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# Constant Harvesting of a Biomass: A study in Population Growth

By ~~XXXXXXXXXX~~

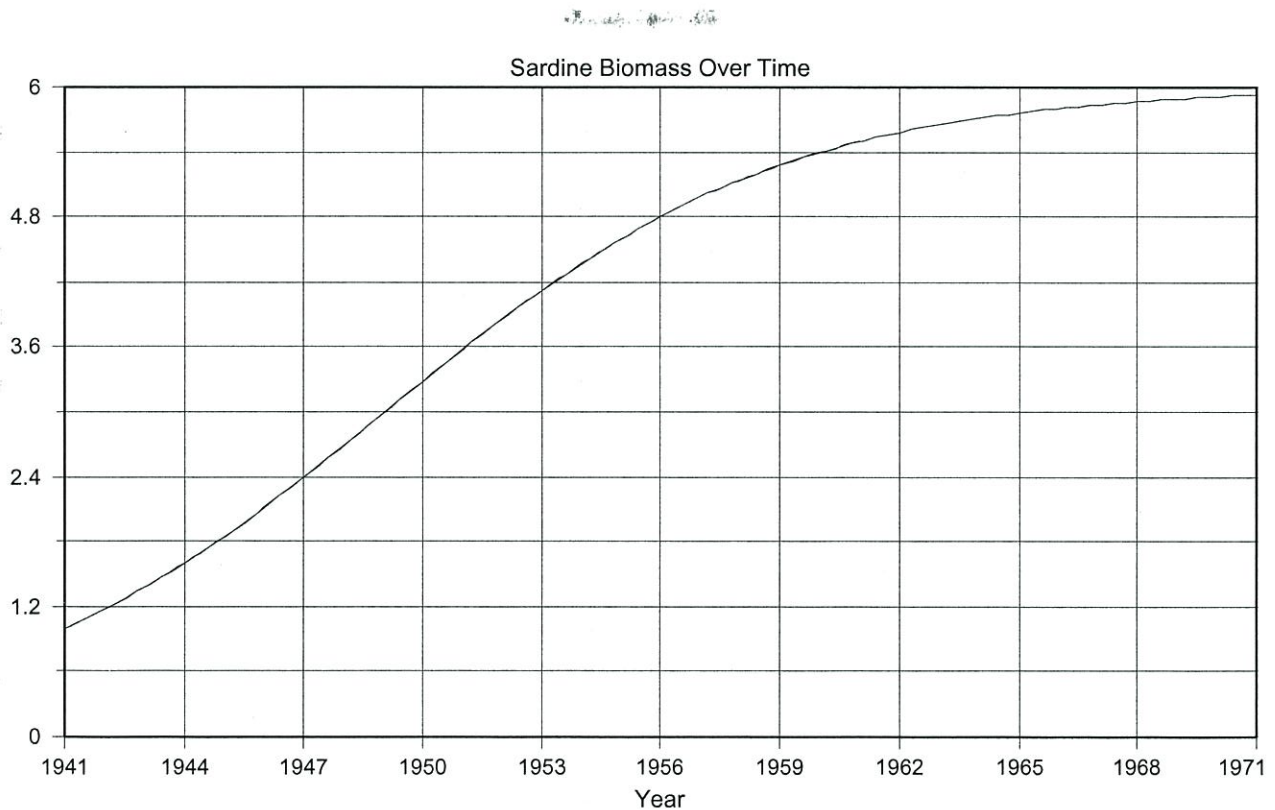
This is a study on population growth and how it is influenced by outside factors. Our model is based on the population of the Pacific sardine which was harvested to near depletion off the West coast of California by the late 1950s. Empirical data was used to help refine the equation used in this model.

We measure the sardine population in terms of its biomass. The biomass is the amount of sardines in this particular habitat and it is measured in millions of tons. There is a limit to the amount of biomass a habitat can support. This is known as the maximum biomass or carrying capacity of the habitat and this sets the upper limit for the population. In our study the maximum biomass is six million tons.

There is also a growth rate factor for the organism. It has been determined that the Pacific sardine grows at a rate of between 10% and 40% per year. Our model uses a 20% growth rate and a starting biomass of one million tons.

### Case 1: No Harvesting

We first would like to see what the population of the Pacific sardine would look like without the outside influence of being harvested. Notice that the population grows at a rate between 10% and 40% per year until leveling off at the carrying capacity of 600 million tons.



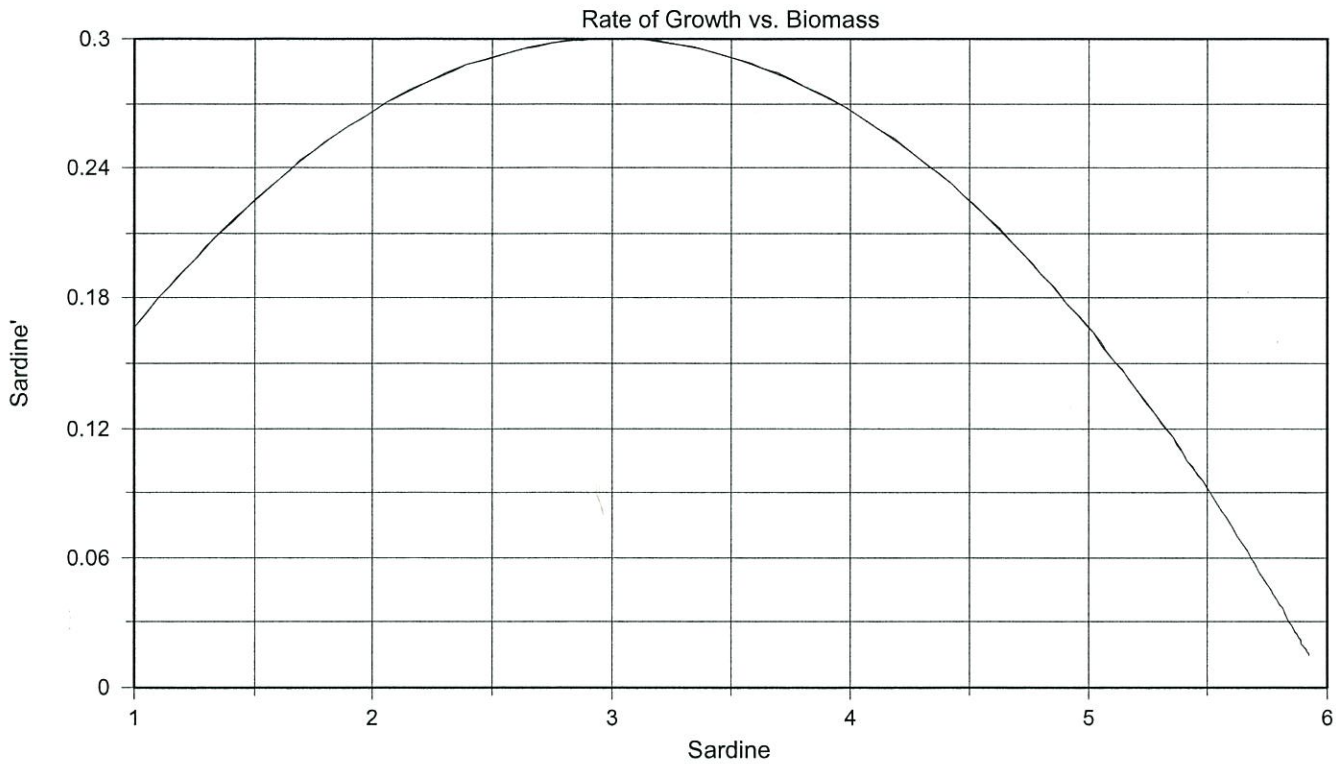
The graphs in this section are based on the equation

*no equation with word and equality*

$$\frac{ds}{dt} =$$

$$(growth\ rate) \times (sardine\ biomass) \times (carrying\ capacity - sardine\ biomass) / carrying\ capacity$$

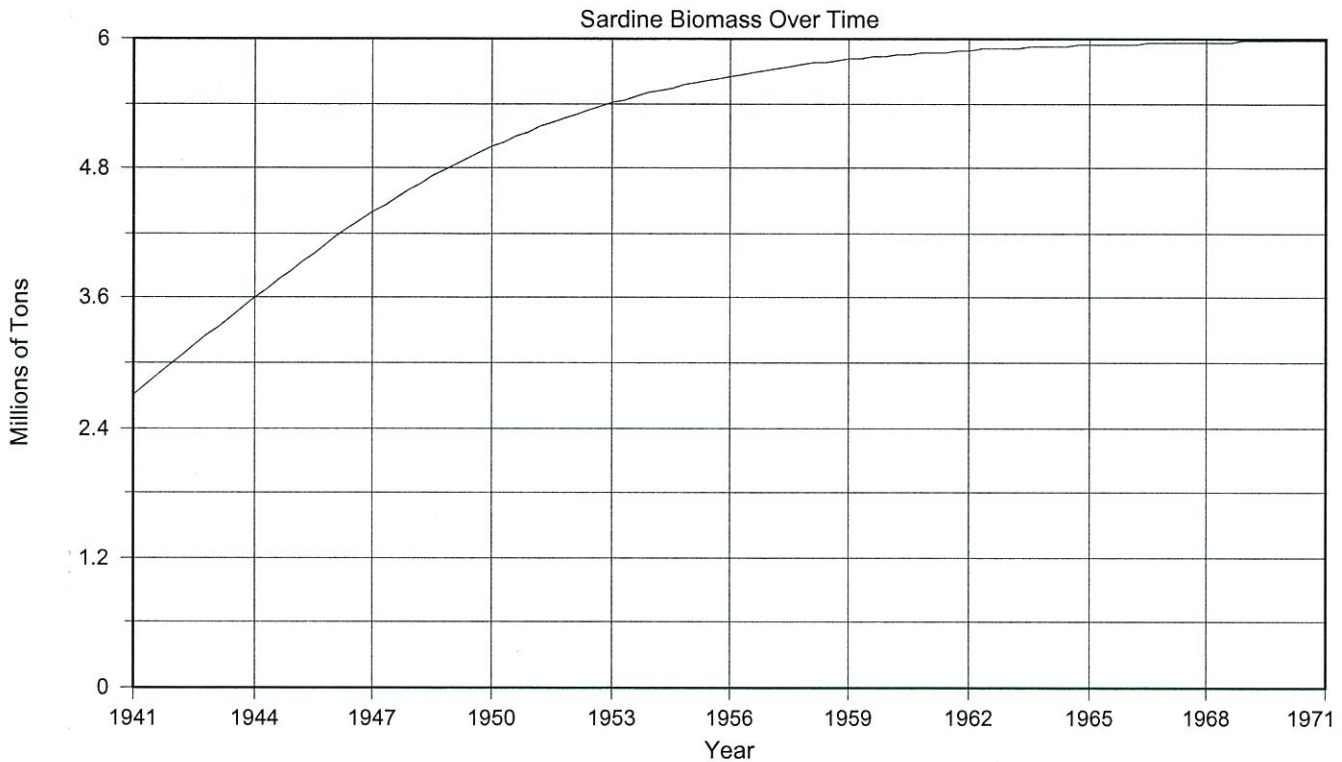
We can see the relationship between biomass and population growth more clearly by plotting the growth rate over the sardine biomass. We expect that the growth rate will rise as the population rises. This peaks at one half of the carrying capacity and diminishes as the population reaches this limit. ✓ good



note Population continues to increase as well since  $\frac{ds}{dt} > 0$   
- it's just not growing as fast as it did when  $P(t)$  was much smaller!

## Case 2: Constant Harvesting

Here we would like to study the effects of constant harvesting on the sardine population. To be more historically accurate we are changing the starting biomass value to what it actually was in 1941, 2.71 million tons. Our first graph is repeated here with this new initial value.



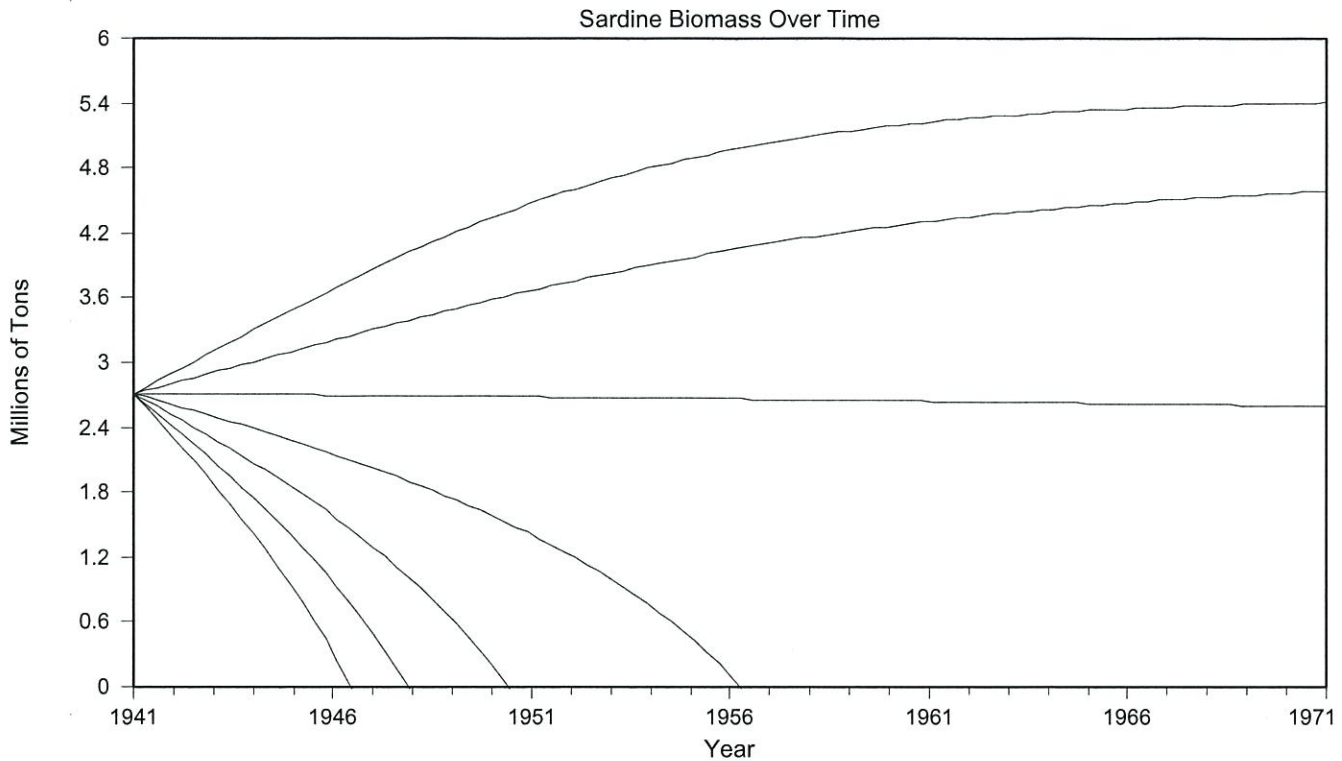
Notice how much more quickly the biomass reaches the carrying capacity of the environment with this higher initial value though the growth rate remains the same.

We now introduce harvesting to this system. We would like to find a harvest amount that would allow the population to sustain itself.

Our formula for the graphs that follow is

$$\frac{ds}{dt} = (\text{growth rate}) \times (\text{sardine biomass}) \times (\text{carrying capacity} - \text{sardine biomass}) / \text{carrying capacity} - \text{harvest}$$

Experimentally we can determine a value for *harvest* that meets our needs. The following graph plots seven different values for *harvest* starting with 0.1 (100,000 tons) and going to 0.7 (700,000) in 100,000 increments. This gives us a good idea of which way to go when we're refining our final value.



The lowest *harvest* value, 0.1, is shown by the top curve of the graph. This shows a population that levels off at around 5.4 million tons. We can see that more sardines can be harvested in this case while still sustaining a stable population.

The third line down has *harvest* value 0.3. We see a very slight downward curve here that over time would lead to a serious decline in population. Our optimal value should be slightly less than this.

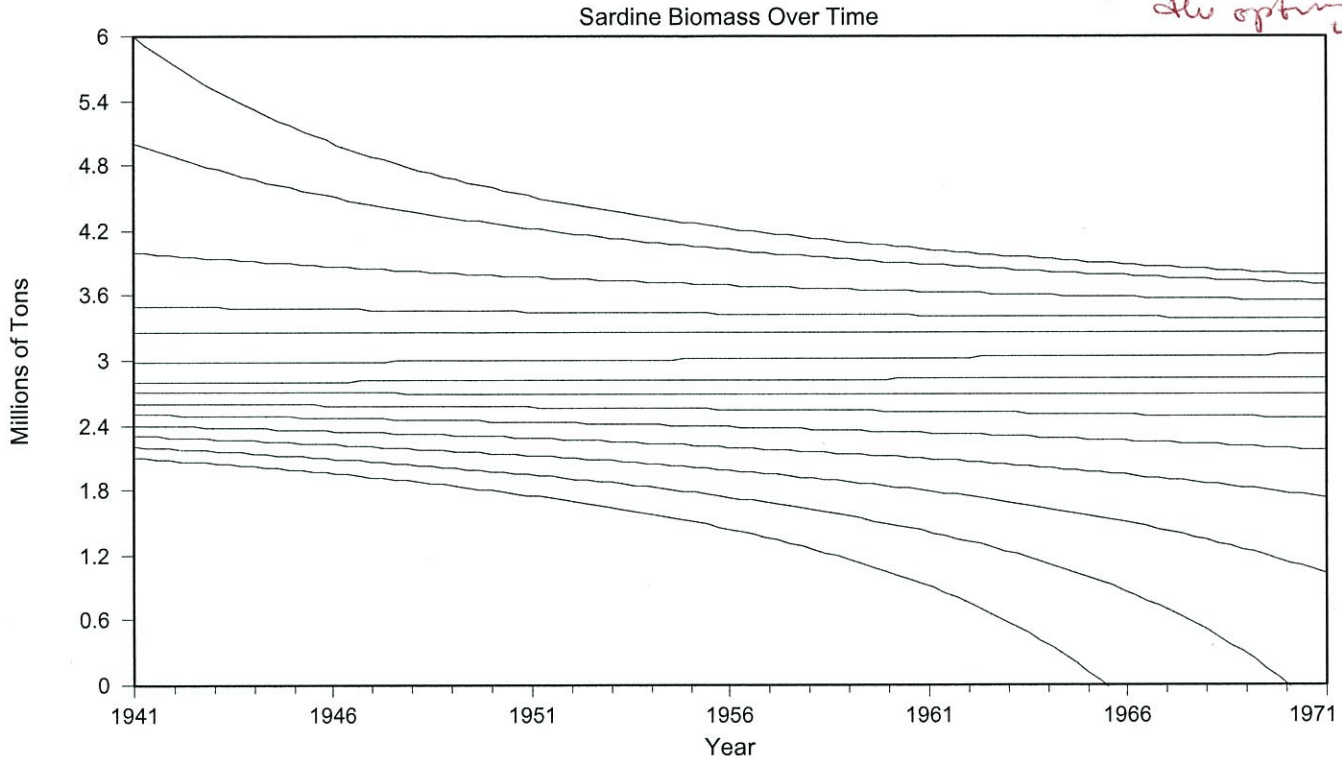
The last four lines are *harvest* values 0.4, 0.5, 0.6, and 0.7 respectively. The graph makes it obvious that harvesting at this rate would be disastrous for the sardine population. — *leads to extinction.*  
 Further experimentation finds that a harvest of 297,500 tons per year will keep the sardine population at a constant level. This agrees with the recommendations dating back to 1929 suggested by fishery researchers.

*↑ positive!*

### Case 3: Different Initial Conditions

Now that we have established an optimal harvest rate, we'd like to know what is affected if the initial conditions change. What happens if the starting biomass is less than in our original model? The following graph keeps the original equation but substitutes several different starting values above and below our optimal value.

*what is h?  
the optimal value  
in prev  
prob?*



Any initial biomass value below 2.71 million tons will only get lower with the passage of time. Lower initial values cannot sustain the harvest rate and can never increase in population. Higher initial values will decrease with harvesting and eventually level off at a level slightly above one half the maximum carrying capacity. From this we see how important the initial conditions are when determining the optimum constant harvest amount. It is better to underestimate the initial biomass when calculating the harvest rate to ensure a self sustaining population.