1. (35%) A wave with the frequency of 1-MHz travels in the -z direction in air. Assume the wave travels at the speed of light (c = $3.0\text{x}10^8$m/s in air). If the wave reaches a peak value of 1.2 π at z =50 m when t=0. Find:

1) Wavelength in air

2) Expression for the instantenous of the wave (time domain)

3) Expression for the wave in the phasor domain

Solution:

1) $\lambda f = c$

$$\lambda = \frac{c}{f} = 300(m) .$$

2) $y(z,t) = 1.2\pi \cos\left(\omega t + \frac{2\pi}{\lambda} z + \phi_0\right)$

$$y(z = 50, t = 0) = 1.2\pi$$

$$\phi_0 = -\frac{2\pi}{\lambda} 50 = -\frac{\pi}{3} .$$

$$y(z,t) = 1.2\pi \cos\left(\omega t + \frac{2\pi}{\lambda} z - \frac{\pi}{3}\right)$$

3) $\tilde{y}(z) = 1.2\pi e^{j\left(\frac{2\pi}{\lambda} z - \frac{\pi}{3}\right)}$

2. (30%) For a lossless transmission line, the characteristic impedance Z_0 = 50 Ω. If the load is Z_L=25+j25 Ω, (1) find out the reflection coefficient; (2) the VSWR?

Solution:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{25 + j25 - 50}{25 + j25 + 50} = 0.45\angle 0.65\pi$$

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = 2.6$$

3. (35%) A 10GHz voltage source is connected to a lossless transmission line is terminated with Z_L = -j75-Ω. The characteristic impedance of the transmission line is 50-Ω. Assuming the phase velocity of the transmission line is 0.8*c*, where *c* is the speed of light. (1) What's the voltage reflection coefficient? (2) Find out the location of the first $|V|_{min}$ from the load, (2) What's the standing wave ratio S? (3) how long the transmission line will make it equivalent to an open circuit?

Solution:

(1) $\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0} = \dfrac{-j75-50}{-j75+50} = 1\angle -0.37\pi$.

(2) |V|min happens at: $2\beta z + \theta_r = (2n+1)\pi$, i.e. $2\beta z - 0.37\pi = (2n+1)\pi$

To make the z negative, n = -1.

$$2\beta z = -\pi + 0.37\pi$$

$$z = \frac{\lambda}{4\pi}(-0.63\pi) = -0.15\lambda$$

(3) S = ∞

(4) $Z_{in} = Z_0 \dfrac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}$

Open circuit means Zin = ∞, i.e. $Z_0 + jZ_L \tan\beta l = 0$.

$$50 + 75\tan\beta l = 0 ,$$

$$\tan\beta l = -\frac{2}{3}$$

$$l = 0.4\lambda$$

Table 2-3: Properties of standing waves on a lossless transmission line.

Voltage maximum Voltage minimum	$\lvert\tilde{V}\rvert_{\max} = \lvert V_0^+\rvert[1 + \lvert\Gamma\rvert]$ $\lvert\tilde{V}\rvert_{\min} = \lvert V_0^+\rvert[1 - \lvert\Gamma\rvert]$ $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$ $S = \frac{\lvert\Gamma\rvert+1}{1-\lvert\Gamma\rvert}$
Positions of voltage maxima (also positions of current minima) Position of first maximum (also position of first current minimum)	$l_{\max} = \frac{\theta_r\lambda}{4\pi} + \frac{n\lambda}{2}, \quad n = 0, 1, 2, \ldots$ $l_{\max} = \begin{cases} \frac{\theta_r\lambda}{4\pi}, & \text{if } 0 \le \theta_r \le \pi \\ \frac{\theta_r\lambda}{4\pi} + \frac{\lambda}{2}, & \text{if } -\pi \le \theta_r \le 0 \end{cases}$
Positions of voltage minima (also positions of first current maxima) Position of first minimum (also position of first current maximum)	$l_{\min} = \frac{\theta_r\lambda}{4\pi} + \frac{(2n+1)\lambda}{4}, \quad n = 0, 1, 2, \ldots$ $l_{\min} = \frac{\lambda}{4}\left(1 + \frac{\theta_r}{\pi}\right)$
Input impedance	$Z_{in} = Z_0\left(\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right)$
Positions at which Z_{in} is real	at voltage maxima and minima
Z_{in} at voltage maxima	$Z_{in} = Z_0\left(\frac{1 + \lvert\Gamma\rvert}{1 - \lvert\Gamma\rvert}\right)$
Z_{in} at voltage minima	$Z_{in} = Z_0\left(\frac{1 - \lvert\Gamma\rvert}{1 + \lvert\Gamma\rvert}\right)$
Z_{in} of short-circuited line	$Z_{in}^{sc} = jZ_0 \tan\beta l$
Z_{in} of open-circuited line	$Z_{in}^{oc} = -jZ_0 \cot\beta l$
Z_{in} of line of length $l = n\lambda/2$	$Z_{in} = Z_L, \quad n = 0, 1, 2, \ldots$
Z_{in} of line of length $l = \lambda/4 + n\lambda/2$ Z_{in} of matched line	$Z_{in} = Z_0^2/Z_L, \quad n = 0, 1, 2, \ldots$ $Z_{in} = Z_0$

$\lvert V_0^+\rvert$ = amplitude of incident wave, $\Gamma = \lvert\Gamma\rvert e^{j\theta_r}$ with $-\pi < \theta_r < \pi$; θ_r in radians.

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Name: ______________________________

Signature: ______________________________