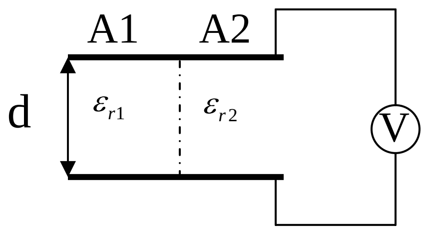
1. (10 points) On a lossless 50-Ω transmission line terminated with a Z_L = 100+j50 Ω. If this transmission line is to be matched to the load using a shorted load stub. Determine the stub length and distance between the load and stub. Two possible answers. You only need to show one of them.

2. (10 points) A 50cm long air spaced lossless 50-Ω line ($\varepsilon_r = 1$) is terminated in an unknown impedance. If the input impedance is Zin = 20 + j30-Ω at frequency 10GHz, Find (1) the reflection coefficient; (2) the location of the first voltage maximum from the load.

3. (10 points) A section of a sphere is described by $1 \le R \le 2$, $0 \le \theta \le \frac{\pi}{2}$, $-\pi < \phi \le \pi$. A vector field $\vec{E} = \frac{1}{2\pi R^2}\hat{R}$. Verify divergence theorem by calculating: $\oiint_s \vec{E} \cdot d\vec{s} = \iiint_v (\nabla \cdot \vec{E}) dv$.

4. (10 points) Determine the line integral of $\vec{F} = 2\hat{x} + z\hat{y} + y\hat{z}$ from P1(-1,3,-2) via P2(2,4,1) to P3 (3,5,6), where points are specified in rectangular coordinate system.

5. (10 points) A parallel-plate capacitor has two dielectric filling regions between the plate, A1 = 10mm^2, A2 = 20mm^2, d = 10mm, V = 10V $\varepsilon_{r1} = 1.5$, $\varepsilon_{r2} = 3$, ignore fringing field effect. $\varepsilon_0 = 8.85 \times 10^{-10} F/m$.



Determine:

(a) Electric field E_1, E_2 of the two regions.

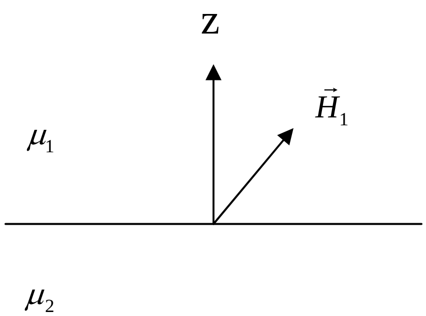
(b) The capacitance of the capacitor.

6. (10 points) In a certain conducting region, the magnetic field is given in cylindrical coordinates by $\vec{H} = \hat{\phi}\frac{4}{r}\left[1-(1+2r)e^{-2r}\right]$. Find out the current density $\vec{J}$.

7. (10 points) The x-y plane separates two magnetic media with magnetic permeability: $\mu_1 = \mu_0, \mu_2 = 2\mu_0$, if we know $\vec{H}_1 = 3\hat{x} + 4\hat{y} + 5\hat{z}$ (A/m). $\mu_0 = 4\pi \times 10^{-7}$ (H/m).

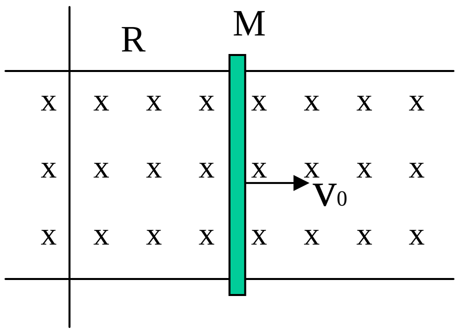
(a) if no surface current, determine: $\vec{H}_2$

(b) if surface current $\vec{J}_s = 1\hat{x} + 2\hat{y}$ (A/m) determine $\vec{H}_2$.



8. (10 points) Write down the Maxwell equations and their integral forms.

9. (10 points) A conducting bar is put in a constant magnetic field B = 3T. The circuit resistance is R =20Ω. If the bar is moving at a speed of V = 10m/s, determine (1) the current generated in the circuit; (2) the force experienced by the conducting bar.



10. (10 points) For an electromagnetic wave of a single frequency f = 100MHz, the magnetic filed is: $\vec{H}(t,z) = \hat{x}\cos(2\pi ft - kz)$. If $\varepsilon = 2\varepsilon_0 = 1.8\times10^{-11} F/m$ and $\mu = \mu_0 = 4\pi\times10^{-7}$ (H/m).

Find out: (1) k ; (2) electric field E(t);

GRADIENT, DIVERGENCE, CURL, & LAPLACIAN OPERATORS

CARTESIAN (RECTANGULAR) COORDINATES (x, y, z)

$$\nabla V = \mathbf{x}\frac{\partial V}{\partial x} + \mathbf{y}\frac{\partial V}{\partial y} + \mathbf{z}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \begin{vmatrix} \mathbf{x} & \mathbf{y} & \mathbf{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \mathbf{x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \mathbf{y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \mathbf{z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

CYLINDRICAL COORDINATES (r, ϕ, z)

$$\nabla V = \mathbf{r}\frac{\partial V}{\partial r} + \boldsymbol{\phi}\frac{1}{r}\frac{\partial V}{\partial \phi} + \mathbf{z}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \frac{1}{r}\begin{vmatrix} \mathbf{r} & \boldsymbol{\phi} r & \mathbf{z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix} = \mathbf{r}\left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \boldsymbol{\phi}\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \mathbf{z}\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial \phi}\right]$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

SPHERICAL COORDINATES (R, θ, ϕ)

$$\nabla V = \mathbf{R}\frac{\partial V}{\partial R} + \boldsymbol{\theta}\frac{1}{R}\frac{\partial V}{\partial \theta} + \boldsymbol{\phi}\frac{1}{R\sin\theta}\frac{\partial V}{\partial \phi}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 A_R) + \frac{1}{R\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \mathbf{A} = \frac{1}{R^2\sin\theta}\begin{vmatrix} \mathbf{R} & \boldsymbol{\theta} R & \boldsymbol{\phi} R\sin\theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & RA_\theta & (R\sin\theta)A_\phi \end{vmatrix}$$

$$= \mathbf{R}\frac{1}{R\sin\theta}\left[\frac{\partial}{\partial \theta}(A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial \phi}\right] + \boldsymbol{\theta}\frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial \phi} - \frac{\partial}{\partial R}(RA_\phi)\right] + \boldsymbol{\phi}\frac{1}{R}\left[\frac{\partial}{\partial R}(RA_\theta) - \frac{\partial A_R}{\partial \theta}\right]$$

$$\nabla^2 V = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

EE 16.360 Final Exam

05/15/2006

Name: ______________________________

Signature: ______________________________