

There are five (5) questions on this exam. Box in your answer to each question. Please fold the exam into your exam booklet. Good luck.

1. (20 points) Solve for  $y$  explicitly in terms of  $x$  if  $y$  satisfies the IVP

$$x^2 \frac{dy}{dx} = xy + y^2; \quad y(1) = -1/2$$

2. (20 points) Solve the following differential equation. You need not solve for  $y$  in terms of  $x$ .

$$(\cos x + \ln y)dx + \left(\frac{x}{y} + e^y\right)dy = 0$$

Determine the constant  $C$  in your solution if  $y(0) = 1$ .

3. (20 points) A tank initially contains 60 gallons of pure water. Brine containing 1 lb. of salt per gallon enters the tank at the rate of 2 gal/min, and the (perfectly mixed) solution leaves the tank at 3 gal/min; thus the tank is empty after one hour. Find the amount of salt in the tank after  $t$  minutes (for  $0 < t < 60$ ).

(5 point Bonus: When does the salt content in the tank reach its maximum value? Note that when  $t = 60$  min the tank is empty, so there is no salt, same as when we started - thus there is a time in between when the salt content is a max.)

4. (20 points) Find an explicit (i.e.,  $y$  explicitly in terms of  $x$ ) particular solution to

$$\frac{dy}{dx} = e^{2x-y}; \quad y(0) = 0.$$

5. (20 points) A projectile is fired straight upward with an initial velocity of 100m/s from the top of a building 20 m high and falls to the ground at the base of the building. Assume only gravity is acting on the projectile (i.e., no air resistance) so that the velocity decreases constantly at the rate of 9.8 m/s every second. Find

- (a) its maximum height above the ground
- (b) when it passes the top of the building
- (c) its total time in the air.