92.236.203

Eng. Diff Eqn's Sp 08 – Test 3 **Name:** ____________________

1. (20 points) Solve the following initial value problem

$$y'' - 5y' + 4y = x, y(0) = 6, y'(0) = 2$$

The char poly. $r^2 - 5r + 4 = 0$ *has* $r = 4, 1$ *as roots, so* $y_c = c_1e^{4x} + c_2e^x$ *is the soln to the homogeneous DE. A particular soln to the nonhomogeneous DE is* $y_p = (1/4)x + 5/16$*, so* $y = c_1e^{4x} + c_2e^x + (1/4)x + 5/16$ *is the general sol'n. The IC's give the two eqns* $c_1 + c_2 + 5/16 = 6$ *and* $4c_1 + c_2 + 1/4 = 2$*. Solving, we get the IVP has* $y = (-21/16)e^{4x} + 7e^x + (1/4)x + 5/16$

2. (20 points) Solve the following system of equations

$$\frac{dx}{dt} - x - y = e^{-t}$$

$$\frac{dy}{dt} + y + x = e^t$$

$$x(0) = -1, y(0) = 1$$

The DE x *satisfies is* $D^2x = e^t$*. Integrating twice we get* $x(t) = e^t + c_1t + c_2$*, and substituting into eqn 1 we get* $y = -e^{-t} + c_1 - c_2 - c_1t$*. The IC's give* $c_1 = 0, c_2 = -2$*. So,* $y = -e^{-t} + 2, x = e^t - 2$*.*

3. (20 points) Solve the following initial-value problem:

$$y^{(3)} - 2y'' + y' = 1 + xe^x; y(0) = y'(0) = 0, y''(0) = 1$$

$r = 0$ *and* $r = 1$ *(mult. 2) are the three roots of the char eqn. so* $y_c = c_1 + c_2e^x + c_3xe^x$*.* ***Note: You cannot solve for the initial conditions with this*** y_c***.*** y_c ***does not satisfy the nonhomogeneous DE. You must first find*** y_p***, then set*** $y = y_c + y_p$***, the general soln of the nonhomogeneous DE, before you use the initial conditions to find the coeffs*** c_1, c_2, c_3***.*** *This* $y_p = Ax + x^2(B + Cx)e^x$ *avoids repeating any term in* y_c*. Substituting into the DE we get* $y_p^{(3)} - 2y_p'' + y_p' = A + (2B + 6C)e^x + 6Cxe^x = 1 + xe^x$*, and so comparing coeffs,* $A = 1, B = -1/3, C = 1/6$*. Thus the general solution is* $y = c_1 + c_2e^x + c_3xe^x + (-x^2/2 + x^3/6)e^x$*. We get the following system*

$$\begin{aligned} c_1 + \quad c_2 \qquad &= y(0) = 0 \\ c_2 + \quad c_3 + 1 \quad &= y'(0) = 0 \\ c_2 + \quad 2c_3 - 1 \quad &= y''(0) = 1 \end{aligned}$$

which has $c_1 = 4, c_2 = -4, c_3 = 3$*. Thus the solution is* $y = 4 - 4e^x + 3xe^x + (-x^2/2 + x^3/6)e^x$

4. (20 points) The differential equation

$$u'' + 3u' + 9u = 5\cos\omega t$$

describes a forced spring system with damping.

(a) Find the steady-state solution (in terms of ω).

(b) Determine the amplitude of the steady state solution.

(c) Determine the amplitude of the steady state solution when $\omega = 2$

$u_p = A\cos(\omega t) + B\sin(\omega t)$ where

$$\begin{aligned} (9-\omega^2)A \quad & +3\omega B &= 5 \\ -3\omega A \quad & +(9-\omega^2)B &= 0 \end{aligned}$$

gives $A = \frac{5(9-\omega^2)}{(9-\omega^2)^2+9\omega^2}$ and $B = \frac{15\omega}{(9-\omega^2)^2+9\omega^2}$, so $u_p = \frac{5}{(9-\omega^2)^2+9\omega^2}[(9-\omega^2)\cos\omega t + 3\omega\sin\omega t]$. This has amplitude $C(\omega) = \frac{5}{\sqrt{(9-\omega^2)^2+9\omega^2}}$. $C(2) = 5/\sqrt{61}$.

5. (20 points) An LRC circuit with $L = 1, R = 2, C = 1/3$ with a battery providing an electromotive force of $(1/2)\sin 2t$ volts has current $I(t)$ which satisfies the differential equation:

$$\frac{d^2I}{dt^2} + 2\frac{dI}{dt} + 3I(t) = \cos 2t$$

(a) Determine a particular solution to this DE.

Substituting $I_p = A\cos 2t + B\sin 2t$ into the DE gives $I_p'' + 2I_p' + 3I_p = (-A + 4B)\cos 2t + (-4A - B)\sin 2t$, so $A = -1/17, B = 4/17$. Thus $I_p = (-1/17)\cos 2t + (4/17)\sin 2t$.

(b) Find the amplitude of the steady state solution (to two decimal digits).

Since I_c decays to 0 as t gets large, then the steady periodic part will just be I_p. Thus Amplitude $= \sqrt{A^2+B^2} = \frac{1}{\sqrt{17}} \approx 0.24$ volts to two decimals.