EECE.3600 Exam I

10/03/2016

Name: ______________________________

Signature: ______________________________

1. (35%) An air spaced lossless 50-Ω line ($\varepsilon_r = 1$) is terminated in a load with impedance of $Z_L = 60 + j60$-Ω at frequency 5GHz, Find (1) the reflection coefficient; (2) the voltage standing wave ratio (S) and (3) the location of the first voltage maximum from the load in centimeters.

Solutions:

(1) $\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0} = \dfrac{60 + j60 - 50}{60 + j60 + 50} = 0.49\angle 0.91$.

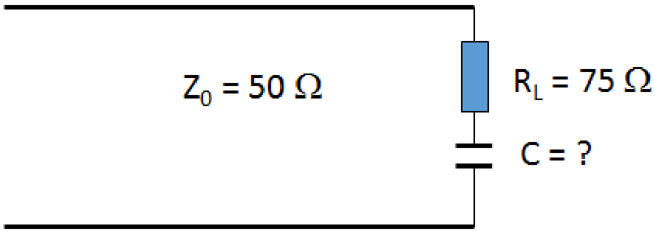
(2) $S = \dfrac{1 + |\Gamma|}{1 - |\Gamma|} = \dfrac{1 + 0.49}{1 - 0.49} = 2.9$.

(3) $l_{\max} = \dfrac{\theta_r}{4\pi}\lambda = 0.072\lambda$.

$$\lambda = \frac{c}{f} = \frac{3\times 10^8\, m/s}{5\times 10^9\, Hz} = 6cm$$

$l_{\max} = 0.43cm$.

2. (30%) A lossless 50-Ω line is terminated with a load. The capacitance is unknown. The voltage standing wave ratio (S) was measured to be 3 at the frequency of 5GHz, Find the capacitance.



Solution:

$$S = \frac{1+|\Gamma|}{1-|\Gamma|} = 3,\ |\Gamma| = 0.5$$

$$|\Gamma| = \left|\frac{Z_L - Z_0}{Z_L + Z_0}\right| = \left|\frac{75 + \frac{1}{j\omega C} - 50}{75 + \frac{1}{j\omega C} + 50}\right| = \left|\frac{25 + \frac{1}{j\omega C}}{125 + \frac{1}{j\omega C}}\right| = \left[\frac{25^2 + \left(\frac{1}{\omega C}\right)^2}{125^2 + \left(\frac{1}{\omega C}\right)^2}\right]^{1/2}$$

$$\frac{25^2 + \left(\frac{1}{\omega C}\right)^2}{125^2 + \left(\frac{1}{\omega C}\right)^2} = \frac{1}{4},\ \left(\frac{1}{\omega C}\right)^2 = 4375\Omega^2,$$

$$\omega C = 0.015\Omega^{-1},\ C = \frac{0.015\Omega^{-1}}{\omega} = 0.48pF\ .$$

3. (35%) A 5GHz voltage source is connected to a lossless transmission line is terminated with Z_L = -j75-Ω. The characteristic impedance of the transmission line is 50-Ω. Assuming the phase velocity of the transmission line is 0.8*c*, where *c* is the speed of light. (1) What's the voltage reflection coefficient? (2) Find out the location of the first $|V|_{min}$ from the load, (3) What's the standing wave ratio S? (4) how long the transmission line will make it equivalent to an open circuit?

Solutions:

(1) $$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{-j75 - 50}{j75 + 50} = 1\angle -1.18.$$

(2) $$l_{\min} = \frac{\theta_r}{4\pi}\lambda + \frac{\lambda}{4} = 0.16\lambda, \quad \lambda = \frac{0.8c}{5GHz} = 4.8cm.$$

$$l_{\min} = 0.16\lambda = 0.77cm.$$

(3) $$S = \frac{1+|\Gamma|}{1-|\Gamma|} = \infty.$$

(4) $$Z_{in} = \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} = \infty,$$

$$Z_0 + jZ_L \tan\beta l = 0, \quad Z_0 + jZ_L \tan\beta l = 0$$

$$50 + 75\tan\beta l = 0, \quad \tan\beta l = -\frac{2}{3}, \quad \tan(\pi - \beta l) = \frac{2}{3}, \quad \pi - \beta l = 0.588,$$

$$\beta l = \pi - 0.588, \quad l = \frac{\pi - 0.588}{4\pi}\lambda = 1.97cm.$$

Another method:

$$Z_{in} = Z_0 \frac{e^{j\beta l} + \Gamma e^{-j\beta l}}{e^{j\beta l} - \Gamma e^{-j\beta l}} = \infty, \quad e^{j\beta l} - \Gamma e^{-j\beta l} = 0, \quad e^{j2\beta l} = \Gamma,$$

$$e^{j2\beta l} = e^{-j1.18}, \quad e^{j2\beta l} = e^{j(2\pi - 1.18)}, \quad 2\beta l = (2\pi - 1.18),$$

$$l = \frac{(2\pi - 1.18)}{4\pi}\lambda = 1.97cm.$$

Voltage maximum Voltage minimum	$\lvert\widetilde{V}\rvert_{\max} = \lvert V_0^+\rvert[1+\lvert\Gamma\rvert]$ $\lvert\widetilde{V}\rvert_{\min} = \lvert V_0^+\rvert[1-\lvert\Gamma\rvert]$
Positions of voltage maxima (also positions of current minima) Position of first maximum (also position of first current minimum)	$l_{\max} = \dfrac{\theta_r\lambda}{4\pi} + \dfrac{n\lambda}{2}, \quad n = 0, 1, 2, \ldots$ $l_{\max} = \begin{cases} \dfrac{\theta_r\lambda}{4\pi}, & \text{if } 0 \le \theta_r \le \pi \\ \dfrac{\theta_r\lambda}{4\pi} + \dfrac{\lambda}{2}, & \text{if } -\pi \le \theta_r \le 0 \end{cases}$
Positions of voltage minima (also positions of first current maxima) Position of first minimum (also position of first current maximum)	$l_{\min} = \dfrac{\theta_r\lambda}{4\pi} + \dfrac{(2n+1)\lambda}{4}, \quad n = 0, 1, 2, \ldots$ $l_{\min} = \dfrac{\lambda}{4}\left(1 + \dfrac{\theta_r}{\pi}\right)$
Input impedance	$Z_{in} = Z_0\left(\dfrac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right)$
Positions at which Z_{in} is real	at voltage maxima and minima
Z_{in} at voltage maxima	$Z_{in} = Z_0\left(\dfrac{1+\lvert\Gamma\rvert}{1-\lvert\Gamma\rvert}\right)$
Z_{in} at voltage minima	$Z_{in} = Z_0\left(\dfrac{1-\lvert\Gamma\rvert}{1+\lvert\Gamma\rvert}\right)$
Z_{in} of short-circuited line	$Z_{in}^{sc} = jZ_0 \tan\beta l$
Z_{in} of open-circuited line	$Z_{in}^{oc} = -jZ_0 \cot\beta l$
Z_{in} of line of length $l = n\lambda/2$	$Z_{in} = Z_L, \quad n = 0, 1, 2, \ldots$
Z_{in} of line of length $l = \lambda/4 + n\lambda/2$ Z_{in} of matched line	$Z_{in} = Z_0^2/Z_L, \quad n = 0, 1, 2, \ldots$ $Z_{in} = Z_0$

$\lvert V_0^+\rvert$ = amplitude of incident wave, $\Gamma = \lvert\Gamma\rvert e^{j\theta_r}$ with $-\pi < \theta_r < \pi$; θ_r in radians.

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$S = \frac{1+\lvert\Gamma\rvert}{1-\lvert\Gamma\rvert}$$