**92.236.201**

**Engineering DE's – Test 2 Spring 09** **Name:** ____________________

Box in your answer to each question. Please fold your exam into the test booklet. Good luck.

1. Consider the differential equation
$$\frac{dx}{dt} = -x(x-2)^2.$$
You are not expected to solve this DE explicitly (nor should you), in order to answer the questions below.
   (a) Find the critical points of this DE
   *$x = 0, 2$ are the two c.p.'s; Only $x = 0$ is stable.*
   (b) Sketch the phase line for this DE
   (c) Determine the stability of the critical points
   (d) Carefully sketch the 4 solution curves $x = x(t)$ for the initial conditions $x(0) = 0$, $x(0) = 2$, $x(0) = 3$, $x(0) = 4$.
   (e) With the help of the phase line, and the solution curves above, determine the $\lim x(t)$ as $t \to \infty$ if $x(0) = 3$.
   $\lim x(t) = 2$.

2. Solve the IVP
$$x'' + 4x' + 8x = 0; \;\; x(0) = 0, x'(0) = 2$$
*Char Poly* $(r^2 + 4r + 8) = 0$ *has* $r = -2 \pm 2i$ *as roots. So,* $x(t) = e^{-2t}(c_1 \cos 2t + c_2 \sin 2t)$*. IC's imply* $c_1 = 0, c_2 = 1$*, and thus* $x(t) = e^{-2t} \sin 2t$

3. Given the initial value problem
$$\frac{dy}{dx} = -xy + y^2, y(0) = 1.$$
Use Euler's numerical algorithm with step size $h = 0.1$ to estimate $y(0.1)$ and $y(0.2)$.
$y(0.1) \approx y(0) + (0.1)f(0, 1) = 1 + (.1)1 = 1.1 = y_1$*, Since* $f(x_1, y_1) = f(.1, 1.1) = -.11 + 1.21 = 1.1$*, then* $y(0.2) \approx y_2 = y_1 + 0.1f(x_1, y_1) = 1.1 + (.1)1.1 = 1.21$.

4. Suppose that the population $P(t)$ (in millions) of a country satisfies the differential equation
$$\frac{dP}{dt} = kP(200 - P)$$
with $k$ constant. Its population in 1940 was 100 million and was then growing at a rate of 1 million per year. In which year will the population exceed 160 million?
*At* $t = 0, P'(0) = 1 = kP(0)(200 - P(0)) = k10^4$*, giving* $k = 10^{-4}$*. Thus* $\frac{dp}{P(200-P)} = 10^{-4}dt$*. Since* $\frac{1}{P(200-P)} = (1/200)[\frac{1}{P} + \frac{1}{200-P}]$*, we have* $(1/200)[\ln P - ln(200 - P)] = 10^{-4}t + C$*. Setting* $P(0) = 100$*, yields* $C = 0$*. Solving for* $t = 50 \ln \frac{P}{200-P}$*. Thus* $P = 160$ *when* $t = 50 \ln 4 \approx 69.14$*. Thus early in the year 2009, the population will exceed 160 million for the first time.*

5. Consider the following differential equations for $u(t)$ the displacement of a spring from its relaxed length at time $t$.

(a) The unforced spring satisfies the homogenous DE

$$u'' + u' + 4u = 0$$

Find the solution for the unforced spring.
$u = e^{-t/2}[c_1 \cos(\sqrt{15}t/2) + c_2 \sin(\sqrt{15}t/2)]$

(b) A motor is turned on and drives the spring so that $u$ satisfies

$$u'' + u' + 4u = \sin t$$

Verify that $u_p(t) = (-1/10)\cos t + (3/10)\sin t$ is a particular solution to this forced spring system.

(c) Write down the general solution to forced spring system

$$u'' + u' + 4u = \sin t.$$

$u = e^{-t/2}[c_1 \cos(\sqrt{15}t/2) + c_2 \sin(\sqrt{15}t/2)] - (1/10)\cos t + (3/10)\sin t$