92.234

Diff Eqns – Test 3 Fall 06 **Name:** _____

Answer 5 all questions. Each problem is worth 20 points. Box in your answer(s) to each question. Good luck.

1. Set up the appropriate form of a particular solution y_p of the differential equation

$$y^{(5)} - y^{(3)} = e^x + 2x^2 - 5.$$

Do not determine the value of the coefficients.

2. Find the solution of the system of differential equations with given initial conditions

$$\begin{aligned} x' &= x + 3y + 4t \\ y' &= x - y \end{aligned}$$

Assume $x(0) = 5, y(0) = 0$.

3. Given the differential equation

$$x'' + 4x' + 4x = 10\cos(3t),$$

find the steady periodic solution and express it in the form $x_{sp} = C\cos(\omega t - \alpha)$. Determine C and α to four decimal place accuracy (if using a calculator).

4. Consider two 10 gallon tanks of water and suppose they are both filled with salty water – however, do not assume that the concentrations of salt in both tanks are identical.

Suppose that fresh water flows into Tank A at the rate of 1 gal/min and into Tank B at the rate of 2 gal/min. At the same time 3 gal/min of salty water flows out of Tank A. Finally suppose that well mixed solutions are exchanged between the two tanks as follows:

1 gal/min is pumped from Tank A to Tank B

3 gal/min is pumped from Tank B to Tank A.

Let $x(t)$ represent the amount of salt in Tank A at time t and $y(t)$ the amount of salt in Tank B at time t.

(a) Without doing any solving of DE's, but just from common sense, can you anticipate what is going to happen to $x(t)$ and $y(t)$ as $t \to \infty$?

(b) Express this exchange of salt between Tanks A and B as a linear system of differential equations.

(c) Using any of the methods which we have developed, determine the general solution. Note: you should obtain $x(t)$ and $y(t)$ which depend on only two parameters c_1 and c_2 which cannot be determined until $x(0)$ and $y(0)$ are specified.

(d) Does the answer above correspond to your answer in (a); i.e., determine the limit as $t \to \infty$ of your expressions for $x(t), y(t)$.

5. Consider the following spring system
   (a) Determine the spring constant k of a spring stretched 0.5m by a force of 2 N.
   (b) Suppose for that spring, a 2 kg mass is attached to the spring and released from rest from the stretched position $x_0 = 1$ m from the relaxed length. Suppose the damping constant c for the system is 4 (N sec/m) (i.e. the resistance force when stretched to x is given by $-4x'$ N.). Find the position $x(t)$ of this mass at time t.
   The DE is $2x'' + 4x' + 4x = 0$ *with IC's* $x(0) = 1$, $x'(0) = 0$. *Then* $x(t) = e^{-t}(\cos t + \sin t)$.