating the base station at each time period involves a considerable overhead and therefore we also consider relocating base station after several time periods.

The second policy for network lifetime optimization is relocating the base station to the centroid of the base station locations obtained over several time periods, where, at each time period the base station location is computed as the centroid of the sensors detecting the targets. We term this policy as *centroid of base station locations (CBS)* as we compute the centroid of the base station locations obtained over a period of time periods. The base station location obtained is termed as the *centroid of centroid locations*. However, for t=1 unit this policy works in the same manner as *CTS*. In case of *CBS* as well, after several time periods, we make a decision whether to relocate the base station to the *centroid of centroid locations* depending on the *energy dissipated/time period* and the *centroid energy* calculations. We simulate the two traffic patterns for the target arrivals, uniform and bursty. In case of the uniform traffic pattern, we consider a uniform load at each time period. In other words, a specific number of nodes detect the target at each time period (contributing the load), but the nodes to be used in target detection are selected randomly. The bursty traffic pattern models a bursty load, i.e., the load for target detection is randomly selected at each time period. Then we take an average of the loads at each time period and simulate the network for the resultant load value. We simulate each of these policies with both the traffic patterns and observe the effects.

A. Centroid of Target Detecting Sensors (CTS) Policy

1) Initialization:
    a) Compute the initial location of the base station, $P_o(x, y)$ as the centroid of all the nodes in the network.
    b) Establish the routing table using Dijkstra's routing.
2) For each time period, t_i do
    a) Calculate the energy dissipated/time period, $E_d^i = \sum_{n=1}^{N} E_n$.
    b) Calculate the *centroid location* of the base station, $P_i(x, y)$ as the centroid of the target detecting sensors.
    c) Calculate the *centroid energy*, $E_c^i = \sum_{n=1}^{N} E_n^c$.
    d) If $(E_d^{i-1} - E_c^{i-1}) > \Delta$ then relocate base station to the *centroid location*, $P_{i-1}(x, y)$ otherwise do nothing.
3) Repeat $Step$ 2 until the network fails.

B. Centroid of Base Station Locations (CBS) Policy

1) Initialization:
    a) Compute the initial location of the base station, $P_o(x, y)$ as the centroid of all the nodes in the network.
    b) Establish the routing table using Dijkstra's routing.
2) For each time period, t_i do
    a) Calculate the energy dissipated/time period, $E_d^i = \sum_{n=1}^{N} E_n$.
    b) Calculate the *centroid of centroid locations* of the base station, $P_i(x, y)$ as the centroid of the base station locations obtained over several time periods.
    c) Calculate the *centroid energy*, $E_c^i = \sum_{n=1}^{N} E_n^c$.
    d) If $(E_d^{i-1} - E_c^{i-1}) > \Delta$ then relocate base station to the *centroid of centroid locations*, $P_{i-1}(x, y)$ otherwise do nothing.
3) Repeat $Step$ 2 until the network fails.

C. CTS and CBS Illustration

To better explain the policies let us consider an example for illustration. We have considered a uniformly distributed random network of 10 nodes and a mobile base station. The

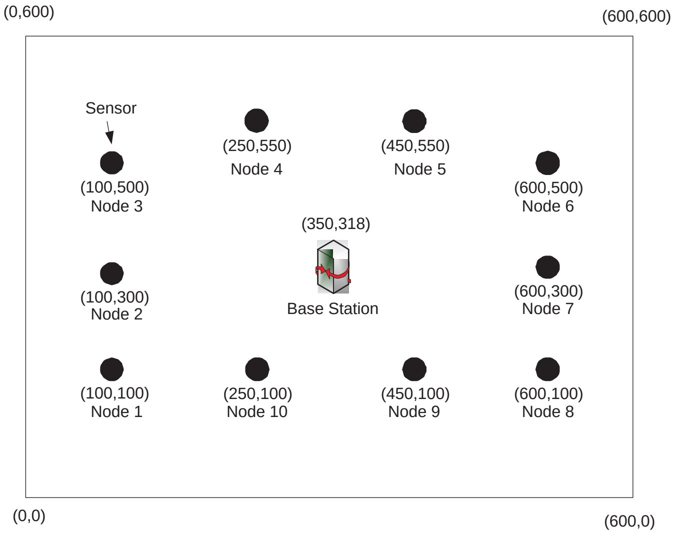


Fig. 1. Network Topology.

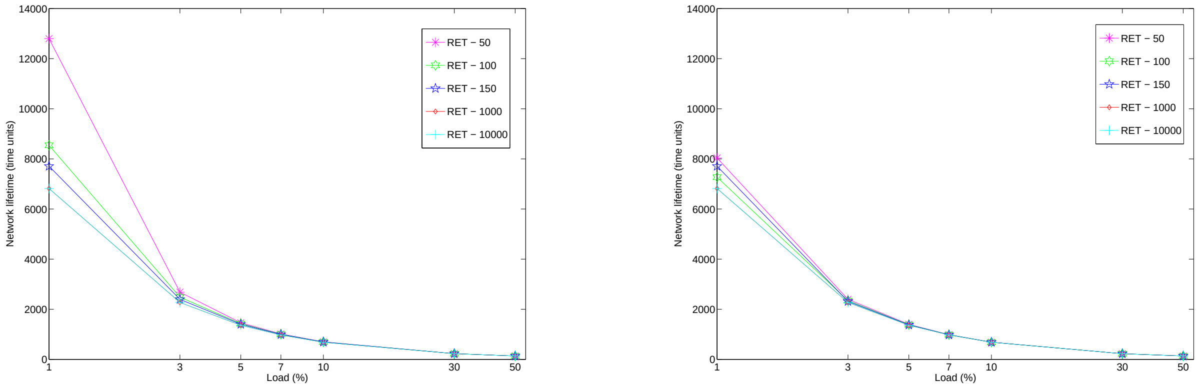
network area is 600×600 m^2 and the transmission range is considered to be 250 m. The total energy in the network is 3000 KJ and the *RET* necessary to relocate the base station is 50 J/m. We have used an uniform load of 20% and a time period t=2 units.

The arrangement of the nodes is as shown in the Fig. 1. The routing table is setup for the network using Dijkstra's shortest path routing algorithm. The initial base station location obtained as the centroid of all the nodes in the network is (350, 318). Now, let us consider the *CTS policy* where the resultant base station location is computed as the centroid of the sensors detecting the targets. At every alternate time period the resultant base station location, the *energy dissipated/time period*, and the *centroid energy* are computed. If the *energy dissipated/time period* is greater than the sum of the *centroid energy* and *RET* for relocating the base station, then the base station is relocated otherwise it is not relocated. This process is repeated at every alternate time period. In this example, we have considered a uniform load of 20% and therefore only 2 nodes (out of 10) detect the target at each time period. In the first time period, Node 2 and Node 10 are randomly selected to detect a target. The *energy dissipated/time period* is computed as the energy required for sending the data from these nodes to the present location of the base station, i.e., (350, 318). The *energy dissipated/time period* for the first time period is 177.7 KJ. As we have considered t=2 units, we make a decision for the base station relocation only at every alternate time period. Therefore, for the first time period we use the *energy dissipated/time period*. Now, in the second time period Node 2 and Node 8 are used to detect a target. At this time period, we compute the *energy dissipated/time period* and the *centroid location* of the base station as the centroid of the Node 2 and Node 8. The *energy dissipated/time period* for this time period is 200.2 KJ and the *centroid location* is (350, 200). The *centroid energy* is computed as the energy required for transmitting data to *centroid location* of the base station from these set of nodes, resulting in 124.76 KJ. Now, the *energy dissipated/time period* is greater then the sum of the *centroid energy* and the *RET* for relocating the base station, which in this case is 5.9 KJ and therefore we decide to relocate. *RET* for each time period is calculated as the euclidian distance between the present location and the *centroid location* of the base station multiplied by *RET*. However, the base station is actually relocated to the *centroid location* in the next time period in the *CTS policy*. This process is continued and we observe through simulation that the network fails at 16^{th} time period.

Now for the same network, we observe the *CBS policy*. It is intuitive to note that *CBS* works in the same manner as *CTS* for t=1 unit. This is because for t=1 unit we relocate the base station at each time period. Therefore, in case of *CBS*, the resultant base station location gets computed as a *centroid location* rather than *centroid of centroid locations*. In case of *CBS*, the centroid is computed as the centroid of the base station locations obtained over several time periods (in this case, 2 time periods), where at each time period the base station location is computed as the centroid of the sensors detecting the targets. In the first time period, again Node 2 and Node 10 are randomly selected to detect a target and the *energy dissipated/time period* is calculated as 177.7 KJ and the *centroid location* for the first time period is the centroid of the Node 2 and Node 10 and is computed as (175, 200). Again, as we have considered t=2 units we make a decision for the base station relocation only at every alternate time period. Therefore, for the first time period we use the *energy dissipated/time period*. In the second time period, Node 2 and Node 8 are randomly selected to detect a target. The actual energy is calculated as 200.2 KJ and *centroid location* for this time period is (350, 200). Now, the *centroid of centroid locations* is computed as the centroid of these two *centroid locations* and the *centroid energy* is the energy required for transmitting the data from Node 2 and Node 8 to this *centroid of centroid locations*. The *centroid energy* comes out to be 103.93 KJ and the *RET* is 7.33 KJ. Clearly, the *energy dissipated/time period* for this time period is greater than the *centroid energy* and the *RET* and so we relocate the base station in the next time period. We continue this process and observe through simulations that for this policy as well, the network fails at 16^{th} time period. Thus, both the policies provide optimized network lifetime. However, in some cases *CTS policy* is better than *CBS policy* because *CTS* computes centroid of the sensors actually detecting the targets. This yields a more accurate resultant base station than *CBS*, which computes the centroid of the base station locations over several time periods.

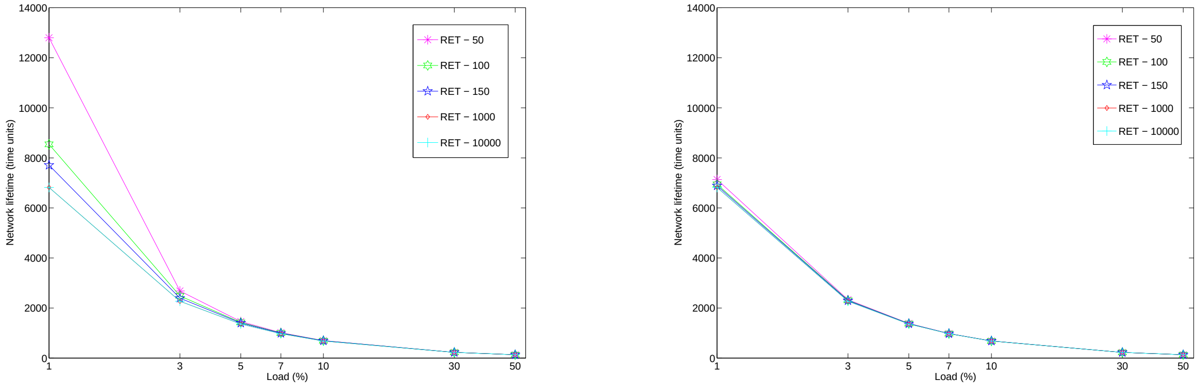
V. SIMULATION RESULTS

To evaluate the two proposed energy-efficient target monitoring policies we create a network. We create a uniformly distributed network of 100 nodes and a mobile base station with full network connectivity. The network area is 500×500 m^2 and the node transmission range is 170 m. The total energy in he network is assumed to be 165000 KJ. We run simulations for relocation time periods of 1, 3, 5, 7, 10, 1000, and 10000 until the network fails. We also look at different *RET*s for relocating the base station. The simulations are run for *RET*s



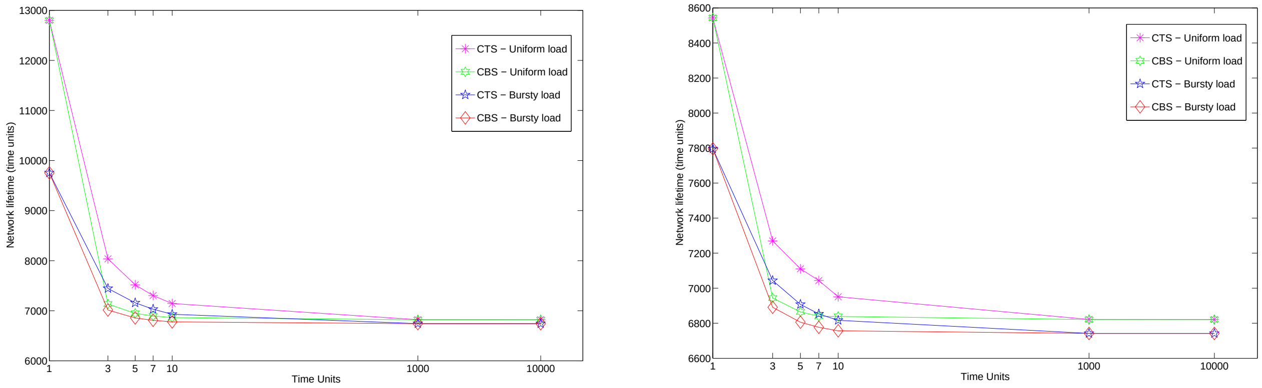
(a) (b)

Fig. 2. CTS network lifetime vs. uniform load (a) for time period=1 and (b) for time period=3.



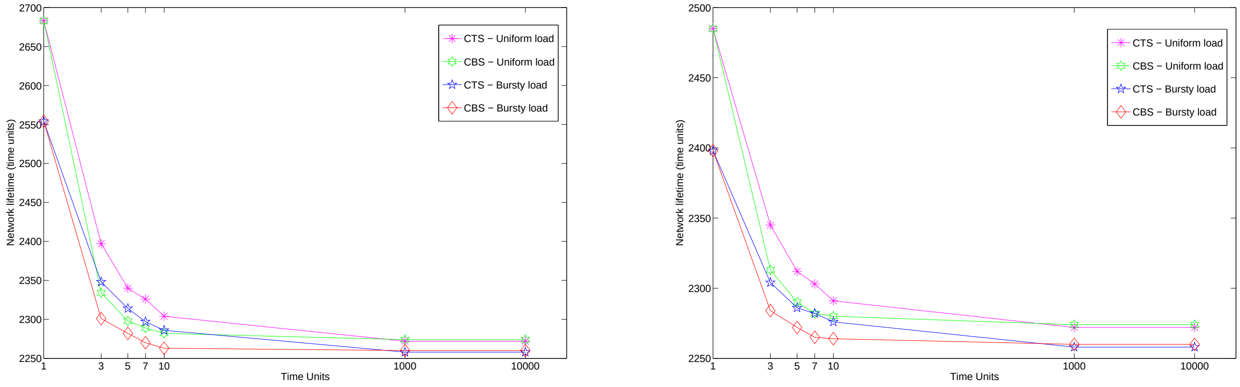
(a) (b)

Fig. 3. CBS network lifetime vs. uniform load (a) for time period=1 and (b) for time period=3.



(a) (b)

Fig. 4. CTS and CBS comparison under 1% load with (a) *RET* = *50* J/m and (b) *RET* =*100* J/m.



(a) (b)

Fig. 5. CTS and CBS comparison under 3% load with (a) *RET* = *50* J/m and (b) *RET* =*100* J/m.

of 50, 100, 150, 1000, and 10000. We also consider uniform and bursty traffic at different loads for both the policies. We consider loads of 1%, 3%, 5%, 7%, 10%, 30%, and 50%.

In case of *CTS*, the resultant base station location is obtained as the centroid of the sensors detecting the targets. We observe through simulations that the highest network lifetime is obtained for t=1 unit, i.e., when at each time period we make a decision whether to relocate the base station or not. However, relocating the base station at each time period involves a considerable overhead and therefore we run simulations for different time period values. As we increase the time period values the network lifetime decreases, since we no longer consider optimal base station location at each time period.

Figure 2 represents *CTS* under uniform load for t=1 unit and t=3 units. We observe that network lifetime is higher for t=1 unit and keeps decreasing with increasing time period values. When $t-3$ units, the decision to relocate the base station is not taken at every time period and therefore the network energy is not used optimally at each time period. Fig. 2 indicates 59.26% improvement in network lifetime under uniform load of 1%, when the decision to relocate the base station is taken at every time period ($t = 1$ unit) as compared to $t = 3$ units. We observe that the network lifetime decreases with increasing *RET* values. We also observe that network fails earlier at high loads when compared to low loads because in the former case, at each time period, the amount of traffic generated is very high.

Figure 3 represents *CBS* under uniform load for $t = 1$ unit and $t = 3$ units. We observe that *CTS* performs better than *CBS* under uniform loads because *CTS* yields a more accurate resultant base station location than *CBS*. In the case of *CTS* the resultant base station location is computed as the centroid of the sensors actually detecting the target, whereas in case of *CBS* the base station location is computed as the centroid of the base station locations obtained over several time periods.

Fig. 4 and Fig. 5 compares *CTS* and *CBS* under uniform and bursty load of 1% and 3% and *RET*s of 50 J/m and 100 J/m. We observe that *CTS* under uniform load provides the highest network lifetime, since *CTS* computes the resultant base station location as the centroid of the sensors actually detecting the target under uniform traffic pattern at each time period. *CTS* under bursty load also provides for an improved network lifetime when compared to *CBS* under uniform and bursty loads. The reason being *CBS* provides the resultant base station location as the centroid of the base station locations obtained over several time period values which is less accurate than *CTS*. We observe that as the time period values increases the network lifetime decreases. Also, as the *RET* increases, the network lifetime decreases.

We have restricted the explanation of our results to uniform and bursty loads of 1% and 3%, *RET*s of 50 J/m and 100 J/m and time periods of 1 unit and 3 units. We have actually simulated our network for time period values of 1, 3, 5, 7, 10, 1000, and 10000. We have also considered load values of 1%, 3%, 5%, 7%, 10%, 30%, and 50% under *RET* values of 50, 100, 150, 1000, and 10000. The results obtained are consistent with the ones presented in this paper.

VI. CONCLUSION

In this paper, we have address the issue of energy-efficient target monitoring using wireless sensor networks. We proposed two policies for energy-efficient target monitoring. Simulation results show that *centroid of target detecting sensors (CTS)* policy provides improved network lifetime. The CTS policy relocates the base station to the centroid of the sensors detecting the targets. Relocating the base station after several time periods to the centroid of the sensors detecting the targets also provides for an increased network lifetime. However, in case of *centroid of base station locations (CBS)* policy, the network lifetime reduces when we move the base station to the centroid of the base station locations obtained over several time period values. This is because the resultant base station location obtained from the centroid of the base station locations over several time period values is not accurate when compared to the base station location obtained from the centroid of the sensors actually detecting the targets. We have also observed that the network lifetime reduces as we increase the *relocating energy threshold* for moving the base station.

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