EECE.3600 Exam I

10/16/2017

Name: ________________________________

Signature: ________________________________

1. (30%) For a wave with frequency f = 1 MHz, the amplitude phasor at different position z can be written as: $\widetilde{A}(z) = 5e^{-j100z}$. (1) What is the instantaneous value of the amplitude $A(z,t)$ at different position z? (2) What is the direction the wave travels? (3) What is the wavelength of the wave?

Solution:

(1) $\widetilde{A}(z) = 5e^{-j100z} = \text{Re}\left[5e^{-j100z}e^{j\omega t}\right] = 5\cos(\omega t - 100z) = 5\cos\left(2\pi 10^6 t - 100z\right)$.

(2) The wave travels in the +z direction.

(3) $\beta = \dfrac{2\pi}{\lambda} = 100$, $\lambda = \dfrac{2\pi}{100} = 0.0628(m)$.

2. (30%) An air spaced lossless 50-Ω line ($\varepsilon_r = 1$) is terminated in a load with impedance of Z_L = 60 + j60-Ω at frequency 5GHz, Find (1) the reflection coefficient; (2) the voltage standing wave ratio (S) and (3) the location of the first voltage maximum from the load in centimeters.

(1) $\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0} = \dfrac{60 + j60 - 50}{60 + j60 + 50} = 0.49\angle 0.91$.

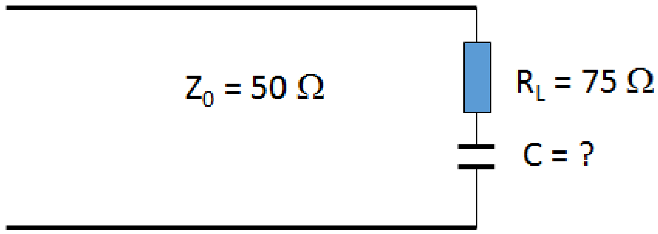
(2) $S = \dfrac{1+|\Gamma|}{1-|\Gamma|} = \dfrac{1+0.49}{1-0.49} = 2.9$.

(3) $l_{\max} = \dfrac{\theta_r}{4\pi}\lambda = 0.072\lambda$.

$$\lambda = \frac{c}{f} = \frac{3\times 10^8\, m/s}{5\times 10^9\, Hz} = 6cm$$

$l_{\max} = 0.43cm$.

3. (40%) A lossless 50-Ω line is terminated with a load. The capacitance is unknown. The voltage standing wave ratio (S) was measured to be S=3 at the frequency of 5GHz. (1) Find the capacitance; (2) what is the shortest length of the transmission line to make the Z_{in} real?



Solution:

(1) $S=\frac{1+|\Gamma|}{1-|\Gamma|}=3$, $|\Gamma|=0.5$.

$$|\Gamma|=\left|\frac{Z_L-Z_0}{Z_L+Z_0}\right|=\left|\frac{75+\frac{1}{j\omega C}-50}{75+\frac{1}{j\omega C}+50}\right|=\left|\frac{25+\frac{1}{j\omega C}}{125+\frac{1}{j\omega C}}\right|=\left[\frac{25^2+\left(\frac{1}{\omega C}\right)^2}{125^2+\left(\frac{1}{\omega C}\right)^2}\right]^{1/2}$$

$$\frac{25^2+\left(\frac{1}{\omega C}\right)^2}{125^2+\left(\frac{1}{\omega C}\right)^2}=\frac{1}{4},\ \left(\frac{1}{\omega C}\right)^2=4375\Omega^2,\left(\frac{1}{\omega C}\right)=66.1\Omega$$

$$\omega C=0.015\Omega^{-1},\ C=\frac{0.015\,\Omega^{-1}}{\omega}=0.48\,pF\ .$$

(2) $Z_L=75+\frac{1}{j\omega C}\Omega=75+j66.1\Omega$

$$\Gamma=\frac{Z_L-Z_0}{Z_L+Z_0}=\left|\frac{75+j66.1-50}{75+j66.1+50}\right|=0.5\angle 0.72.$$

Z_{in} is real at $|V|_{max}$ or $|V|_{min}$.

$$l_{max}=\frac{\theta_r}{4\pi}\lambda=0.057\lambda,\ l_{min}=\frac{\theta_r}{4\pi}\lambda+\frac{1}{4}\lambda=0.307\lambda$$

Voltage maximum Voltage minimum	$|\widetilde{V}|_{\text{max}} = |V_0^+|[1 + |\Gamma|]$ $|\widetilde{V}|_{\text{min}} = |V_0^+|[1 - |\Gamma|]$
Positions of voltage maxima (also positions of current minima)	$l_{\text{max}} = \dfrac{\theta_{\text{r}}\lambda}{4\pi} + \dfrac{n\lambda}{2}, \quad n = 0, 1, 2, \ldots$
Position of first maximum (also position of first current minimum)	$l_{\text{max}} = \begin{cases} \dfrac{\theta_{\text{r}}\lambda}{4\pi}, & \text{if } 0 \le \theta_{\text{r}} \le \pi \\ \dfrac{\theta_{\text{r}}\lambda}{4\pi} + \dfrac{\lambda}{2}, & \text{if } -\pi \le \theta_{\text{r}} \le 0 \end{cases}$
Positions of voltage minima (also positions of first current maxima)	$l_{\text{min}} = \dfrac{\theta_{\text{r}}\lambda}{4\pi} + \dfrac{(2n+1)\lambda}{4}, \quad n = 0, 1, 2, \ldots$
Position of first minimum (also position of first current maximum)	$l_{\text{min}} = \dfrac{\lambda}{4}\left(1 + \dfrac{\theta_{\text{r}}}{\pi}\right)$
Input impedance	$Z_{\text{in}} = Z_0 \left(\dfrac{Z_{\text{L}} + jZ_0 \tan \beta l}{Z_0 + jZ_{\text{L}} \tan \beta l}\right)$
Positions at which Z_{in} is real	at voltage maxima and minima
Z_{in} at voltage maxima	$Z_{\text{in}} = Z_0 \left(\dfrac{1 + |\Gamma|}{1 - |\Gamma|}\right)$
Z_{in} at voltage minima	$Z_{\text{in}} = Z_0 \left(\dfrac{1 - |\Gamma|}{1 + |\Gamma|}\right)$
Z_{in} of short-circuited line	$Z_{\text{in}}^{\text{sc}} = jZ_0 \tan \beta l$
Z_{in} of open-circuited line	$Z_{\text{in}}^{\text{oc}} = -jZ_0 \cot \beta l$
Z_{in} of line of length $l = n\lambda/2$	$Z_{\text{in}} = Z_{\text{L}}, \quad n = 0, 1, 2, \ldots$
Z_{in} of line of length $l = \lambda/4 + n\lambda/2$ Z_{in} of matched line	$Z_{\text{in}} = Z_0^2/Z_{\text{L}}, \quad n = 0, 1, 2, \ldots$ $Z_{\text{in}} = Z_0$
$|V_0^+|$ = amplitude of incident wave, $\Gamma = |\Gamma|e^{j\theta_{\text{r}}}$ with $-\pi < \theta_{\text{r}} < \pi$; θ_{r} in radians.	

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$