1. (30 %) A 12cm long lossless 50-Ω line is terminated with Z_L = -j25-Ω. We know that the interval of voltage maximums is 20cm. Find the following:

(a) The standing wave ratio (S)

(b) Where is the first $|V|_{max}$ from the load?

Solution : (a) $\lambda/2 = 20\,cm \Rightarrow \lambda = 40\,cm$

$$\Gamma = \frac{Z_L - 50}{Z_L + 50} \Rightarrow \begin{array}{l} |\Gamma| = 1 \\ \theta_r = -126^\circ \end{array}$$

$$S = \frac{1+|\Gamma|}{1-|\Gamma|} = \infty$$

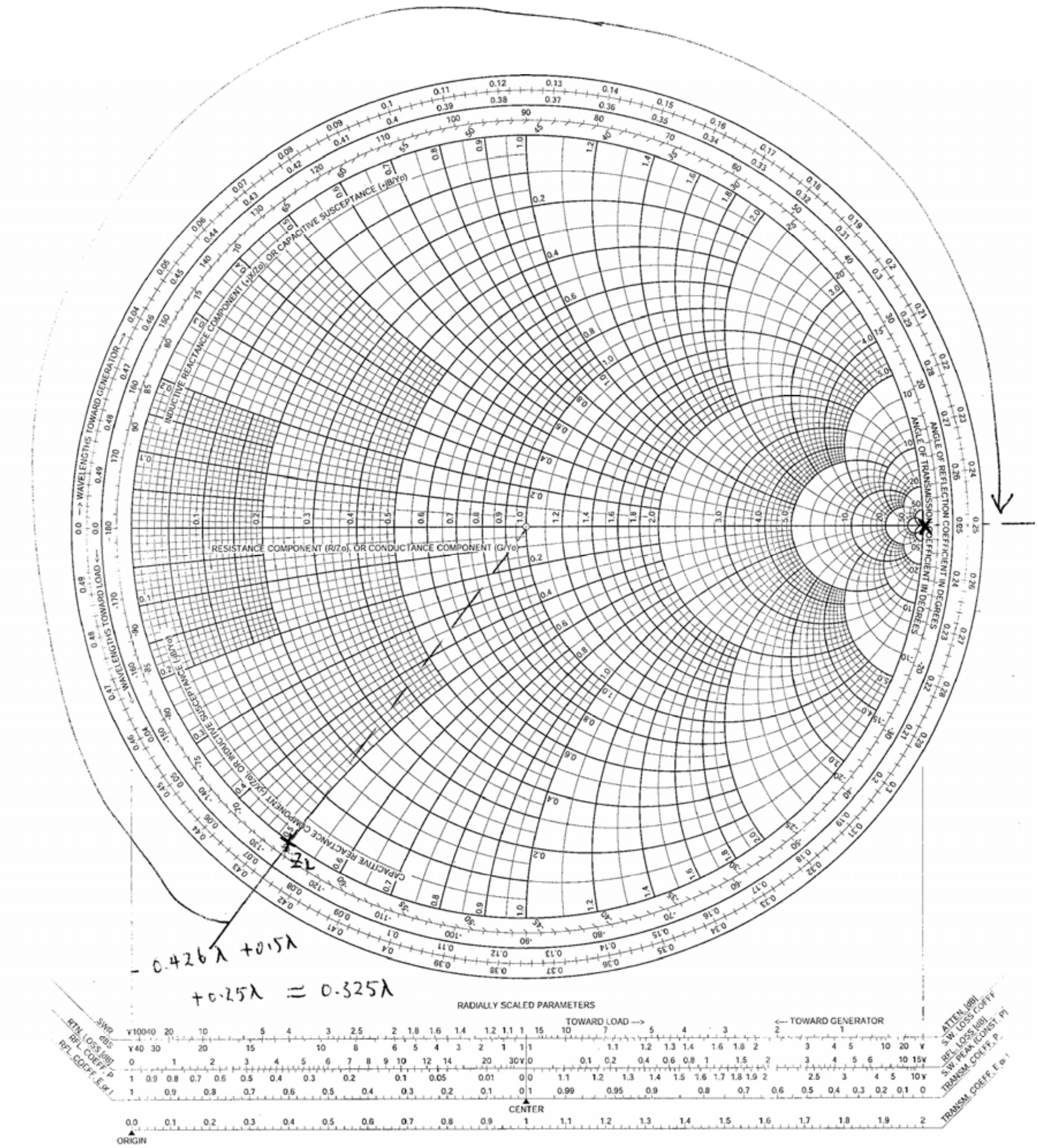
(b). $$2\beta L + \theta_r = -2\pi$$

$$\Rightarrow 2\beta L = -360^\circ + 126^\circ$$

$$\Rightarrow L = -0.325\lambda = 0.325 \cdot 40\,cm$$

$$= -13\,cm.$$

the voltage ~~ma~~ magnitude keeps increasing until it reaches the source.



$-0.426\lambda + 0.5\lambda$

$+0.25\lambda = 0.325\lambda$

2. (30%) On a lossless 50-Ω transmission line terminated with a $Z_L = 60+j80\ \Omega$. If this transmission line is be matched to the load using a shorted load stub. Determine the stub length and distance between the load and stub. Two possible answers. You only need to show one of them.

Solution (1):

$$z_L = \frac{Z_L}{Z_0} = 1.2 + j1.6 \quad \text{point A}$$

admitance point B:

the distance between the load and the short stub:

$$d = 0.174\lambda + (0.5 - 0.44)\lambda = 0.234\lambda$$

point C reading: $1.0 + 1.4j$.

the length ~~between~~ of the short stub:

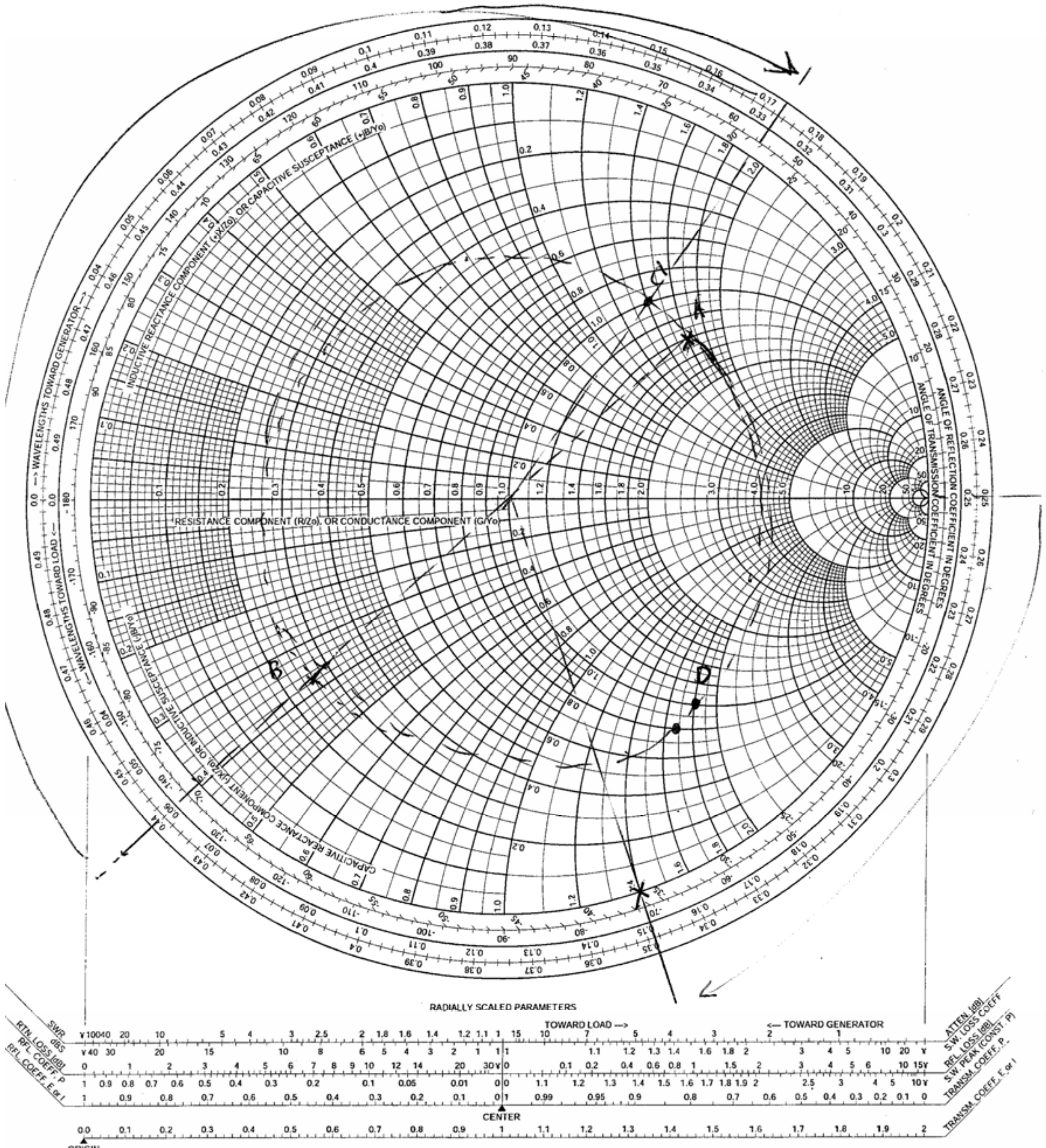
$$l = 0.348\lambda - 0.25\lambda = 0.098\lambda.$$

Solution (2):

$$d = 0.334\lambda + (0.5 - 0.44\lambda) = 0.394\lambda$$

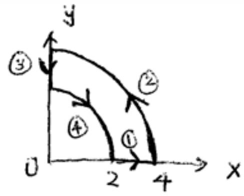
$$l = 0.50\lambda + 0.15\lambda - 0.25\lambda \approx 0.40\lambda$$

Solution 1:



A
B
C
D
RADIALLY SCALED PARAMETERS
TOWARD LOAD —>
<— TOWARD GENERATOR
CENTER
ORIGIN
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
INDUCTIVE REACTANCE COMPONENT (+jX/Zo), OR CAPACITIVE SUSCEPTANCE (+jB/Yo)
CAPACITIVE REACTANCE COMPONENT (-jX/Zo) OR INDUCTIVE SUSCEPTANCE (-jB/Yo)
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES
WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
SWR
dBS
RTN. LOSS [dB]
RFL. COEFF, P
RFL. COEFF, E or I
ATTEN. [dB]
S.W. LOSS COEFF
RFL. LOSS [dB]
S.W. PEAK (CONST. P)
TRANSM. COEFF, P
TRANSM. COEFF, E or I

3. (40%) Verify Stokes's theorem for the vector field $\vec{A} = \hat{r} r\cos\phi + \hat{\phi}\sin\phi$ along the semicircular contour shown below:



Solution: (a) $\oint \vec{A}\cdot d\vec{l}$

$$= \int_{①} \vec{A}\cdot d\vec{l} + \int_{②} \vec{A}\cdot d\vec{l} + \int_{③} \vec{A}\cdot d\vec{l} + \int_{④} \vec{A}\cdot d\vec{l}$$

$$\int_{①} \vec{A}\cdot d\vec{l} = \int_{①} \left(\hat{r}\, r\cos\phi + \hat{\phi}\sin\phi\right)\cdot \hat{r}\, dr \Big|_{\phi=0}$$

$$= \int_2^4 r\, dr = \frac{r^2}{2}\Big|_2^4 = 6$$

$$\int_{②} \vec{A}\cdot d\vec{l} = \int_{②} \left(\hat{r}\, r\cos\phi + \hat{\phi}\sin\phi\right)\cdot \hat{\phi}\, r\, d\phi \Big|_{r=4}$$

$$= \int_0^{\frac{\pi}{2}} r\sin\phi\, d\phi \Big|_{r=4} = -4\cos\phi\Big|_0^{\frac{\pi}{2}} = +4$$

$$\int_{③} \vec{A}\cdot d\vec{l} = \int \left(\hat{r}\, r\cos\phi + \hat{\phi}\sin\phi\right)\cdot(-\hat{r})\, dr \Big|_{\phi=\frac{\pi}{2}} = 0$$

$$\int_{④} \vec{A}\cdot d\vec{l} = \int_{0}^{\frac{\pi}{2}} \left(\hat{r}\, r\cos\phi + \hat{\phi}\sin\phi\right)(-\hat{\phi})\, r\, d\phi \Big|_{r=2}$$

$$= \int_0^{\frac{\pi}{2}} - r\sin\phi\, d\phi \Big|_{r=2} = +2\cdot\cos\phi\Big|_0^{\frac{\pi}{2}} = -2$$

$$\Rightarrow \oint \vec{A}\cdot d\vec{l} = 6 + (+4) + \emptyset + (-2) = 8$$

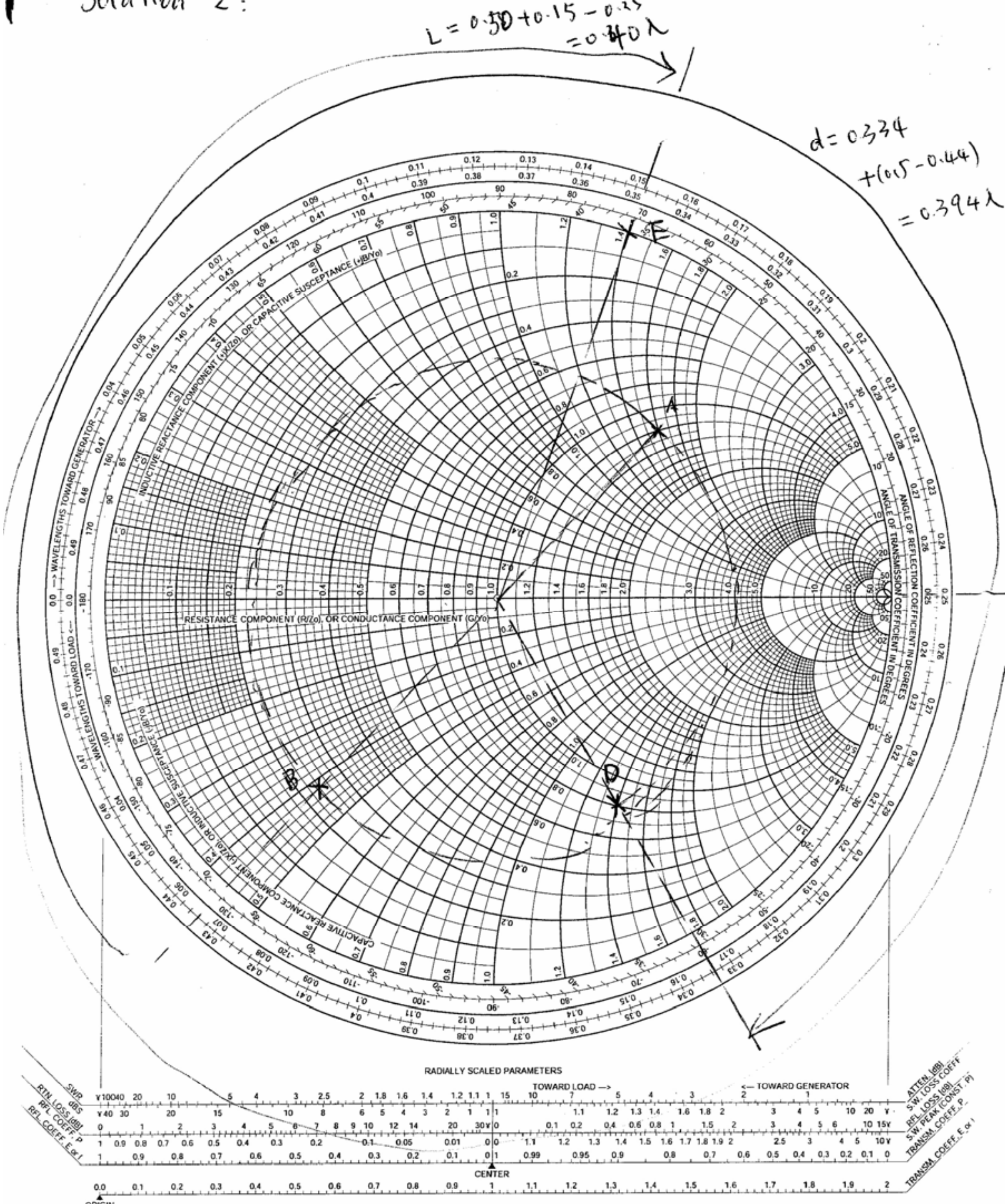
$$\iint_S (\nabla\times\vec{A})\cdot d\vec{s} = \iint \left[\left(\frac{1}{r}\right)\left[\frac{\partial}{\partial r}(r\sin\phi) - \frac{\partial\, r\cos\phi}{\partial\phi}\right]\right]\cdot r\, d\phi\, dr$$

$$= \int_0^{\frac{\pi}{2}}\int_2^4 (\sin\phi + r\sin\phi)\, d\phi\, dr$$

$$= 2\cdot(-\cos\phi)\Big|_0^{\frac{\pi}{2}} + \frac{r^2}{2}\Big|_2^4 \cdot (-\cos\phi)\Big|_0^{\frac{\pi}{2}}$$

$$= 2 + 6 = 8$$

Solution 2:



L = 0.50 + 0.15 − 0.25
= 0.40λ
d = 0.334
+(0.15 − 0.44)
= 0.394λ
A
B
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E
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