EEECE.3600 Exam I

03/05/2018

Name: ______________________________

Signature: ______________________________

1. (30%) A wave with the frequency of 1-MHz travels in the -z direction in air. Assume the wave travels at the speed of light (c = 3.0×10^8 m/s in air). If the wave reaches a peak value of 1.5 at z =100 m when t = 0.2 μs. Find:

1) (10 points) Wavelength in air

2) (10 points) Expression for the instantaneous of the wave (time domain)

3) (10 points) Expression for the wave in the phasor domain

Solution:

1). $\lambda f = c, \lambda = \frac{c}{f} = \frac{3\times 10^8 m/s}{10^6 Hz} = 300m$

2). $y(z,t) = 1.5\cos\left(\omega t + \frac{2\pi}{\lambda} z + \varphi_0\right)$

$1.5 = 1.5\cos\left(2\pi 10^6 \times 0.2 \times 10^{-6} + \frac{2\pi}{300} 100 + \varphi_0\right) \varphi_0 = 0.933\pi$

$y(z,t) = 1.5\cos\left(\omega t + \frac{2\pi}{\lambda} z + 0.933\pi\right)$

3) $\tilde{y}(z) = 1.2\pi e^{j\left(\frac{2\pi}{\lambda} z + 0.933\pi\right)}$

2 (35%) An air spaced lossless 50-Ω line ($\varepsilon_r = 1$) is terminated in a load with impedance of Z_L = 100 + j60-Ω at frequency 5GHz. Find (1) the reflection coefficient; (2) the voltage standing wave ratio (S) and (3) the location of the first voltage maximum from the load in centimeters.

Solutions:

(1) $\Gamma = \frac{100+j60-50}{100+j60+50} = 0.48\angle 28.4°$

(2) $S = \frac{1+|\Gamma|}{1-|\Gamma|} = 2.85$

(3) $l_{max} = \frac{\theta_r}{4\pi}\lambda = 0.04\lambda = 0.24cm$

3. (35%) A lossless 50-Ω line λ/8 is terminated in an unknown impedance. If the input impedance is Z_{in} = -j60-Ω. Find the load Z_L.

Solutions:

$$\beta L = \frac{2\pi}{\lambda}\frac{\lambda}{8} = \frac{\pi}{4},$$

$$\frac{Z_{in}}{Z_0} = \frac{Z_L + jZ_0 tan\beta L}{Z_0 + jZ_L tan\beta L} = \frac{Z_L + jZ_0}{Z_0 + jZ_L} = -j1.2$$

$$Z_L = j550\Omega$$

Voltage maximum Voltage minimum	$\lvert\widetilde{V}\rvert_{\max} = \lvert V_0^+\rvert[1+\lvert\Gamma\rvert]$ $\lvert\widetilde{V}\rvert_{\min} = \lvert V_0^+\rvert[1-\lvert\Gamma\rvert]$
Positions of voltage maxima (also positions of current minima) Position of first maximum (also position of first current minimum)	$l_{\max} = \dfrac{\theta_r\lambda}{4\pi} + \dfrac{n\lambda}{2}, \quad n = 0, 1, 2, \ldots$ $l_{\max} = \begin{cases} \dfrac{\theta_r\lambda}{4\pi}, & \text{if } 0 \le \theta_r \le \pi \\ \dfrac{\theta_r\lambda}{4\pi} + \dfrac{\lambda}{2}, & \text{if } -\pi \le \theta_r \le 0 \end{cases}$
Positions of voltage minima (also positions of first current maxima) Position of first minimum (also position of first current maximum)	$l_{\min} = \dfrac{\theta_r\lambda}{4\pi} + \dfrac{(2n+1)\lambda}{4}, \quad n = 0, 1, 2, \ldots$ $l_{\min} = \dfrac{\lambda}{4}\left(1 + \dfrac{\theta_r}{\pi}\right)$
Input impedance	$Z_{in} = Z_0 \left(\dfrac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right)$
Positions at which Z_{in} is real	at voltage maxima and minima
Z_{in} at voltage maxima	$Z_{in} = Z_0 \left(\dfrac{1+\lvert\Gamma\rvert}{1-\lvert\Gamma\rvert}\right)$
Z_{in} at voltage minima	$Z_{in} = Z_0 \left(\dfrac{1-\lvert\Gamma\rvert}{1+\lvert\Gamma\rvert}\right)$
Z_{in} of short-circuited line	$Z_{in}^{sc} = jZ_0 \tan\beta l$
Z_{in} of open-circuited line	$Z_{in}^{oc} = -jZ_0 \cot\beta l$
Z_{in} of line of length $l = n\lambda/2$	$Z_{in} = Z_L, \quad n = 0, 1, 2, \ldots$
Z_{in} of line of length $l = \lambda/4 + n\lambda/2$ Z_{in} of matched line	$Z_{in} = Z_0^2/Z_L, \quad n = 0, 1, 2, \ldots$ $Z_{in} = Z_0$

$\lvert V_0^+\rvert$ = amplitude of incident wave, $\Gamma = \lvert\Gamma\rvert e^{j\theta_r}$ with $-\pi < \theta_r < \pi$; θ_r in radians.

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$S = \frac{1+\lvert\Gamma\rvert}{1-\lvert\Gamma\rvert}$$