Analytical Blocking Probability Model for Hybrid Immediate and Advance Reservations in Optical WDM Networks

Joan Triay, *Member, IEEE*, Cristina Cervelló-Pastor, and Vinod M. Vokkarane, *Senior Member, IEEE*

***Abstract*—Immediate reservation (IR) and advance reservation (AR) are the two main reservation mechanisms currently implemented on large-scale scientific optical networks. They can be used to satisfy both provisioning delay and low blocking for delay tolerant applications. Therefore, it seems reasonable that future optical network provisioning systems will provide both mechanisms in hybrid IR/AR scenarios. Nonetheless, such scenarios can increase the blocking of IR if no quality of service (QoS) policies are implemented. A solution could be to quantify such blocking performance based on the current network load and implement mechanisms that would act accordingly. However, current blocking analytical models are not able to deal with both IR and AR. In this paper, we propose an analytical model to compute the network-wide blocking performance of different IR/AR classes within the scope of a multi-service framework for optical WDM networks. Specifically, we calculate the blocking on two common optical network scenarios using the fixed-point approximation analysis: on wavelength conversion capable and wavelength-continuity constrained networks. Performance results show that our model provides good accuracy compared to simulation results, even in a scenario with multiple reservation classes defined by different book-ahead times.**

***Index Terms*—Analytical model, immediate reservation, advance reservation, WDM, optical networks.**

I. Introduction

THE growing demand of network bandwidth is not restricted to happen only on the Internet. Recently, we have also seen a number of initiatives fostering the development of high-capacity optical networks to support large-scale scientific experimentation. Examples are the European GÉANT network [1] and the US Energy Sciences network (ESnet) [2]. The purpose of these networks is to enable the transport of data generated by large-scale experiments, such as those from the Large Hadron Collider Computing Grid Project. These research facilities generate huge amounts of data from live experiments. Usually, this data needs to be processed or stored elsewhere demanding very high-bandwidth capacity connections or virtual circuits for a specified duration. These connections are commonly handled over optical wavelength-division multiplexing (WDM) networks by creating *lightpaths*. The joint allocation of Grid and network resources is known as *resource co-allocation*. The interconnection of Grids over optical networks is also referred as Lambda Grids [3].

The connection requests generated by these scientific applications can be classified as either delay-sensitive or delay-tolerant. Additionally, with respect to their provisioning, applications can also be tolerant or sensitive to connection blocking. For instance, immediate reservation can be used by delay-sensitive applications that require immediate service provisioning but at the expense of higher blocking. In contrast, other services like video-conference, Grid computing, off-site backup, and e-Health tele-surgery are tolerant to delay as long as the resources are provisioned before a deadline. In the latter case, allocation of network resources is realized at a "future" time, typically after a book-ahead time. This book-ahead improves the service allocation probability of delay-tolerant applications when considering the co-existence of IR and AR due to the greater probability of having idle resources at a future time. Furthermore, enhanced AR mechanisms, such as flexible start times or batch processing of requests [4] can further help improve the resources utilization in the network.

Under this scenario, we foresee that the aforementioned applications will need to be handled by a mixture of immediate and advance reservations, thus requiring effective frameworks for handling IR/AR co-existence. Most existing studies in this area [5]–[7] have analyzed the problem but only with two classes sharing the same network resources: one IR and one AR. We believe that the number of reservation classes in a real scenario could be greater than in the aforementioned studies. In this paper, we define a reservation class as the group of connection requests with a specific book-ahead time. Recall that for immediate reservation the book-ahead is always zero.

Computing and predicting the expected blocking probability before processing an incoming connection could be a useful tool. Network operators might use such a mechanism to trigger customized policies and reduce blocking of higher priority requests. The majority of blocking models for all-optical networks assume the connections start as soon as they are processed by the network provisioning controller. As such, these can effectively be used to analyze IR blocking. However, they are not suitable for AR because future bandwidth availability information is not present in common Erlang loss models.

In this paper, we propose an analytical model to calculate the network-wide blocking probability of hybrid immediate and advance reservation co-scheduling in optical WDM networks. As well as this, we formulate the analysis for two

First submitted on Oct. 16, 2011. Accepted on Dec. 11, 2012.

Portions of this paper appeared in IEEE GLOBECOM 2011.

Joan Triay and Cristina Cervelló-Pastor are with the Dept. of Telematics Engineering at Universitat Politècnica de Catalunya (UPC), Esteve Terradas 7, 08860, Castelldefels, Spain. Joan Triay was also with the Dept. of Computer and Information Science, University of Massachusetts, Dartmouth, MA, USA. E-mail: joan.triay@ieee.org, cristina@entel.upc.edu

Vinod M. Vokkarane is with the Dept. of Computer and Information Science, University of Massachusetts, Dartmouth, MA, USA. E-mail: vvokkarane@ieee.org

common scenarios: (a) wavelength conversion (WC) capable networks and (b) wavelength-continuity constrained (WCC) optical networks. In comparison with past works, our proposal is able to flexibly model the network resources sharing between immediate and advance reservations by taking into account future state transition probabilities. We assess the results of our model under different traffic scenarios and number of wavelengths, and evaluate its correctness against simulation results on two different network topologies. Overall, results demonstrate the feasibility of the model to predict the blocking probability regardless of the complexity of the number of reservation classes and IR/AR parametrizations. Furthermore, the results corroborate the theory that under a scenario without class prioritization, the greater the book-ahead of the class, the lower its blocking probability.

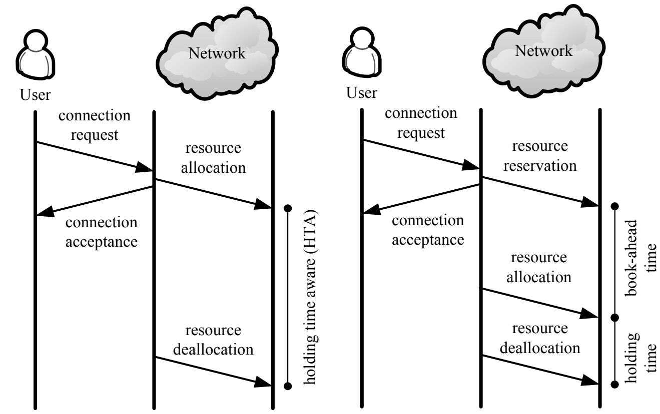
The remainder of this paper is as follows: Section II introduces immediate and advance reservations in electronic and optical networks and points out some of their applications. In Section III we review the state of the art related to this paper. Section IV introduces the network model assumptions and later we describe the proposed IR/AR blocking probability analytical model in Sections V and VI. Results are shown in Section VII, and finally, Section VIII concludes this paper.

II. Background: Immediate and Advance Reservations, and Their Applications

In immediate reservation, the resource provisioning and allocation for the connection request starts as soon as the call arrives into the system (refer Fig. 1(a)). If the reservation is successful, then the resources are allocated and the user/client is positively acknowledged to start transmitting data.

We can consider two general IR request types: IR with and without specified duration. The former does not specify the holding time, thus the connection uses the network resources until the tear-down request is explicitly sent by the user or upper-layer application. The latter does provide the duration, and it is also known as holding-time-aware (HTA) IR. Fig. 1(a) shows an example wherein the holding time is announced, so the provisioning system can deallocate the resources without an explicit tear-down request. In this work, we assume that IR requests always specify their holding time [8]. This assumption holds true for a great number of applications, especially in the field of large-scale experimentation. For instance, from the size of the experimentation data and the bandwidth provided per wavelength, we can compute the connection duration necessary to transmit the data set.

In general, advance reservation allows the allocation of bandwidth to start after a book-ahead time (see Fig. 1(b)). In this case, the reservation blocking is resolved at the connection arrival thanks to the knowledge that the centralized provisioning scheduler has about the future availability of network resources. It is worth noting that IR can also be treated as a special case of AR, but with a zero book-ahead. Commonly, AR requests announce their expected holding time. Requests that specify a start time and duration are denoted as specified-start specified deadline (STSD). Another type is flexible AR, wherein the start time of the resource allocation is flexible as long as it fits within a specified time window and before a maximum deadline. In spite of the benefits of flexible AR, the model under evaluation only handles IR and fixed AR as many of the applications, especially in scientific Grid environments, require strict start and end reservation times. For a more exhaustive classification of AR, we refer to [9].



(a) Immediate reservation. (b) Advance reservation.

Fig. 1. Reservation types.

AR is conceptually similar to the offset approach of optical burst switching (OBS) with just-enough-time (JET) [10]. Nonetheless, the latter uses a distributed per-hop reservation in comparison to the widely used centralized reservation approach in wavelength-routed WDM networks. Moreover, the offset time in OBS greatly depends on the optical-cross connection time and the number of nodes along the route, whereas the book-ahead time of AR is usually specified by the user or the service-to-network interface. One last significant difference is the duration of the reservation, in the order of milliseconds for a burst, and of minutes, hours or even days in the case of wavelength-routed optical networks.

Grid and large-scale experimentation applications can benefit from advance reservation. It is often the case that the connection duration is known in advance due to the task-driven traffic generated by such applications. Also, many Grid applications involve delay-tolerant background or recurring tasks. A concrete example is Fusion Energy Sciences (FES) research. Magnetic fusion experiments operate in a pulsed-mode where in any given day, between 25 and 35 plasma 10-second long pulses are taken on 10 to 20 minute intervals. Throughout the experiment, control adjustments are made by the experimental facility in order to decide on the next pulse parametrization. This process requires sending out the data over the Grid to be stored and processed. Such recurrent connections can be submitted as AR requests.

Network provisioning frameworks, such as On-demand Secure Circuits and Advance Reservation System (OSCARS) [11], Dynamic Resource Allocation Controller (DRAC) [12] or G-Lambda [3] have immediate and advance reservation capabilities. OSCARS, which is currently in production and supporting the setup of layer 2/3 virtual connections in the U.S. ESnet network, incorporates a resource manager that stores information about all current and future connections for both IR and AR requests.

III. Related Work and Contributions

The analysis of the blocking probability in all-optical networks was initiated by Birman in [13]. The author introduced a model to compute the immediate reservation blocking probability on wavelength-routed optical networks using a generalized reduced-load approximation for two routing schemes: fixed routing and least-loaded routing. On the foundations of this work, the authors of [14] presented an analytical model using inclusion-exclusion principle of combinatorics to evaluate the performance of all-optical WDM networks with no wavelength conversion. Their aim was to cope with the significant link load correlation under wavelength-continuity constraint, especially on topologies like optical rings. The authors also evaluated their model for fixed-alternate path and least-loaded routing. Again, the authors assumed all connection requests are reserved and provisioned as the request arrives. More recently, the authors in [15] also developed a fixed-point approximation algorithm to compute the approximate blocking probabilities on multi-class WDM networks. Each class is defined by its required number of wavelengths and expected holding time. No advance reservation was considered by the authors.

One of the first works to provide some analytical results on resource sharing (co-scheduling) for book-ahead and instantaneous connection requests was [6]. Greenberg et al. introduced an admission control algorithm in which a call is admitted if an approximate interrupt probability is below a threshold. In their model the authors assume that time-scales between IR and AR are widely separated and that the IR holding time is much shorter than the book-ahead time of AR. However, this might not be always the case. Therefore, their model lacks the ability to consider diverse reservation classes with different book-ahead times. Similarly, the authors of [16] proposed an analytical model to remark the benefits of book-ahead bandwidth-sharing when compared to immediate reservation call-blocking. Such an analysis is realized by means of a discrete-time Markov chain (DTMC) model. However, due to the complexity of their model in terms of programmability, the authors provide analytical results only for one channel, thus making its application unfeasible on true WDM scenarios. Additionally, the authors consider a single-link analysis, whereas our model computes the network-wide blocking probability. Finally, [17] introduces a model to compute the blocking probability in optical burst-switched networks. Although it is applied to the sub-wavelength case (vs. full-wavelength), the model interestingly introduces knowledge about future connection arrival slot distributions and state transition probabilities. Nevertheless, [17] considered a single-link analysis, whereas our contribution goes beyond it by extending the analysis to the network-wide blocking computation and completing the analysis with greater number of wavelengths, while also overcoming some of the limitations of other referenced works, like the inability to handle diverse IR/AR classes.

IV. Network Model

We assume a multi-layer application-aware framework with a centralized network resource provisioning system. The centralized model is extensively used, especially in hybrid immediate and advance reservation capable optical Grid networks. Production networks like ESnet [11], and others devised in recent projects [18] make use of this approach. Although multi-domain features can also be defined, within each domain the centralized approach is the norm.

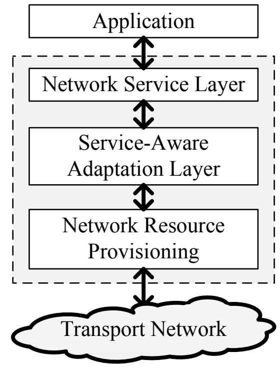


Fig. 2. Multi-layer application-aware service framework.

In the proposed application-aware service framework, requests are handled by the network service layer (see Fig. 2). This layer is responsible for translating the request into a proper network service setup call. The call is then forwarded to the service-aware adaptation layer module in order to be mapped onto an existing reservation class according to the delay constraints and the book-ahead time. At the end of this stage, connection requests are forwarded to the network provisioning system with a specific book-ahead corresponding to one of the available reservation classes.

The network under consideration in this analysis can be represented by a graph $G = (V, E, W, H)$, where V is the set of network nodes, E represents the links interconnecting the nodes, and W stands for the number of wavelengths per link. H stands for the time horizon of the resource's state information (future availability) held by the centralized network provisioning system. Moreover, we also assume that time is divided into time-slots, and applications request to set up a lightpath between the source and destination node for a certain duration or number of time-slots. This assumption is reasonable in optical circuit-switched networks where connections are active for a specified time period of seconds, minutes or even hours. The size of the time-slot is not determinant for the present analysis. However, its size needs to be considered in real implementations in order to satisfy physical device specifications (e.g., optical cross-connect switching time, configuration delay, etc.) and increase channel utilization given an average connection duration.

In this paper, we assume a first-fit slot and wavelength assignment (FF-SWA) policy. In FF-SWA, the first wavelength in increasing index order with enough free slots to allocate the connection request is assigned and reserved. Fig. 3 illustrates an example using FF-SWA. Let the book-ahead time and duration of the connection be defined by α and τ, respectively. From α, we can compute the starting slot for this connection as $t_a = t_{now} + \alpha$, where t_{now} denotes the current arriving slot of the connection request. With this information, FF-SWA allocates the lowest index wavelength that fits the connection request duration. In the example, this is λ_3. With current connection scheduling knowledge, first-fit performs better than

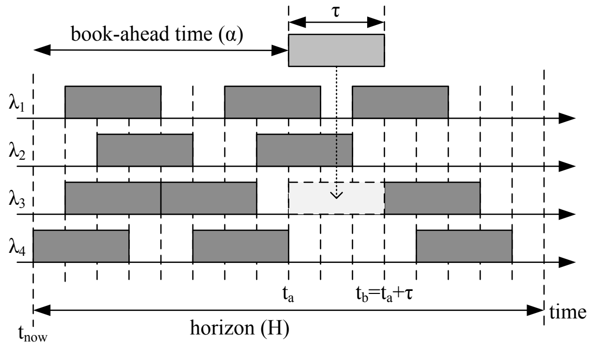


Fig. 3. First-fit slot and wavelength assignment (FF-SWA) policy.

random, because in the former case, capacity usage is packed across fewer wavelengths, which leaves more wavelength scheduling options under continuity constrained scenarios. The description of other scheduling algorithms is out of the scope of this work, but references can be found in [19].

Also related to the wavelength assignment, we propose two blocking probability models. Firstly, we assume wavelength conversion between an input and output link at intermediate optical cross-connects. This allows a connection to be reserved a lightpath that can make use of a different wavelength at every link along the path. Secondly, we propose a model for the wavelength-continuity constrained network, i.e., network nodes do not have wavelength converters, hence the same wavelength must be used on all the links between the source and destination nodes.

A. Problem Statement, Assumptions, and Notation

The immediate/advance reservation system to be analyzed in this paper is characterized as follows:

- We have an all-optical network which is represented by a graph $G = (V, E, W, H)$.
- We assume time is divided into time-slots of width δ and connections request for a multiple number of them.
- A list of pre-computed paths between source-destination node pairs. For simplicity, we assume a single fixed shortest-hop route computed using Dijkstra's algorithm over a non-hierarchical network graph. The route between the source s and destination node d is denoted by $r(s, d)$. The hop length of the route is symbolized by $h(r)$.
- A set of IR/AR reservation classes $C = \{c\}_i$. Every connection request is defined by: (a) the book-ahead time α, and (b) the mean service holding time τ. We assume that α and τ are also integer multiples of δ.
- A stochastic request arrival process. We model connection arrivals to be a Poisson process with mean arrival rate per time-slot and per class λ_c. In the analysis, the total offered load to the network is uniformly distributed among source-destination pairs. The arrival rate between a source and destination is denoted by $\lambda_c^{s,d}$. Finally, the connection arrival rate on to a link j is designated by λ_c^j.
- We also assume that the holding time is geometrically distributed based on the time-slot assumption.
- For simplicity, we assume the Poisson arrivals see time averages (PASTA) property [20]. PASTA defines that the probability of the state as seen by an external random observer is the same as the probability of the state seen by an arriving connection. This is not strictly true because of the assumed slotted approach which states that connections start at slot boundaries [17].

We can formally define the problem as follows.

Definition Given a network, $G = (V, E, W, H)$ and the current state of the network resources denoted by $U[E, W, H]$. Let an incoming request $R_i = (\alpha_i, \tau_i)$, where α_i represents the book-ahead time and τ_i the holding time (i.e., duration), we must allocate the time-slots if the connection can be granted (i.e., it does not overlap with any existing reserved interval on the path), otherwise block the request.

V. IR/AR Analytical Loss Model

In order to differentiate delay-sensitive and delay-tolerant connection requests and to calculate the blocking probability in an IR/AR hybrid scenario, we need to take into account the temporal evolution of the system's state. As introduced previously, most blocking analytical models make the assumption that all requests need to be allocated and reserved immediately upon arriving. Obviously, this is not suitable for AR connections which request resources "in the future".

Three are the main works that set the grounds of the present mathematical analysis over which we develop our main contribution, i.e., a network-wide blocking model for hybrid IR/AR: the Markov chain with transitional probabilities analysis presented in [17], the network-wide analysis from [15], and the reduced-load Erlang fixed-point approximation in [21].

A. Link Blocking Analysis

We start by deriving the link blocking analysis. The blocking computed here will be later used to calculate the network-wide blocking probability for the hybrid IR/AR scenario. For notational convenience, we will use α and τ to generalize for any IR/AR class.

1) Traffic Arrival Process and State Probabilities: In order to process different IR and AR connection requests, we define the random variable A with a probability distribution $f(\alpha) = \Pr(A = \alpha)$, for $\alpha \geq 0$. The probability function determines the probability that a certain arriving connection request has a book-ahead of α. We make use of the index (n) to denote the state of a particular random variable at slot n.

To compute the arrival process of a connection at a certain time instant requesting for bandwidth from the n-th time-slot, we define the number of requests whose start time is at slot n to be Poisson distributed with mean,

$$\lambda_{(n)} = \begin{cases} \lambda & \text{for } n \leq 0 \\ \lambda\left(1 - \displaystyle\sum_{\alpha=0}^{n-1} f(\alpha)\right) & \text{for } n > 0. \end{cases} \tag{1}$$

First part of (1) shows that the sum of requests made from all previous slots for current time-slot $n = 0$ is Poisson with rate λ [22], i.e., the mean arrival rate of the system seen by an immediate request is λ. For $n > 0$, that is, for a given arrival at time-slot 0 (the present time-slot) the probability to request starting time-slot n (in the future) is $f(n)$, and as a result, the process rate for time-slot n is $\lambda f(n)$. This shows that from the point of view of a reference connection arriving to the system, the number of requests for time-slot n is the sum of requests made in the past in addition to requests made at current time-slot that demand a reservation starting at time-slot n. In order words, we only need to consider the traffic load for slot n seen at the time of the request. This is because any reservation request that will be generated in the future for time-slot n (i.e., $0 \leq \alpha \leq n-1$) will not affect the probability of accepting/blocking the reference reservation request. Therefore, the latter is defined by discarding from the total arrival load the connection requests to be made from time-slot 1 to n for time-slot n.

In this work, we are interested in computing the blocking probabilities when a set of pre-defined IR and AR classes with specific book-ahead times are defined by the service-aware adaptation layer (see Fig. 2). Given this assumption, we can also assume that the book-ahead time is a constant value for each class as it is also used in [6]. To simplify the notation, we will use indistinctly $\lambda_{(n)}$ or λ_c to define the arrival rate of a specific class c. Moreover, we assume there is no special priority treatment among the different classes, hence the arrival time and book-ahead time are the only class differentiating parameters. That is, there is no special QoS policy to differentiate classes.

For instance, let define three reservation classes $c \in C = \{IR0, AR0, AR1\}$ with their respective book-ahead times: α_{IR0}, α_{AR0}, and α_{AR1}. Also, let us consider $\alpha_{IR0} = 0$ (i.e., IR requests have zero book-ahead time) and $\alpha_{IR0} < \alpha_{AR0} < \alpha_{AR1}$. Moreover, assume that the input traffic ratio is distributed as 50%, 25%, and 25%, respectively. In such a case, the arrival mean rate seen at each one of the defined book-ahead times is given by,

$IR0$	$\lambda_{(0)} = \lambda$
$AR0$	$\lambda_{(\alpha_{AR0})} = \lambda(1 - \sum_{\alpha=0}^{\alpha_{AR0}-1} f(\alpha)) = \lambda(1-0.50) = 0.5\lambda$
$AR1$	$\lambda_{(\alpha_{AR1})} = \lambda(1-0.50-0.25) = 0.25\lambda$

The example shows that the number of requests for time-slot α_{AR0} seen by a reference arrival at current time-slot 0 is 0.5λ and that for α_{AR1} is 0.25λ, i.e., for the AR1 traffic is only considered this traffic, for the AR0 traffic is considered both the AR1 and AR0 traffic, and for IR0, all traffic is considered.

Due to the slotted approach used in this paper, we also assume that the connection request's duration has a geometric distribution T [23] with probability $g(\tau) = \Pr(T = \tau) = q(1-q)^{\tau}$ and mean $\overline{T} = 1/q$. Recall that τ states for the duration of the connection and it can take values multiple of the slot duration, thus $\tau = 1, 2, \ldots$.

Following the example of [17], two Markov chains are necessary to model our wavelength reservation system. The first chain models the transition state of a single wavelength channel along the time horizon. Therefore, it will be useful for computing the probability that a connection can have enough free continuous slots for its duration on a single given wavelength. The second chain is related to the state probability that a connection will find a free wavelength on the system regardless of how many slots are free, which is already counted by the first chain.

Because of the first-fit slot and wavelength assignment policy, we can no longer assume wavelength assignments are uniformly distributed among those on the output link. We approximate the average arrival rate per wavelength, $\tilde{\lambda}$, by taking the relationship between the case wherein the number of arriving connections would request to reserve each one of the available wavelengths and the possible number of first-fit (ordered) permutations of these wavelengths. That is, having W as the number of wavelengths on the output link, the approximation considers the conditional joint probability of cases when all wavelengths are free λ/W, when all are free but one $\lambda/(W-1)$, but two $\lambda/(W-2)$, and so on, which results in $\tilde{\lambda} = \lambda^W/W!$. This approximation has shown to provide good results on link-based computation [24].

First, we define the model for the single wavelength channel. Let $S^{(n)}$ define the state of a wavelength at time-slot n,

$$S^{(n)} = \begin{cases} 1 & \text{if occupied,} \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

The wavelength channel occupancy is modeled by a non-homogenous Markov chain, i.e., with time dependent transition probabilities, and with transitional probability matrix $Q^{(n)}$,

$$Q^{(n)} = \begin{bmatrix} p_{0,0}^{(n)} & p_{0,1}^{(n)} \\ p_{1,0}^{(n)} & p_{1,1}^{(n)} \end{bmatrix}. \tag{3}$$

The transition probabilities of matrix $Q^{(n)}$ are defined as $p_{i,j}^{(n)} = \Pr(S^{(n+1)} = j | S^{(n)} = i)$ for $\{i,j\} = \{0,1\}$, and computed as,

$$p_{i,j}^{(n)} = \begin{cases} e^{-\tilde{\lambda}_{(n+1)}} & \text{if } i=j=0 \\ 1 - e^{-\tilde{\lambda}_{(n+1)}} & \text{if } i=0, j=1 \\ q e^{-\tilde{\lambda}_{(n+1)}} & \text{if } i=1, j=0 \\ 1 - q e^{-\tilde{\lambda}_{(n+1)}} & \text{if } i=j=1\ . \end{cases} \tag{4}$$

The second Markov chain is used to compute the transition probabilities of the number of used wavelengths on the output link throughout time. Let $Z^{(n)}$ be the number of channels reserved at slot n; hence, $Z^{(n)} = 0, 1, \ldots, W$. Again, the variable can be modeled as a non-homogeneous Markov chain with a transitional probability matrix $X^{(n)}$,

$$X^{(n)} = \begin{bmatrix} x_{0,0}^{(n)} & x_{0,1}^{(n)} & \cdots & x_{0,W}^{(n)} \\ x_{1,0}^{(n)} & x_{1,1}^{(n)} & \cdots & x_{1,W}^{(n)} \\ \vdots & \vdots & \ddots & \vdots \\ x_{W,0}^{(n)} & x_{W,1}^{(n)} & \ldots & x_{W,W}^{(n)} \end{bmatrix}, \tag{5}$$

where $x_{i,j}^{(n)}$ represents the transition probability defined as follows,

$$x_{i,j}^{(n)} = \Pr(Z^{(n+1)} = j | Z^{(n)} = i), \tag{6}$$

and with $\{i,j\} = \{0, 1, \ldots, W\}$. The state probabilities are computed as shown in (7) (on the next page).

The state probabilities of $Z^{(n)}$ are given by vector Π as,

$$\Pi = \begin{bmatrix} \pi_0 & \pi_1 & \ldots & \pi_W \end{bmatrix}. \tag{8}$$

$$x_{i,j}^{(n)} = \begin{cases} \sum_{z=0}^{i} \binom{i}{z} q^z (1-q)^{i-z} e^{-\lambda_{(n+1)}} \frac{\lambda_{(n+1)}^{j-i+z}}{(j-i+z)!} & \text{if } j \leq W-1 \text{ and } i \leq j, \\ \sum_{z=0}^{j} \binom{i}{i-j+z} q^{i-j+z} (1-q)^{j-z} e^{-\lambda_{(n+1)}} \frac{\lambda_{(n+1)}^{z}}{z!} & \text{if } j \leq W-1 \text{ and } i \geq j, \text{ and} \\ 1 - \sum_{z=0}^{W-1} x_{i,z}^{(n)} & \text{if } j = W. \end{cases} \quad (7)$$

To calculate Π we solve the following set of equations,

$$\pi_j = \sum_{i=0}^{W} x_{i,j} \pi_i \quad \text{for } 0 \leq j \leq W. \quad (9)$$

Note that in this case, $x_{i,j}$ is computed as in $x_{i,j}^{(n)}$, but now using the mean total arrival rate to the link λ instead of $\lambda_{(n)}$ due to the Markov and memoryless property to describe the present state.

2) Link Blocking Probability Computation: To compute the blocking probability for an arriving connection request with book-ahead α and duration τ, we need to consider the following two terms: first, the probability that at least one wavelength is free at the beginning of the connection, and second, the probability that this same wavelength is free for the entire connection holding time. For the former, let $Z^{(n)}$ denote the number of wavelengths reserved on the link at time-slot n, and its probability distribution be,

$$v_z^{(n)} = \Pr(Z^{(n)} = z), \quad (10)$$

where $z = 0, 1, \ldots, W$. To compute the number of wavelengths reserved at a specific book-ahead time α, we have to track the transition probability of the wavelengths used in all slots between the connection arrival and time-slot α, the starting resource allocation slot. To this end, (11) counts all possible state transitions in between the connection arrival and time-slot α reaching state z wavelengths reserved on the link.

$$v_z^{(\alpha)} = \Pi X^{(0)} \cdots X^{(\alpha-2)} X_{:,z}^{(\alpha-1)}. \quad (11)$$

In (11), $X_{:,z}^{(\alpha-1)}$ stands for column z of matrix $X^{(\alpha-1)}$. We note that the probability of blocking a connection corresponds to having W wavelengths reserved, $v_W^{(\alpha)}$.

The second term refers to the wavelength reservation state transition over the connection holding time, i.e., from the book-ahead time-slot α to the ending slot $(\alpha + \tau - 1)$. Such property can be expressed making use of the *first-passage-time* (FPT) analysis for a discrete random distribution. FPT can be used to model problems where computing the probability that a failure or a change in the system occurs within finite time [25]. In this respect, let $T_s(\alpha, \tau)$ represent the probability that at slot α, the state $s \in S^{(n)}$ remains for a duration equal to τ slots. If we want to compute that the wavelength remains free for τ duration, then we have to compute $T_0(\alpha, \tau)$, which is simply the probability that the wavelength is free from α to $(\alpha + \tau - 1)$ as given by,

$$\begin{aligned} T_0(\alpha, \tau) &= p_{0,0}^{(\alpha)} \cdot p_{0,0}^{(\alpha+1)} \cdots p_{0,0}^{(\alpha+\tau-2)} \\ &= e^{-\tilde{\lambda}_{(\alpha+1)}} e^{-\tilde{\lambda}_{(\alpha+2)}} \cdots e^{-\tilde{\lambda}_{(\alpha+\tau-1)}}. \end{aligned} \quad (12)$$

All the wavelength state probabilities $p_{0,0}^{(n)}$ (12) have been previously derived in (3).

At this stage, we have all the necessary elements to calculate the blocking probability of class c on link j, L_c^j. As specified in (13), this is simply 1.0 minus the probability that the system can allocate such a connection request, that is, the probability that there is at least one free wavelength at the specified book-ahead time that remains idle for the whole connection duration,

$$L_c^j = BP(\alpha, \tau) = 1 - T_0(\alpha, \tau) \sum_{z=0}^{W-1} v_z^{(\alpha)}. \quad (13)$$

B. Route Blocking Computation under Wavelength Conversion

As introduced, we assume a fixed routing policy, i.e., there is only one route between any origin and destination pair for any class. Therefore, if the arriving connection cannot be allocated along this route due to insufficient free resources, the connection is blocked. Recall that under WC, different wavelengths can be assigned on different links along the path.

In order to compute the end-to-end path blocking we must consider the specific offered load to each link of such a path. We obtain the arrival rate λ_c^j of a specific class c into link j by combining the contributions of same class c requests from all routes $r_c(s,d)$ that traverse such a link. Hence,

$$\lambda_c^j = \sum_{s,d | j \in r_c(s,d)} \lambda_c^{s,d}. \quad (14)$$

Once we have the contributed arrival rate into every link we can compute the blocking probabilities. We will consider first the case of a single link route, then of a two-link route and finally we will generalize for routes of any length. We will omit the class subscript to simplify the notation.

1) One-Hop Route: If we consider the route to be composed of a single link j, i.e., the source and destination nodes are “neighbors”, then the route is simply $r(s,d) = \{j\}$. In such a case, to compute the blocking probability we only need to take into account the link blocking computation derived in Section V-A2 using the contributed arrival rate into the link. As a result,

$$L_c^{s,d} = L_c^j. \quad (15)$$

2) Two-Hop Route: Under the wavelength conversion case, we can assume that the wavelength allocation on a two-hop route is independent between the two links. As a result, the probability that a connection request of class c gets blocked is the probability that the connection is not blocked in any of the corresponding links. If we denote the two links as j_1 and j_2, so that, $r(s,d) = \{j_1, j_2\}$, then the blocking probability is,

$$L_c^{s,d} = 1 - (1 - L_c^{j_1})(1 - L_c^{j_2}). \quad (16)$$

3) General Case: If we extend the previous analysis, we can simply derive the blocking probability for a route of any length in the wavelength conversion scenario as the probability that the connection is not blocked in any of the links j along the route r. Hence, we can compute the blocking as

$$L_c^{s,d} = 1 - \prod_{j:j\in r} (1 - L_c^j). \tag{17}$$

C. Route Blocking Computation under Wavelength-Continuity Constraint

Under the wavelength-continuity constraint, the lightpath reserved for the connection request must use the same wavelength along the route from the origin to the destination node. Due to this restriction, the probability to set up such a lightpath is decreased in comparison to the wavelength conversion case. Such a constraint can be resolved by considering a conditional probability among the links conforming the route [13].

First of all, we define some extra notation:

- We denote m_j the number of idle wavelengths on link j. Therefore, $m_j \in \Lambda$, where $\Lambda = \{0, 1, \ldots, W\}$.
- The random variable Y_j defines the number of idle wavelengths on link j able to allocate the connection request for its duration. We assume it is statistically independent among links [13].
- We define $q_j(m_j) \equiv \Pr(Y_j = m_j)$, to be the probability distribution of the number of free wavelengths on link j.

We note that Y_j is related to the number of reserved wavelengths on the output link j, as we defined in (10), and the probability that these same wavelengths are idle for the connection holding time. Therefore, if we remove the slot notation (n) from (10) we can define $Y_j \equiv (W - Z_j)$, and use the $v_z^{(\alpha)}$ states and the $T_0(\alpha, \tau)$ about the wavelength occupancy to infer the number of idle wavelengths. As a result, we define the following notation

$$q_j(m_j) \equiv v_{W-m_j} \cdot f(m_j; W, T_0(\alpha, \tau)), \tag{18}$$

that is, the probability to have m_j idle wavelengths on link j is the same as the probability to have $W - m_j$ reserved wavelengths, and m_j wavelengths idle from the book-ahead α and for the connection holding time τ. $f(m_j; W, T_0(\alpha, \tau))$ is the probability mass function of the Binomial distribution, which is used to compute the probability that m_j idle wavelengths are free for the specified book-ahead and holding times. That is,

$$\begin{aligned} f(m_j; W, T_0(\alpha, \tau)) &= \Pr(Y_j = m_j) \\ &= \binom{W}{m_j} T_0(\alpha, \tau)^{m_j} (1 - T_0(\alpha, \tau))^{W - m_j}. \end{aligned} \tag{19}$$

1) Definition of the Wavelength-Continuity Constraint: If we note the fixed route between nodes s and d as $r(s, d)$, which is constituent of the following links $\{j_1, j_2, \ldots, j_{h(r)}\}$, the probability that n common wavelengths are available on route r can be defined as,

$$\begin{aligned} p_n(m_{j_1}, m_{j_2}, \ldots, m_{j_{h(r)}}) \equiv \Pr(Y_r = n | Y_{j_1} = m_{j_1}, \\ , Y_{j_2} = m_{j_2}, \ldots, Y_{j_{h(r)}} = m_{j_{h(r)}}). \end{aligned} \tag{20}$$

The probability is conditional that the same n wavelengths are free on all constituent links conditioned on the event that every link j_i has m_{j_i} idle wavelengths. In (20), the expression on the right side is conditioned on the set of disjoint events $\{Y_{r^-} = k | k = n, n+1, \ldots, m_{j_{h(r)-1}}\}$, where route r^- is formed by links $\{j_1, j_2, \ldots, j_{h(r)-1}\}$, that is, as route r but removing its last link. In our reservation system, we need at least one idle wavelength, $n = 1$, in order to avoid blocking.

For a simple two-link route $r = \{j_1, j_2\}$ we have $p_n(m_{j_1}, m_{j_2}) = \Pr(Y_r = n | Y_{j_1} = m_{j_1}, Y_{j_2} = m_{j_2})$. In this example, we calculate the probability that there are n common idle wavelengths conditioned to the event that there are m_{j_1} idle wavelengths on the first link and m_{j_2} on the second one. This probability can be derived combinatorially [13] as

$$p_n(m_{j_1}, m_{j_2}) = \begin{cases} \dfrac{\binom{m_{j_1}}{n}\binom{W - m_{j_1}}{m_{j_2} - n}}{\binom{W}{m_{j_2}}} & \text{if } n \le m_{j_1}, m_{j_2} \le W, \\ & m_{j_1} + m_{j_2} - n \le W \\ 0 & \text{otherwise .} \end{cases} \tag{21}$$

In the first part of (21), the denominator corresponds to the number of combinations that m_{j_2} wavelength can be selected on the second link, while the numerator defines the number of combinations of wavelengths that can be selected on the second link so that n of the selected ones are also on the first link. For a $h(r)$-hop route [15], we obtain the following recursive relation

$$\begin{aligned} &p_n(m_{j_1}, m_{j_2}, \ldots, m_{j_{h(r)}}) = \\ &\quad \sum_{k=n}^{k^*} p_n(k, m_{j_{h(r)}}) p_k(m_{j_1}, m_{j_2}, \ldots, m_{j_{h(r)-1}}), \end{aligned} \tag{22}$$

where $k^* = \min\{m_{j_1}, m_{j_2}, \ldots, m_{j_{h(r)-1}}\}$.

As in the previous case (wavelength conversion), we need to consider the total contributed offered load to each link (14). Again, we will develop first the case of a one-hop route and then generalize for $h(r)$ hops.

2) One-Hop Route: For a single link scenario, the computation of the blocking probability is the same as in the WC scenario. Therefore,

$$L_c^{s,d} = L_c^j. \tag{23}$$

The sum probability of blocking states for link j for class c call arrivals can also be denoted as,

$$L_c^j \equiv q_j(0). \tag{24}$$

3) Two-Hop Route: Let now consider the route $r = \{j_1, j_2\}$ and assume random variables Y_{j_1} and Y_{j_2} to be independent. We first need to compute the probability that there is at least one available wavelength along the route wherein the reservation can be allocated given that the number of free wavelengths on link j_1 is m_{j_1}. This is represented by $\Pr(Y_r \ge 1 | Y_{j_1} = m_{j_1})$. Knowing this, we can then compute the blocking probability for class c on route $r(s, d)$ as follows (see Appendix A for the whole derivation):

$$L_c^{s,d} = \Pr(Y_r < 1) = 1 - \Pr(Y_r \ge 1) = 1 - (1 - L_c^{j_1})(1 - L_c^{j_2}) +$$

$$
\begin{aligned}
&+ \sum_{m_{j_1}=1}^{W} \sum_{m_{j_2}=1}^{W} q_{j_1}(m_{j_1}) q_{j_2}(m_{j_2}) p_0(m_{j_1}, m_{j_2}) \\
&= 1 - \prod_{j:j\in r} (1 - L_c^j) + \\
&+ \sum_{m_{j_1}=1}^{W} \sum_{m_{j_2}=1}^{W} q_{j_1}(m_{j_1}) q_{j_2}(m_{j_2}) p_0(m_{j_1}, m_{j_2}) \quad (25)
\end{aligned}
$$

4) General Case: The previous two-hop route expression can be generalized for routes of any number of hops $h(r)$. If in the general case we have a route $r(s,d) = \{j_1, j_2, \dots, j_{h(r)}\}$, then the blocking probability of class c is

$$
\begin{aligned}
L_c^{s,d} &= \Pr(Y_r < 1) = 1 - \prod_{j:j\in r} (1 - L_c^j) + \\
&+ \sum_{m_{j_1}=1}^{W} \cdots \sum_{m_{j_{h(r)}}=1}^{W} q_{j_1}(m_{j_1}) \dots q_{j_{h(r)}}(m_{j_{h(r)}}) \cdot \\
&\cdot p_0(m_{j_1}, m_{j_2}, \dots, m_{j_{h(r)}}). \quad (26)
\end{aligned}
$$

As introduced in [15], equation (26) is a generalization of (17). That is, (26) is composed of two terms: (a) a term that is present regardless of having or not having wavelength conversion, and (b) a term related to operating under wavelength-continuity constraint, which increases the blocking probability with respect to having wavelength conversion capability.

VI. Reduced Load Erlang Fixed-Point Approximation for Network-Wide Blocking

Once we have computed the blocking probability of all routes r and all classes c, we can calculate the average network blocking probability per class L_c^G, which is simply defined as

$$L_c^G = \frac{\sum_{s,d} \lambda_c^{s,d}/\mu \cdot L_c^{s,d}}{\sum_{s,d} \lambda_c^{s,d}/\mu}. \quad (27)$$

Analogously, the total average network blocking probability L^G is given by

$$L^G = \frac{\sum_{s,d} \sum_c \lambda_c^{s,d}/\mu \cdot L^{s,d}}{\sum_{s,d} \sum_c \lambda_c^{s,d}/\mu}. \quad (28)$$

Typically, the blocking probabilities and the arrival rate to a link are related since blocking determines the traffic carried by the network, which in turn determines the blocking [14]. We use the reduced-load Erlang fixed-point approximation (EFPA) algorithm [21] to obtain the network-wide approximate blocking probability for each IR/AR class and for both scenarios, with and without wavelength conversion.

In the reduced-load EFPA, the contributed load into a network link is reduced due to blocking on other links pertaining to the route under consideration. For instance, let us consider two traffic flows between source-destination pairs (s_1,d_1) and (s_2,d_2) which have one link in common in their respective lightpaths as shown in Fig. 4. The load into the link between nodes n_1 and n_2 of class c, λ_c^j, is equal to the current load into the link contributed by the route loads minus the load

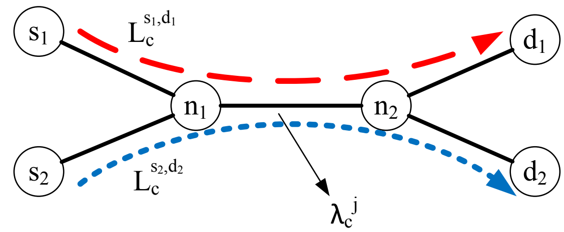


Fig. 4. Reduced-load link approximation.

Algorithm 1 Reduced-load Erlang fixed-point approximation.

1: Initialize all route blocking probabilities to zero, i.e., $\hat{L}_c^{s,d} = 0 \quad \forall\, s, d$.
2: Set error threshold ϵ and $end \leftarrow$ **false**.
3: Decompose traffic load to source-destination pairs and routes (see Section IV-A).
4: **repeat**
5: Decompose and compute the arrival load to links based on the reduced-load due to blocking using (30).
6: Compute per link blocking probability for each class c, L_c^j, as specified in Section V-A using (13).
7: **for all** route $r(s,d)$ **and** $c \in C$ **do**
8: Compute the route blocking probability, $L_c^{s,d}$, under WC (17) or WCC (26).
9: **end for**
10: **if** $\max_c^{s,d} |\hat{L}_c^{s,d} - L_c^{s,d}| < \epsilon$ **then**
11: $end \leftarrow$ **true**.
12: **else**
13: Update $\hat{L}_c^{s,d} \leftarrow L_c^{s,d}$.
14: $end \leftarrow$ **false**.
15: **end if**
16: **until** end is **true**

blocked on the remaining links of both routes. Therefore, for this specific example we have

$$\lambda_c^j = \lambda_c^{s_1,d_1} \prod_{\substack{l\in r(s_1,d_1):\\ l\neq j}} (1 - L_c^l) + \lambda_c^{s_2,d_2} \prod_{\substack{l\in r(s_2,d_2):\\ l\neq j}} (1 - L_c^l). \quad (29)$$

Generalizing for any link and origin-destination pair, then

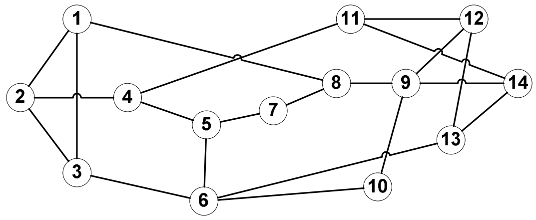
$$\lambda_c^j = \sum_{\substack{r(s,d):\\ j\in r}} \Big(\lambda_c^{s,d} \prod_{\substack{l\in r(s,d):\\ l\neq j}} (1 - L_c^l)\Big). \quad (30)$$

Algorithm 1 shows the steps involved in the fixed-point approximation. After the initialization stage, the traffic load decomposition process in step (5) is iterated until the maximum route blocking probability difference for any class is under a specified error threshold (see step (10)). Within the loop, we first calculate the link blocking probability (step (6)). Then, using this information, we compute for all routes and classes the route blocking probability as shown in step (8).

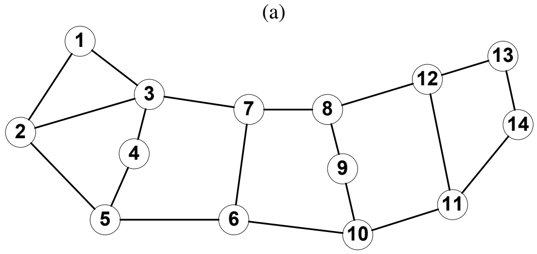
The approximation algorithm is not guaranteed to converge. However, in practice, we find that it usually converges within a few iterations, especially for low and medium network loads.

VII. Numerical Results

In this section we assess the analytical blocking model proposed in the paper and compare its results with others



(a)



(b)

Fig. 5. Network topologies used: (a) NSFNET, and (b) ESnet.

TABLE I
NETWORK TOPOLOGIES AND CHARACTERISTICS.

Network	# nodes	# links	Avg. nodal degree	Avg. path length	Std dev. path length
NSFNET	14	21	3.0	2.14	0.766
ESnet	14	19	2.71	2.65	1.282

obtained from simulation. To this end, we implemented an event-based simulator to check the performance of the hybrid IR/AR system. In order to deeply assess the model, we used two different network topologies: the well-known National Science Foundation network (NSFNET) (see Fig. 5(a)), and the DOE Energy Sciences Network (ESnet) (on Fig. 5(b)). Although both topologies are similar in number of nodes and links, they have different network characteristics, as shown in Table I. Most notably, the average path length (in number of hops) of NSFNET is smaller than ESnet, so is its path length standard deviation (i.e., ESnet topology has longer paths to interconnect nodes between U.S. West and East coasts). Unless specified otherwise, the number of wavelengths on the network is 8. This value allowed us to get results in a reasonable amount of time, especially when using simulation.

In the simulations, we assume a Poisson arrival process with a total average rate of λ connections/slot and a geometric mean holding time of τ slots. For a given simulation set, we changed the arrival rate in order to generate the desired offered load ($\rho = \lambda/\tau$). Moreover, we simulated the arrival of different IR/AR classes according to different input traffic class ratios in correspondence to the number of classes considered in the model, similarly as we did in (1). Table II shows the traffic scenarios that we considered in our performance analysis: the first with two reservation classes, one IR (denoted IR0) and one AR (AR0); and the second and third scenarios with three classes, one IR (IR0) and two AR (AR0 and AR1). Also, we

TABLE II
2 AND 3-CLASS IR/AR RESULTS PARAMETERIZATIONS.

	IR0			**AR0**			**AR1**		
Case	%	α	τ	%	α	τ	%	α	τ
1	50	0	5	50	200	5	–	–	–
2	33.3	0	5	33.3	100	5	33.3	200	5
3	33.3	0	20	33.3	100	20	33.3	200	20

evaluated two mean holding times to check their influence with respect to the book-ahead times of the AR classes. The percentage of load assigned to each class is also shown in Table II. We must note that the only difference among the classes has to do with the book-ahead time of each one of them. That is, all classes use the same scheduling policy (FF-SWA) (refer to Section IV) to allocate the time-slots and wavelength.

The simulation results were averaged over 30 batches of 10^5 connections each. Very narrow 95% confidence intervals were obtained, which have been omitted on the graphs to improve their readability.

We divide the performance analysis into three subsections. First, we analyze the proposed model for the case where the optical WDM network is wavelength-conversion capable. Second, we assess the results for the wavelength-continuity case. And finally, we evaluate the results as a function of the number of wavelengths per link and the lightpath length.

A. Results under Wavelength Conversion

The graphs in Fig. 6 show the connection blocking probability performance on NSFNET and ESnet when wavelength conversion is present in the network. We illustrate the results for the three aforementioned traffic scenarios introduced in Table II. If we focus our attention on the first two graphs, Fig. 6(a) and 6(b), we can observe that the analytical results accurately match the simulation results. As such, when the average holding time for both IR and AR traffic is 5 slots, the IR0 class fits almost perfectly; so does the average total blocking probability. Also, by comparing these two graphs, the blocking probability is lower in NSFNET than in ESnet due to the longer average path length of the second. The more hops a connection has to traverse, the more resources need to be used and the higher the chances of the connection being blocked. As we introduced in the model, we assume that there is a single path between every source and destination pair. Therefore, upon blocking another path cannot be probed.

The rest of graphs in Fig. 6 show the blocking probability when adding a new AR class (AR1) to the system according to Cases 2 and 3 in Table II. We can see that in both cases, and for both topologies, the simulation results also match well the model. The results also corroborate the theory that reservation classes with longer book-ahead time experience less blocking. Moreover, when the holding time is 5 slots (Case 2), the higher blocking classes, IR0 and AR0, analytical results fit almost perfectly to simulation (refer to Fig. 6(c) and 6(d)). On the contrary, when we set the mean connection duration to 20 slots as in Case 3 (refer to Fig. 6(e) and 6(f)), the lowest blocking class in the model (i.e., AR1) resembles better the simulation. In the latter case, the model slightly underestimates the higher blocking of IR0 and AR0. This is related to the link-independence assumption and the assumed Poisson overflow traffic of the first-fit wavelength assignment [26].

In summary, we can conclude that in the wavelength conversion scenario, the model captures the simulation results with a very acceptable resemblance, and in particular with higher accuracy when the IR traffic holding time is two orders shorter than the book-ahead of to the AR classes.

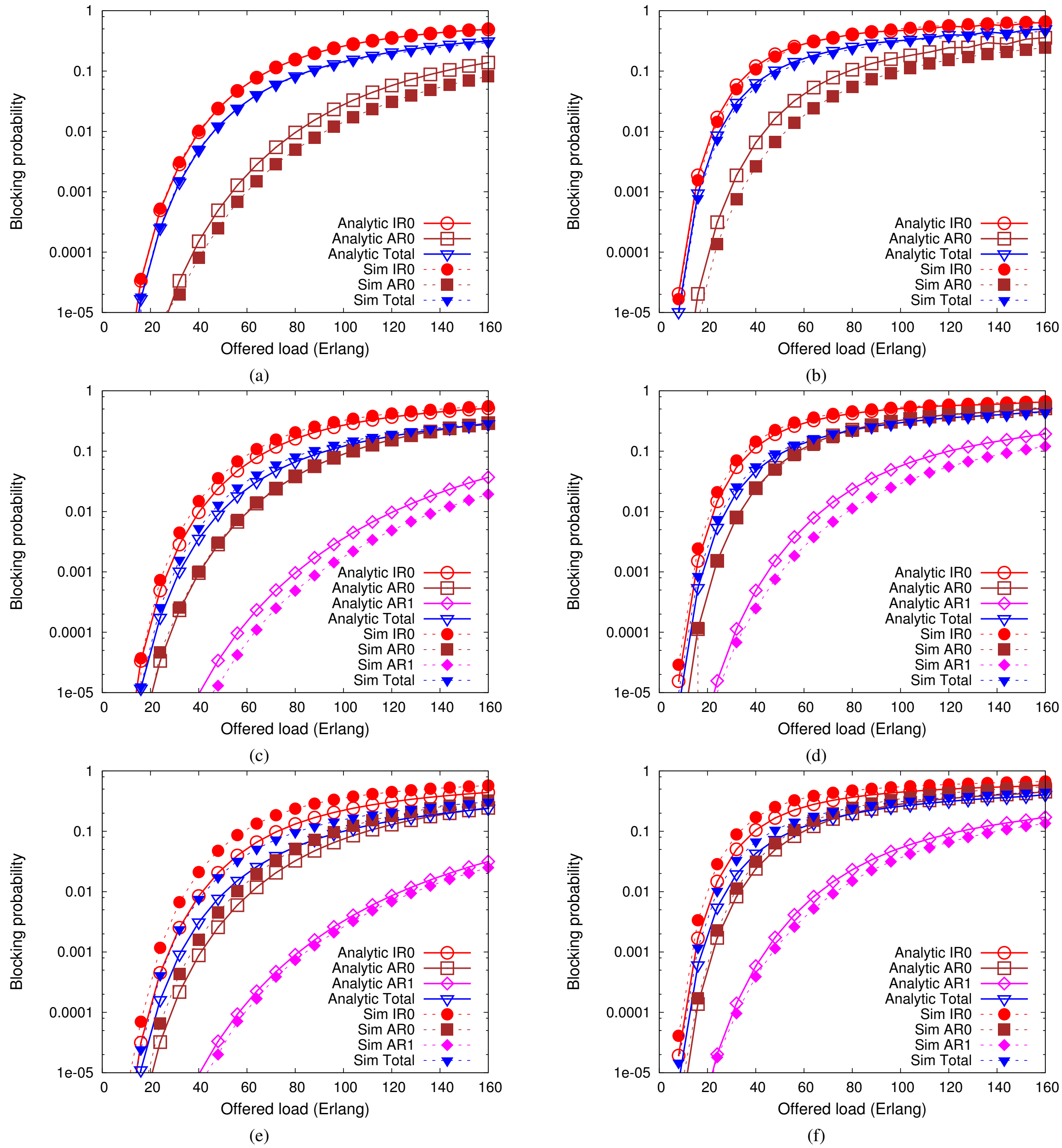


Fig. 6. Blocking probability under wavelength conversion: (a) Case 1 on the NSFNET, (b) Case 1 on the ESnet, (c) Case 2 on the NSFNET, (d) Case 2 on the ESnet, (e) Case 3 on the NSFNET, and (f) Case 3 on the ESnet.

B. Results under Wavelength-Continuity Constraint

The second part of the performance analysis compares the model and the simulation results for the wavelength-continuity constrained case. Again, we show the results for both the NSFNET and ESnet topologies.

The first two graphs in Fig. 7 show the results for the case with one IR class and one AR class. Now, as opposed to the wavelength conversion results, AR0 shows a better match between the model and the simulation results. Also, comparing between Fig. 7(a) and 7(b), the average total blocking matches better on the NSFNET. This is related to the average longer path of the ESnet that produces a greater overestimation of the approximate analytical blocking probability.

Fig. 7(c) and 7(d) show the results for Case 2, and Fig. 7(e) and 7(f) for Case 3 on the NSFNET and ESnet topologies, respectively. When we add a second AR class, the analytical results also provide a very good approximation to the simulation results. Such class addition to the system does not worsen the computation of the blocking probability achieving accurate results for both cases with mean holding times of 5 and 20 slots. As a matter of fact, in the last four graphs (i.e., Cases 2 and 3), the matching of the average total blocking is even better than in Case 1. Only the intermediate class (AR0) shows some discrepancy between simulation and analysis for low-

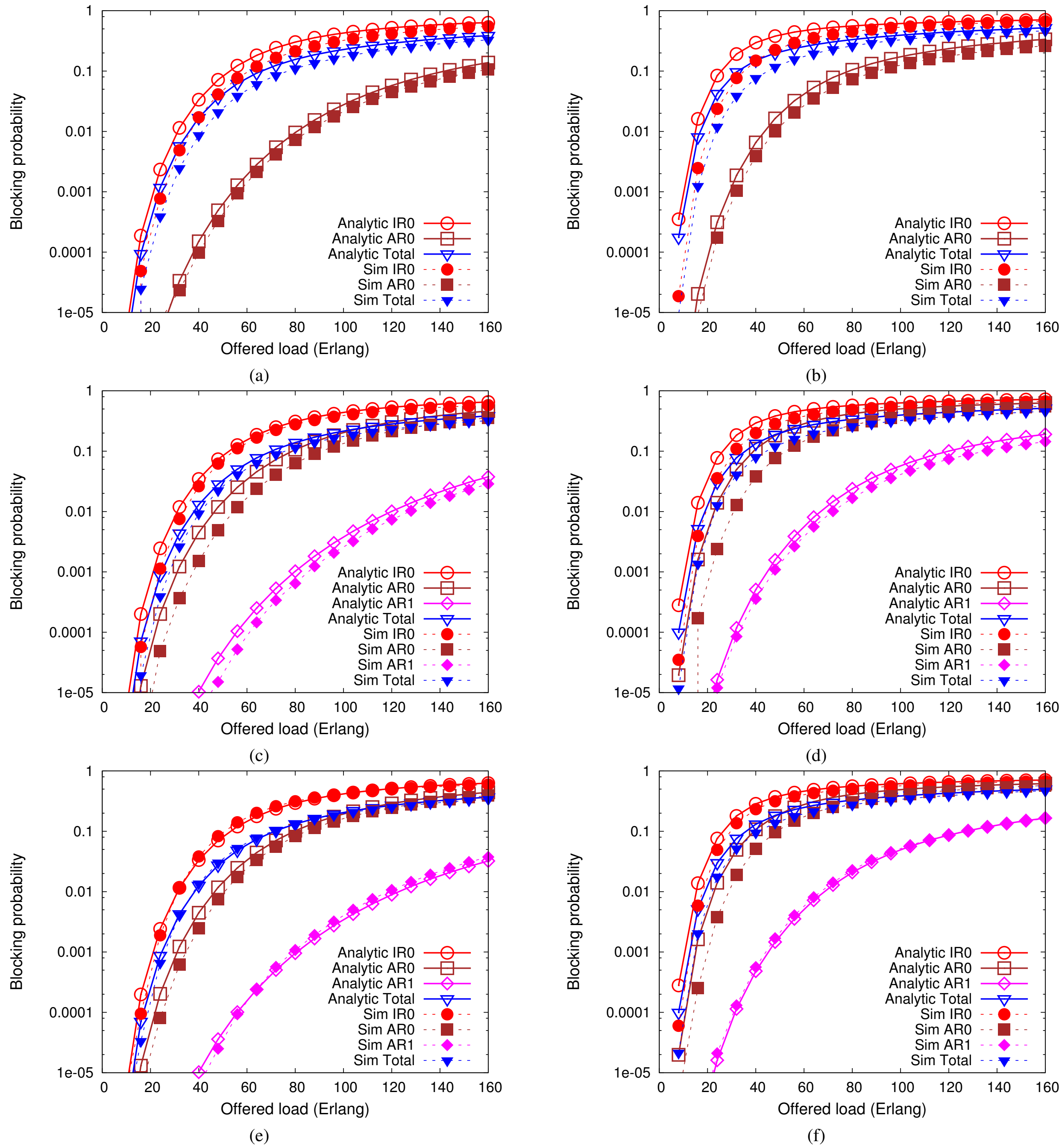


Fig. 7. Blocking probability under wavelength-continuity constraint: (a) Case 1 on the NSFNET, (b) Case 1 on the ESnet, (c) Case 2 on the NSFNET, (d) Case 2 on the ESnet, (e) Case 3 on the NSFNET, and (f) Case 3 on the ESnet.

to-medium offered loads. In general, we can observe that in the WCC case, the best comparison results are obtained when the mean holding time for the different reservation classes is longer and closer (one order of magnitude of difference) to the book-ahead times assigned to each class (recall that IR has a book-ahead of 0).

It is worth noting that comparing the results between the WC capable network and the WCC counterpart, and for both topologies and same traffic case, the blocking probability on the latter is higher. As we analyzed in Section V-C4, the blocking probability in the WCC case is the sum of the blocking probability contributed from the wavelength conversion blocking and a term that depends on the wavelength-continuity constraint. Also, in the WC case, the analytical model approximates better the IR blocking than AR, while for WCC is the other way round. We argue that the link-independence and wavelength conversion, which does not influence so much on the overflow traffic, make the model to overestimate [26] the blocking of AR under WC. This behavior is not so noticeable when we consider WCC.

To gain more insight into the correctness of the model, the final set of results in this subsection shows the blocking probability analysis for three different wavelength scenarios: 8, 16, and 32 wavelengths per link. We picked three representative

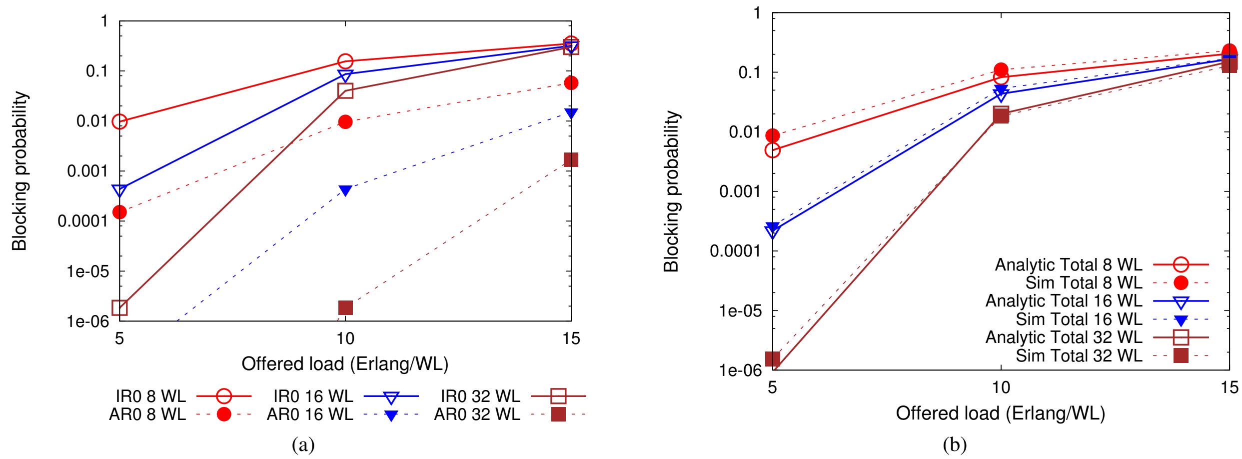


Fig. 8. Blocking probability under wavelength-continuity constraint on the NSFNET for 8, 16, and 32 wavelengths under Case 1: (a) IR/AR comparison from the model, and (b) total blocking comparison between simulation and the model.

offered traffic loads, namely 5, 10, and 15 Erlang/Wavelength. For each wavelength case, we generated the corresponding offered load in order to obtain a "fair" comparison among them. For instance, if we consider a network with 16 wavelengths per link, a value of 10 Erlang/WL is equivalent to a total offered load to the network of 160 Erlang, while for a case of 32 wavelengths per link, this represents an offered load of 320 Erlang. Fig. 8(a) shows the connection blocking probability per reservation class computed from the analytical model when we have two reservation classes as defined for Case 1 in Table II. As expected, the model shows that increasing the number of wavelengths on the network drops the connection blocking probability. This holds true for both reservation classes under consideration. It is also worth noting that the AR class yields a much better performance when we increase the number of wavelengths available.

Finally, Fig. 8(b) shows the comparison of the average total blocking probability between simulation and analysis for the three wavelength scenarios. We can see that for the three loads considered, simulation and analysis reach very close values, which corroborates the proposed model.

C. Path Length Performance Results

In this subsection, we compare the blocking performance as a function of the path length. To narrow the scope of the results, we consider the Case 1 scenario and use only the NSFNET topology. For such network, the shortest path is at most three hops long. Also, we show the results for two different offered traffic loads and 8, 16, and 32 wavelengths per link.

The first two graphs, Fig. 9(a) and 9(b), show the results for 5 and 10 Er/wl when considering the wavelength conversion capable optical network. The graphs show a very good match for any path length and for any number of wavelengths considered, especially at 5 Er/wl. At higher loads, 10 Er/wl, the model matches better with the simulation results when considering fewer number of wavelengths, i.e., 8 and 16 wavelengths. These results under wavelength conversion corroborate the results we obtained previously from both network topologies.

The following two graphs in Fig. 9(c) and 9(d) show the same set of results but for the wavelength-continuity constrained case. Again, at low loads we can observe that the model provides a very good approximation to the simulation results. At higher loads (refer Fig. 9(d)), the approximation is better when the number of wavelengths is small.

Finally, we can see from all graphs that the blocking probability is higher for longer paths (1-hop vs. 2-hop vs. 3-hop lightpaths). This happens regardless the number of wavelengths on the network and the traffic load.

VIII. Conclusion

Delay-sensitive and delay-tolerant applications require the network to provision the demanded bandwidth at the right time in order to facilitate the best user-experience. To satisfy this, IR and AR reservation mechanisms can be utilized. However, IR/AR co-existence requires one to thoroughly analyze the required service-level for traffic demands. In this paper, we have introduced an analytical model to compute the approximate network-wide blocking probability in hybrid IR/AR co-scheduling optical networks. The model uses two probability transition Markov chains to model future state transitions; the first keeps track of the time-slot availability for the connection duration, and the second of the wavelength availability at the reservation book-ahead time. Later, we compute the blocking on the network using a reduced-load fixed-point approximation analysis for two common scenarios, with and without wavelength conversion. Results obtained from two different network topologies demonstrate that with this model we can approximately compute the blocking probability in the network even in the case when multiple immediate and advance reservation classes are present and the number of wavelengths on the network varies.

Acknowledgement

This work has been partially supported by the Department of Energy (DOE) COMMON project under grant DE-SC0004909 and the Government of Spain under project TEC2010-20527-C02-01. Joan Triay was a visiting researcher at University of Massachusetts sponsored by a Fulbright fellowship.

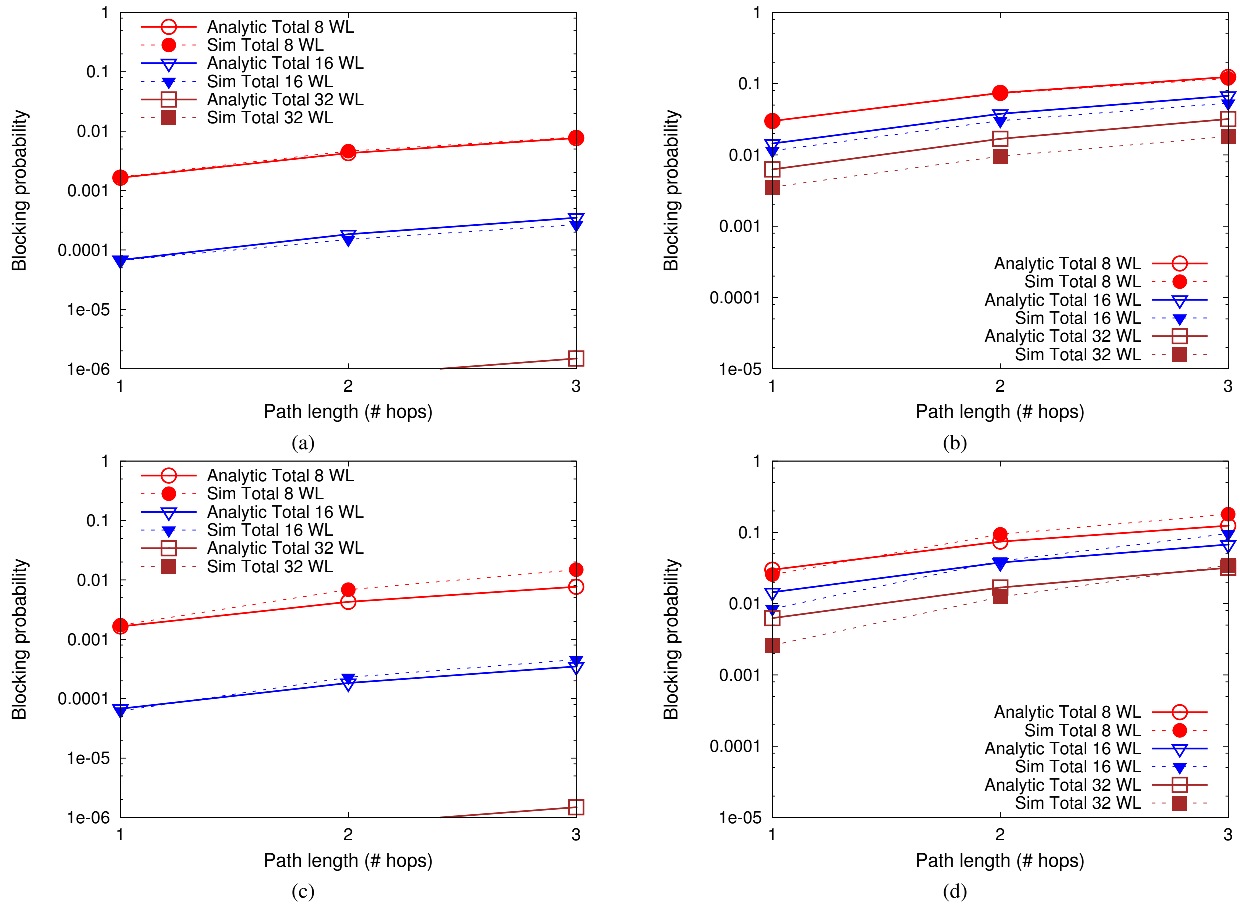


Fig. 9. Blocking probability as a function of the path length on the NSFNET for 8, 16, and 32 wavelengths under Case 1: (a) 5 Er/wl offered load with wavelength conversion, (b) 10 Er/wl with wavelength conversion, (c) 5 Er/wl under WCC, and (d) 10 Er/wl under WCC.

Appendix A
Blocking Probability on Two Link Route under Wavelength-Continuity Constraint

If we consider the route $r = \{j_1, j_2\}$ and we assume random variables Y_{j_1} and Y_{j_2} to be independent, then we have [15]

$$\Pr(Y_r \geq 1 | Y_{j_1} = m_{j_1}) =$$

$$= \sum_{m_{j_2}=0}^{W} \Pr(Y_r \geq 1 | Y_{j_1} = m_{j_1}, Y_{j_2} = m_{j_2}) \cdot \Pr(Y_{j_2} = m_{j_2} | Y_{j_1} = m_{j_1})$$

$$= \sum_{m_{j_2}=1}^{W} (1 - \Pr(Y_r < 1 | Y_{j_1} = m_{j_1}, Y_{j_2} = m_{j_2})) \cdot \Pr(Y_{j_2} = m_{j_2})$$

$$= \sum_{m_{j_2}=1}^{W} \Pr(Y_{j_2} = m_{j_2}) \cdot (1 - \Pr(Y_r = 0 | Y_{j_1} = m_{j_1}, Y_{j_2} = m_{j_2}))$$

$$= \sum_{m_{j_2}=1}^{W} q_{j_2}(m_{j_2}) \left(1 - p_0(m_{j_1}, m_{j_2})\right), \tag{31}$$

which is the probability that there is at least one available wavelength along the route wherein the reservation can be allocated given that the number of free wavelengths on link j_1 is m_{j_1}. The blocking probability for class c on route $r(s,d)$ is the following [15]:

$$L_c^{s,d} = \Pr(Y_r < 1) = 1 - \Pr(Y_r \geq 1)$$

$$= 1 - \sum_{m_{j_1}=0}^{W} \Pr(Y_r \geq 1 | Y_{j_1} = m_{j_1}) \Pr(Y_{j_1} = m_{j_1})$$

$$= 1 - \sum_{m_{j_1}=1}^{W} \Pr(Y_r \geq 1 | Y_{j_1} = m_{j_1}) \Pr(Y_{j_1} = m_{j_1}) \tag{32}$$

and by (31), we get

$$L_c^{s,d} = 1 - \sum_{m_{j_1}=1}^{W} \sum_{m_{j_2}=1}^{W} q_{j_1}(m_{j_1}) q_{j_2}(m_{j_2}) \cdot (1 - p_0(m_{j_1}, m_{j_2}))$$

$$= 1 - \sum_{m_{j_1}=1}^{W} \sum_{m_{j_2}=1}^{W} q_{j_1}(m_{j_1}) q_{j_2}(m_{j_2}) +$$

$$+ \sum_{m_{j_1}=1}^{W} \sum_{m_{j_2}=1}^{W} q_{j_1}(m_{j_1}) q_{j_2}(m_{j_2}) p_0(m_{j_1}, m_{j_2})$$

$$= 1 - (1 - q_{j_1}(0))(1 - q_{j_2}(0))$$

$$+ \sum_{m_{j_1}=1}^{W} \sum_{m_{j_2}=1}^{W} q_{j_1}(m_{j_1}) q_{j_2}(m_{j_2}) p_0(m_{j_1}, m_{j_2}) \tag{33}$$

and again from (24) and knowing that $r = \{j_1, j_2\}$, we end up having

$$L_c^{s,d} = 1 - (1 - L_c^{j_1})(1 - L_c^{j_2}) +$$

$$+\sum_{m_{j_1}=1}^{W}\sum_{m_{j_2}=1}^{W} q_{j_1}(m_{j_1})q_{j_2}(m_{j_2})p_0(m_{j_1},m_{j_2})$$
$$=1-\prod_{j:j\in r}(1-L_c^j)+$$
$$+\sum_{m_{j_1}=1}^{W}\sum_{m_{j_2}=1}^{W} q_{j_1}(m_{j_1})q_{j_2}(m_{j_2})p_0(m_{j_1},m_{j_2}). \quad (34)$$

REFERENCES

[1] Delivery of Advanced Network Technology to Europe (DANTE), “GÉANT,” [Online]. Available: http://www.dante.net.

[2] Lawrence Berkeley National Laboratory, “Energy Sciences Network (ESnet),” [Online]. Available: http://www.es.net.

[3] A. Takefusa *et al.*, “G-lambda: coordination of a Grid scheduler and lambda path service over GMPLS,” *Future Generation Computer Systems*, vol. 22, no. 8, pp. 868–875, 2006.

[4] C. Bouras and K. Stamos, “An adaptive admission control algorithm for bandwidth brokers,” in *Proc. of the Third IEEE International Symposium on Network Computing and Applications*, Washington, DC, USA, 2004, pp. 243–250.

[5] K. Liu, K. Nahrstedt, and S. Chen, “Routing with topology abstraction in delay-bandwidth sensitive networks,” *IEEE/ACM Trans. Netw.*, vol. 12, no. 1, pp. 17–29, Feb. 2004.

[6] A. Greenberg, R. Srikant, and W. Whitt, “Resource sharing for book-ahead and instantaneous-request calls,” *IEEE/ACM Trans. Netw.*, vol. 7, no. 1, pp. 10–22, Feb. 1999.

[7] I. Ahmad, J. Kamruzzaman, and S. Aswathanarayaniah, “A dynamic approach to reduce preemption in book-ahead reservation in QoS-enabled networks,” *Computer Communications*, vol. 29, no. 9, pp. 1443–1457, May 2006.

[8] M. Tornatore, A. Baruffaldi, H. Zhu, B. Mukherjee, and A. Pattavina, “Holding-time-aware dynamic traffic grooming,” *IEEE J. Sel. Areas Commun.*, vol. 26, no. 3, pp. 28–35, Apr. 2008.

[9] J. Zheng and H. Mouftah, “Routing and wavelength assignment for advance reservation in wavelength-routed WDM optical networks,” in *IEEE Int. Conf. on Communications (ICC) 2002*, vol. 5, New York, NY, USA, Aug. 2002, pp. 2722–2726.

[10] Y. Chen, C. Qiao, and X. Yu, “Optical burst switching: a new area in optical networking research,” *IEEE Netw.*, vol. 18, no. 3, pp. 16–23, May 2004.

[11] I. Monga, C. Guok, W. Johnston, and B. Tierney, “Hybrid networks: lessons learned and future challenges based on ESnet4 experience,” *IEEE Commun. Mag.*, vol. 49, no. 5, pp. 114–121, may 2011.

[12] Ciena, “Dynamic Resource Allocation Controller (DRAC),” [Online]. Available: https://www.opendrac.org.

[13] A. Birman, “Computing approximate blocking probabilities for a class of all-optical networks,” *IEEE J. Sel. Areas Commun.*, vol. 14, no. 5, pp. 852–857, Jun. 1996.

[14] A. Sridharan and K. N. Sivarajan, “Blocking in all-optical networks,” *IEEE/ACM Trans. Netw.*, vol. 12, no. 2, pp. 384–397, Apr. 2004.

[15] K. Kuppuswamy and D. C. Lee, “An analytic approach to efficiently computing call blocking probabilities for multiclass WDM networks,” *IEEE/ACM Trans. Netw.*, vol. 17, no. 2, pp. 658–670, Apr. 2009.

[16] X. Zhu and M. Veeraraghavan, “Analysis and design of book-ahead bandwidth-sharing mechanisms,” *IEEE Trans. Commun.*, vol. 56, no. 12, pp. 2156–2165, Dec. 2008.

[17] A. M. Kaheel, H. Alnuweiri, and F. Gebali, “A new analytical model for computing blocking probability in optical burst switching networks,” *IEEE J. Sel. Areas Commun.*, vol. 24, no. 12, pp. 120–128, Dec. 2006.

[18] N. Charbonneau, V. M. Vokkarane, C. Guok, and I. Monga, “Advance reservation frameworks in hybrid IP-WDM networks,” *IEEE Commun. Mag.*, vol. 49, no. 5, pp. 132–139, may 2011.

[19] E. Escalona, S. Spadaro, J. Comellas, and G. Junyent, “Advance reservations for service-aware GMPLS-based optical networks,” *Computer Networks*, vol. 52, no. 10, pp. 1938–1950, 2008.

[20] R. W. Wolff, “Poisson arrivals see time averages,” *Operations Research*, vol. 30, no. 2, pp. 223–231, Mar. 1982.

[21] F. P. Kelly, “Blocking probabilities in large circuit-switched networks,” *Advances in Applied Probability*, vol. 18, no. 2, pp. 473–505, 1986.

[22] P. Emstad and B. Feng, “Traffic models for reservation systems,” in *Proc. 13th Int. Teletraffic Congress*, Copenhagen, Denmark, Jun. 1991, pp. 647–652.

[23] R. Nelson, *Probability, stochastic processes, and queueing theory: the mathematics of computer performance modeling*. New York, NY, USA: Springer-Verlag New York, Inc., 1995.

[24] J. Triay, C. Cervelló-Pastor, and V. Vokkarane, “Analytical model for hybrid immediate and advance reservation in optical WDM networks,” in *2011 IEEE Global Telecommunications Conference (GLOBECOM 2011)*, Houston, TX, USA, Dec. 2011, pp. 1–6.

[25] S. Redner, *A Guide to First-Passage Processes*. Cambridge University Press, 2001.

[26] A. Alyatama, “Wavelength decomposition approach for computing blocking probabilities in WDM optical networks without wavelength conversions,” *Computer Networks*, vol. 49, no. 6, pp. 727–742, Dec. 2005.

Joan Triay received a B.Eng. and a M.Eng. in Telecommunications Engineering and a M.Sc. in Telematics in 2004, 2006, and 2007, respectively. In 2011, he received the Ph.D. degree in Telematics Engineering from the Universitat Politècnica de Catalunya (UPC), Spain. In 2007, he was awarded with a 4-year predoctoral FI scholarship from the Government of Catalonia and the European Social Fund to undertake his Ph.D. at UPC. He was a visiting fellow at the University of Essex (UK) from June 2009 to August 2010 thanks to a BE-DGR fellowship. He was also a visiting researcher at University of Massachusetts, Dartmouth (USA) from September 2010 to August 2011 sponsored by a Fulbright fellowship. His research interests include, but are not limited to, future optical network architectures and optical control plane design.

Cristina Cervelló-Pastor received her M.Sc. degree in Telecom Engineering in 1989 and Ph.D. degree in Telecommunication Engineering in 1998, both from the Universitat Politècnica de Catalunya (UPC), Barcelona, Spain. She is currently an Associate Professor in the Department of Telematics Engineering at UPC, which she joined in 1989, and leader of the optical networks research group within the BAMPLA research line. Her research trajectory has been centered on the field of routing in high speed networks and the development of new protocols and services in OBS/OPS, taking part in diverse national and European projects (FEDERICA, ATDMA, A@DAN, Euro-NGI, Euro-FGI, Euro-NF) and being responsible of various public and private funding R&D projects, some of them with the i2CAT Foundation. In parallel she has presented several patent proposals about OBS networks.

Vinod M. Vokkarane (S'02-M'04-SM'09) received the B.E. degree with Honors in Computer Science and Engineering from the University of Mysore, India in 1999, and the M.S. and Ph.D. degrees in Computer Science from the University of Texas at Dallas in 2001 and 2004, respectively. Dr. Vinod Vokkarane is an Associate Professor of Computer and Information Science at the University of Massachusetts, Dartmouth. He is currently a Visiting Scientist at the Research Laboratory of Electronics (RLE) at Massachusetts Institute of Technology (MIT). He is a recipient of the Texas Telecommunication Engineering Consortium Fellowship 2002-03 and the University of Texas at Dallas Computer Science Dissertation of the Year Award 2003-04. Dr. Vokkarane is the co-author of a book, Optical Burst Switched Networks, Springer, 2005. He has served as the Technical Program Committee chair of IEEE INFOCOM HSN 2011 workshop, IEEE GLOBECOM 2011 Optical Networks and Systems (ONS) Symposium, and IEEE ICC 2012 ONS Symposium. He is also serving as the Associate Editor of IEEE Communication Letters. His primary areas of research include design and analysis of architectures and protocols for optical and wireless networks.