EECE.3600 Final Exam

12/20/2017

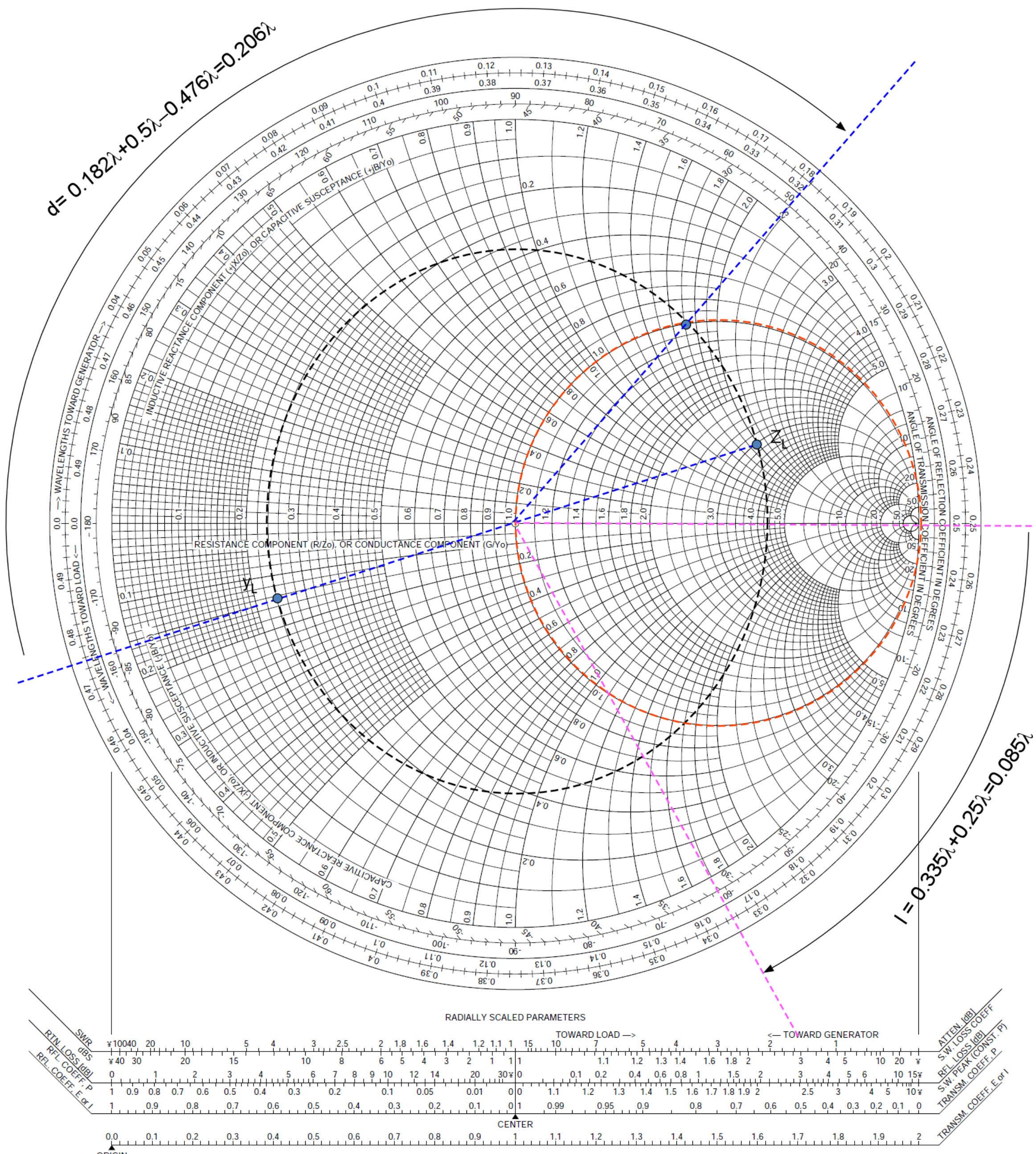
Name: ______________________________

Signature: ______________________________

1. (10 points) On a lossless 50-Ω transmission line terminated with a $Z_L = 150+j100\ \Omega$. If this transmission line is matched to the load using a shorted load stub. Determine the stub length and distance between the load and stub. Two possible answers. You only need to show one of them.

Solution:

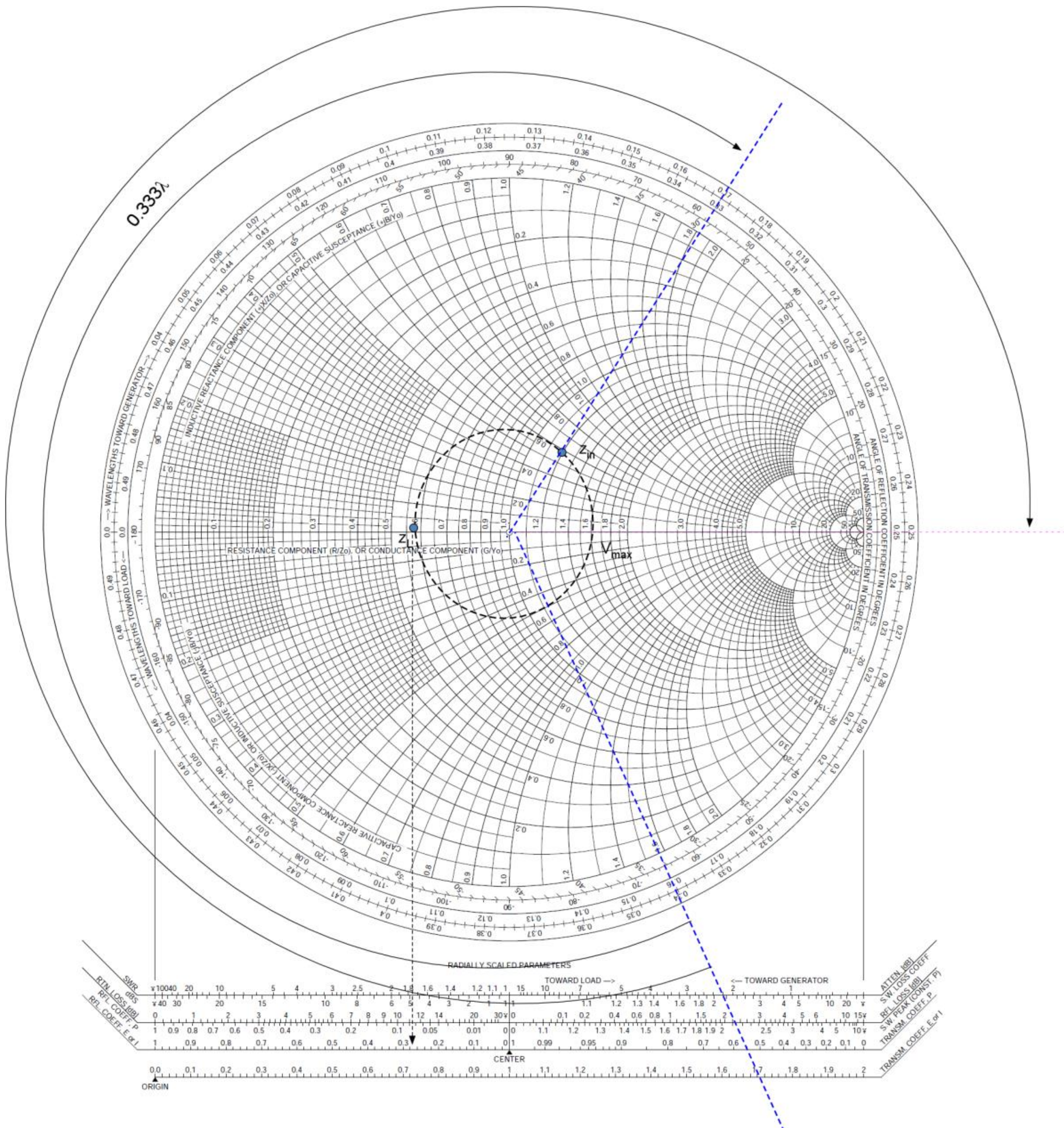
See Smith Chart



2. (10 points) A 20-cm long air spaced lossless 50-Ω line (ε_r = 1) is terminated in an unknown impedance. If the input impedance is Z_{in} = 60 + j30-Ω at frequency 5GHz, Find (1) the reflection coefficient; (2) the standing wave ratio (S) and (3) the location of the first voltage maximum from the load.

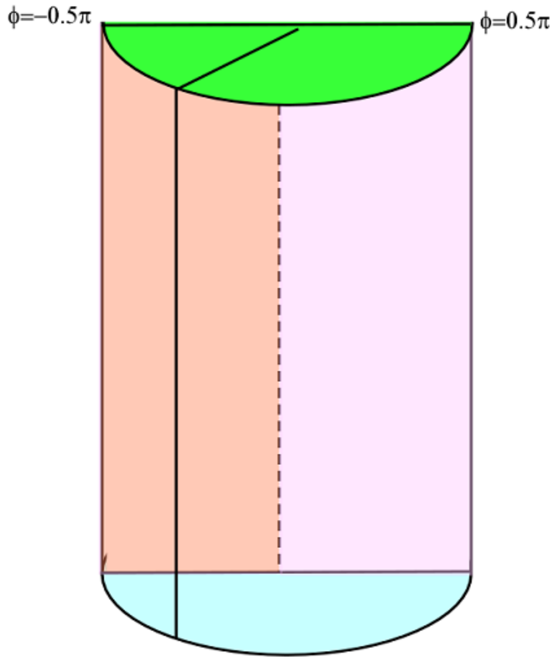
$\lambda = \dfrac{c}{f} = \dfrac{3\times10^{8} m/s}{5\times10^{9} Hz} = 6cm$. $z_{in} = \dfrac{Z_{in}}{Z_0} = 1.2 + j0.6$

$l = \dfrac{20cm}{6cm} = 3.333\lambda$. $S = 1.7$, $\Gamma = 0.3\angle-68^{0}$. Location at 0.41λ from load.



3. (10 points) For a vector filed $\vec{A} = r^2\hat{r} + 3r\phi\hat{\phi} - 2\hat{z}$, verify the divergence theorem

$\oiint_s \vec{A}\cdot d\vec{s} = \iiint_v (\nabla\cdot\vec{A})dv$, on a section of a cylinder bounded by $r=1,\ -\frac{\pi}{2}\le\phi\le\frac{\pi}{2},\ 1\le z\le 3.$



Solution:

$$\oiint_s \vec{A}\cdot d\vec{s} = \iint_{top}\vec{A}\cdot d\vec{s} + \iint_{bottom}\vec{A}\cdot d\vec{s} + \iint_{side}\vec{A}\cdot d\vec{s} + \iint_{left}\vec{A}\cdot d\vec{s} + \iint_{right}\vec{A}\cdot d\vec{s},$$

$$\iint_{top}\vec{A}\cdot d\vec{s} = \iint_{top} -2\hat{z}\cdot\hat{z}rdrd\phi = -\pi, \quad \iint_{bottom}\vec{A}\cdot d\vec{s} = \iint_{bottom} -2\hat{z}\cdot(-\hat{z})rdrd\phi = \pi,$$

$$\iint_{side}\vec{A}\cdot d\vec{s} = \iint_{side} r^2\hat{r}\cdot\hat{r}rd\phi dz\Big|_{r=1} = 2\pi,$$

$$\iint_{left}\vec{A}\cdot d\vec{s} = \iint_{left} 3r\phi\hat{\phi}\cdot\hat{\phi}drdz\Big|_{\phi=\pi/2} = \frac{3\pi}{2},$$

$$\iint_{right}\vec{A}\cdot d\vec{s} = \iint_{right} 3r\phi\hat{\phi}\cdot\left(-\hat{\phi}\right)drdz\Big|_{\phi=-\pi/2} = \frac{3\pi}{2},$$

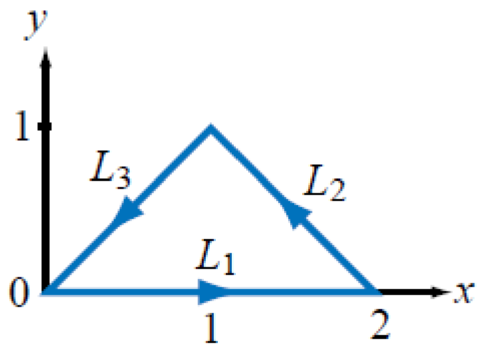
$$\oiint_s \vec{A}\cdot d\vec{s} = \iint_{top}\vec{A}\cdot d\vec{s} + \iint_{bottom}\vec{A}\cdot d\vec{s} + \iint_{side}\vec{A}\cdot d\vec{s} + \iint_{left}\vec{A}\cdot d\vec{s} + \iint_{right}\vec{A}\cdot d\vec{s} = 5\pi.$$

$$\nabla\cdot\vec{A} = \frac{1}{r}\frac{\partial}{\partial r}\left(rr^2\right) + \frac{1}{r}\frac{\partial}{\partial\phi}\left(3r\phi\right) = 3r+3,$$

$$\iiint_v (\nabla\cdot\vec{A})dv = \int_1^3 dz\int_{-\pi/2}^{-\pi/2} d\phi\int_0^1 (3r+3)rdr = 2\pi\left(r^3 + \frac{3}{2}r^2\right)\Bigg|_0^1 = 5\pi,$$

$$\oiint_s \vec{A}\cdot d\vec{s} = \iiint_v (\nabla\cdot\vec{A})dv.$$

4. (10 points) Verify Stokes's theorem for the vector field $\vec{A} = \hat{x}y^3 + \hat{y}x^3$ along the path shown below:



Solution:

$$\oint \vec{A}\cdot d\vec{l} = \int_{L_1} \vec{A}\cdot d\vec{l} + \int_{L_2} \vec{A}\cdot d\vec{l} + \int_{L_3} \vec{A}\cdot d\vec{l}\ .$$

$$\int_{L_1} \vec{A}\cdot d\vec{l} = \int_{L_1} \left(\hat{x}y^3 + \hat{y}x^3\right)\cdot \hat{x}dx\,|_{y=0} = \int_{L_1} y^3 dx\,|_{y=0} = 0\,.$$

$$\int_{L_2} \vec{A}\cdot d\vec{l} = \int_2^1 \left(\hat{x}y^3 + \hat{y}x^3\right)\cdot \hat{x}dx\,|_{y=2-x} + \int_0^1 \left(\hat{x}y^3 + \hat{y}x^3\right)\cdot \hat{y}dy\,|_{x=2-y}$$

$$= \int_2^1 (2-x)^3 dx + \int_0^1 (2-y)^3 dy$$

$$= \int_1^2 (2-x)^3 d(2-x) - \int_0^1 (2-y)^3 d(2-y) \quad .$$

$$= \frac{1}{4}(2-x)^4\Big|_1^2 - \frac{1}{4}(2-y)^4\,|_0^1$$

$$= -\frac{1}{4} - \frac{1}{4}(1-16) = \frac{14}{4}$$

$$\int_{L_3} \vec{A}\cdot d\vec{l} = \int_1^0 \left(\hat{x}y^3 + \hat{y}x^3\right)\cdot \hat{x}dx\,|_{y=x} + \int_1^0 \left(\hat{x}y^3 + \hat{y}x^3\right)\cdot \hat{y}dy\,|_{x=y}$$

$$= \int_1^0 x^3 dx + \int_1^0 y^3 dy \quad .$$

$$= -\frac{1}{4}x^4\Big|_0^1 - \frac{1}{4}y^4\,|_0^1$$

$$= -\frac{2}{4}$$

$$\oint \vec{A}\cdot d\vec{l} = \int_{L_1} \vec{A}\cdot d\vec{l} + \int_{L_2} \vec{A}\cdot d\vec{l} + \int_{L_3} \vec{A}\cdot d\vec{l} = 0 + \frac{14}{4} - \frac{2}{4} = 3\,.$$

$$\nabla\times\vec{A}=\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z}\\ y^3 & x^3 & 0\end{vmatrix}=\hat{x}0+\hat{y}0+\hat{z}\left(3x^2-3y^2\right)$$

$$\iint_s\left(\nabla\times\vec{A}\right)\cdot d\vec{s}=\iint_s\hat{z}\left(3x^2-3y^2\right)\cdot\hat{z}dxdy$$

$$=\int_0^1 dx\int_0^x\left(3x^2-3y^2\right)dy+\int_1^2 dx\int_0^{2-x}\left(3x^2-3y^2\right)dy$$

$$=\int_0^1 2x^3dx+\int_1^2\left[3x^2(2-x)-(2-x)^3\right]dx$$

$$=\frac{2}{4}+\int_1^2 3x^2(2-x)dx-\int_1^2(2-x)^3dx$$

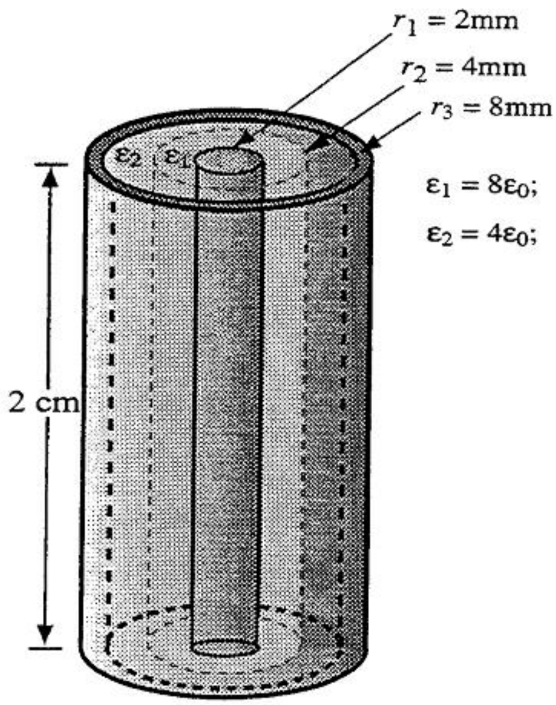
$$=\frac{2}{4}+2x^3\Big|_1^2-\frac{3}{4}x^4\Big|_1^2+\frac{1}{4}(2-x)^4\Big|_1^2$$

$$=\frac{2}{4}+14-\frac{3}{4}15-\frac{1}{4}$$

$$=3$$

$$\oint\vec{A}\cdot d\vec{l}=\int_s\left(\nabla\times\vec{A}\right)\cdot d\vec{s}$$

5. (10 points) A coaxial capacitor with inner connector radius r_1 = 2mm, and outer connector radius r_3= 8mm, is filled with two different materials as shown in the following figure. The length of the capacitor is 2cm. Calculate (1) if the surface charge density σ=1.0x10^{-10}C/cm^2, calculate the electric field inside the capacitor; (2) the capacitance of the capacitor. $\varepsilon_0 = 8.85\times10^{-12} F/m$.



Solution:

Using Guass's Law, the E fields are:

$$\vec{E} = \begin{cases} \dfrac{\sigma r_1}{\varepsilon_1 r}, & r_1 < r < r_2 \\ \dfrac{\sigma r_1}{\varepsilon_2 r}, & r_2 < r < r_3 \end{cases}$$

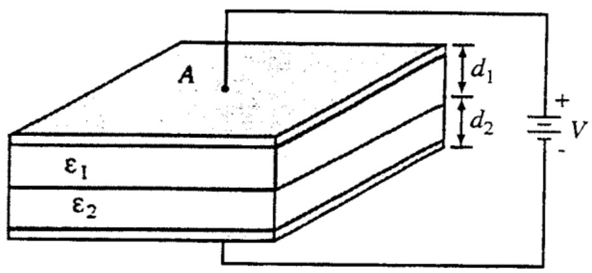
The voltage is:

$$V_{drop} = \oint \vec{E}\cdot\hat{r}dr = \frac{\sigma r_1}{\varepsilon_1}\ln\left(\frac{r_2}{r_1}\right) + \frac{\sigma r_1}{\varepsilon_2}\ln\left(\frac{r_3}{r_2}\right),$$

$$Q = 2\pi r_1 L\sigma,$$

$$C = \frac{2\pi r_1 \sigma L}{\frac{\sigma r_1}{\varepsilon_1}\ln\left(\frac{r_2}{r_1}\right) + \frac{\sigma r_1}{\varepsilon_2}\ln\left(\frac{r_3}{r_2}\right)} = \frac{2\pi L}{\frac{1}{\varepsilon_1}\ln\left(\frac{r_2}{r_1}\right) + \frac{1}{\varepsilon_2}\ln\left(\frac{r_3}{r_2}\right)} = \frac{2\pi\varepsilon_0 L}{\frac{1}{4}\ln\left(\frac{r_2}{r_1}\right) + \frac{1}{8}\ln\left(\frac{r_3}{r_2}\right)} \text{=4.3 pF.}$$

6. (10 points) A Capacitor shown in the following figure consists of two dielectric layers. d1=d2 = 10cm, and A = 200cm^2. Determine the capacitance of the capacitor.
$\varepsilon_1 = 4\varepsilon_0 = 3.5\times10^{-11} F/m,\ \varepsilon_2 = 2\varepsilon_0 = 1.8\times10^{-11} F/m$.



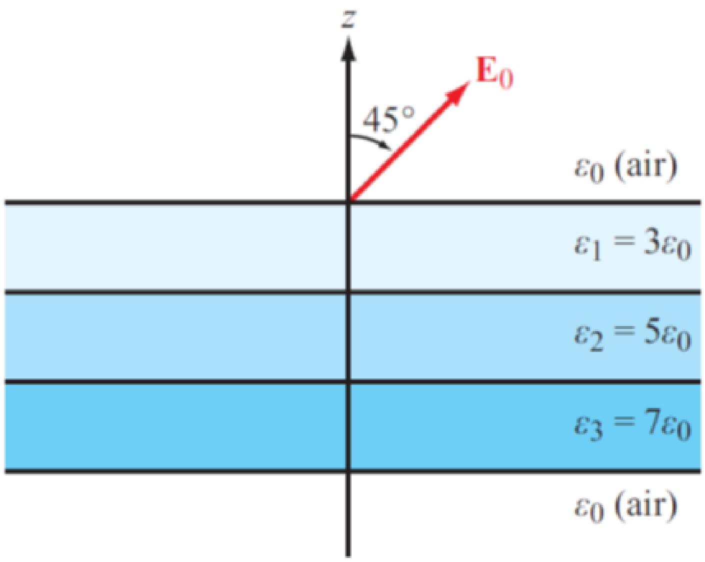
Solution:
Assuming the surface charge density of σ, the flux density D is:
$\vec{D}_1 = \vec{D}_2 = -\hat{z}\sigma$.

$\vec{E}_1 = -\hat{z}\dfrac{\sigma}{\varepsilon_1},\ \vec{E}_2 = -\hat{z}\dfrac{\sigma}{\varepsilon_2}$. $V = \vec{E}_1 d_1 + \vec{E}_2 d_2 = \dfrac{\sigma d_1}{\varepsilon_1} + \dfrac{\sigma d_2}{\varepsilon_2}$

$$C = \frac{Q}{V} = \frac{A\sigma}{\dfrac{\sigma d_1}{\varepsilon_1} + \dfrac{\sigma d_2}{\varepsilon_2}} = \frac{A}{\dfrac{d_1}{\varepsilon_1} + \dfrac{d_2}{\varepsilon_2}}$$

$$C = \frac{A}{\dfrac{d_1}{\varepsilon_1} + \dfrac{d_2}{\varepsilon_2}} = \frac{200\times10^{-4} m^2}{\dfrac{10\times10^{-2} m}{3.5\times10^{-11} F/m} + \dfrac{10\times10^{-2} m}{1.8\times10^{-11} F/m}} = \frac{2\times10^{-11} F}{\dfrac{10}{3.5} + \dfrac{10}{1.8}} = 2.4 pF .$$

7. (10 points) Determine the electric field intensity vectors in each layer.



Solutions:

Layer ε1:

$$E_{1t} = E_{0t} = \frac{\sqrt{2}}{2} E_0, \ \varepsilon_1 E_{1z} = \varepsilon_0 E_{0z}, \ E_{1z} = \frac{\varepsilon_0}{\varepsilon_1} E_{0z} = \frac{\sqrt{2}}{6} E_0,$$

Layer ε2:

$$E_{2t} = E_{1t} = \frac{\sqrt{2}}{2} E_0, \ \varepsilon_2 E_{2z} = \varepsilon_1 E_{1z} = \varepsilon_0 E_{0z}, \ E_{2z} = \frac{\varepsilon_0}{\varepsilon_2} E_{0z} = \frac{\sqrt{2}}{10} E_0,$$

Layer ε3:

$$E_{3t} = E_{2t} = \frac{\sqrt{2}}{2} E_0, \ \varepsilon_3 E_{3z} = \varepsilon_2 E_{2z} = \varepsilon_0 E_{0z}, \ E_{3z} = \frac{\varepsilon_0}{\varepsilon_3} E_{0z} = \frac{\sqrt{2}}{14} E_0,$$

Layer ε0:

$$E_{4t} = E_{3t} = \frac{\sqrt{2}}{2} E_0, \ \varepsilon_0 E_{4z} = \varepsilon_3 E_{3z} = \varepsilon_0 E_{0z}, \ E_{4z} = \frac{\varepsilon_0}{\varepsilon_0} E_{0z} = \frac{\sqrt{2}}{2} E_0.$$

8. (10 points) If the inner conductor of a coaxial cable carries a current of I, calculate the magnetic field inside the coaxial cable. The inner radius of the coaxial cable is r_1, and the outer radius is r_2. Assuming the permeability of material inside the coaxial cable is μ_0.

Solution:

Applying Ampere's law:

$$\oint \vec{H} \cdot d\vec{l} = I, \; H(r)2\pi r = I,$$

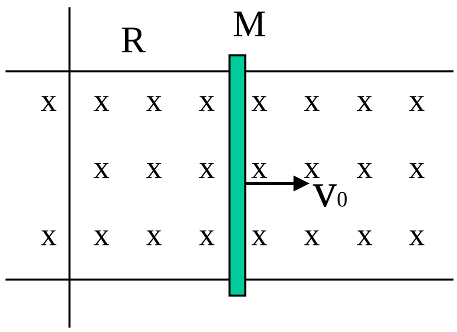
$$H(r) = \frac{I}{2\pi r}, \; r_1 < r < r_2$$

$$\vec{H} = \hat{\phi}\frac{I}{2\pi r}, \; r_1 < r < r_2$$

$$\vec{B} = \mu_0 \vec{H} = \hat{\phi}\frac{I\mu_0}{2\pi r}, \; r_1 < r < r_2$$

9. (10 points) Write down the Maxwell equations and their integral forms.

10. (10 points) A conducting bar is put in a constant magnetic field B = 0.1T. The circuit resistance is R =20Ω. The bar width is 20cm. The mass of the bar is m = 1kg. The circuit and the metal bar are on a flat horizontal surface. If the bar has an initial speed (t = 0) of V0 = 4m/s, determine:
(1) the current generated in the bar at t = 0;
(2) the force experienced by the conducting bar at t = 0;
(3) (extra 5 points) the speed of the bar at time t = 20s;
(4) (extra 5 points) the current generated at t =20s?
(5) (extra 5 points) prove that the kinetic energy loss of the metal bar is converted to the resistor heating power with a 100% energy conversion efficiency.



(1) $V = -\frac{\partial}{\partial t}\left(B \cdot W \cdot L(t)\right) = BWv_0$

$I = \frac{V}{R} = \frac{BWv_0}{R} = 4(mA)$, counter clockwise

(2) $F = BWI$ = 8×10^{-5} (N), pointing to the left.

(3) $F = -m\frac{d}{dt}v = \frac{B^2W^2}{R}v$,

$$v = v_0 e^{-\frac{W^2B^2}{mR}t}$$

(4) $I = \frac{WBv_0}{R}e^{-\frac{W^2B^2}{mR}t}$

Voltage maximum Voltage minimum	$\lvert\widetilde{V}\rvert_{\max} = \lvert V_0^+\rvert[1+\lvert\Gamma\rvert]$ $\lvert\widetilde{V}\rvert_{\min} = \lvert V_0^+\rvert[1-\lvert\Gamma\rvert]$
Positions of voltage maxima (also positions of current minima) Position of first maximum (also position of first current minimum)	$l_{\max} = \dfrac{\theta_r\lambda}{4\pi} + \dfrac{n\lambda}{2}, \quad n = 0, 1, 2, \ldots$ $l_{\max} = \begin{cases} \dfrac{\theta_r\lambda}{4\pi}, & \text{if } 0 \le \theta_r \le \pi \\ \dfrac{\theta_r\lambda}{4\pi} + \dfrac{\lambda}{2}, & \text{if } -\pi \le \theta_r \le 0 \end{cases}$
Positions of voltage minima (also positions of first current maxima) Position of first minimum (also position of first current maximum)	$l_{\min} = \dfrac{\theta_r\lambda}{4\pi} + \dfrac{(2n+1)\lambda}{4}, \quad n = 0, 1, 2, \ldots$ $l_{\min} = \dfrac{\lambda}{4}\left(1 + \dfrac{\theta_r}{\pi}\right)$
Input impedance	$Z_{in} = Z_0 \left(\dfrac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right)$
Positions at which Z_{in} is real	at voltage maxima and minima
Z_{in} at voltage maxima	$Z_{in} = Z_0 \left(\dfrac{1+\lvert\Gamma\rvert}{1-\lvert\Gamma\rvert}\right)$
Z_{in} at voltage minima	$Z_{in} = Z_0 \left(\dfrac{1-\lvert\Gamma\rvert}{1+\lvert\Gamma\rvert}\right)$
Z_{in} of short-circuited line	$Z_{in}^{sc} = jZ_0 \tan\beta l$
Z_{in} of open-circuited line	$Z_{in}^{oc} = -jZ_0 \cot\beta l$
Z_{in} of line of length $l = n\lambda/2$	$Z_{in} = Z_L, \quad n = 0, 1, 2, \ldots$
Z_{in} of line of length $l = \lambda/4 + n\lambda/2$ Z_{in} of matched line	$Z_{in} = Z_0^2/Z_L, \quad n = 0, 1, 2, \ldots$ $Z_{in} = Z_0$

$\lvert V_0^+\rvert$ = amplitude of incident wave, $\Gamma = \lvert\Gamma\rvert e^{j\theta_r}$ with $-\pi < \theta_r < \pi$; θ_r in radians.

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$S = \frac{1+\lvert\Gamma\rvert}{1-\lvert\Gamma\rvert}$$

GRADIENT, DIVERGENCE, CURL, & LAPLACIAN OPERATORS

CARTESIAN (RECTANGULAR) COORDINATES (x, y, z)

$$\nabla V = \mathbf{x}\frac{\partial V}{\partial x} + \mathbf{y}\frac{\partial V}{\partial y} + \mathbf{z}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \begin{vmatrix} \mathbf{x} & \mathbf{y} & \mathbf{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \mathbf{x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \mathbf{y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \mathbf{z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

CYLINDRICAL COORDINATES (r, ϕ, z)

$$\nabla V = \mathbf{r}\frac{\partial V}{\partial r} + \boldsymbol{\phi}\frac{1}{r}\frac{\partial V}{\partial \phi} + \mathbf{z}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \frac{1}{r}\begin{vmatrix} \mathbf{r} & \boldsymbol{\phi} r & \mathbf{z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix} = \mathbf{r}\left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \boldsymbol{\phi}\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \mathbf{z}\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial \phi}\right]$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

SPHERICAL COORDINATES (R, θ, ϕ)

$$\nabla V = \mathbf{R}\frac{\partial V}{\partial R} + \boldsymbol{\theta}\frac{1}{R}\frac{\partial V}{\partial \theta} + \boldsymbol{\phi}\frac{1}{R\sin\theta}\frac{\partial V}{\partial \phi}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 A_R) + \frac{1}{R\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \mathbf{A} = \frac{1}{R^2\sin\theta}\begin{vmatrix} \mathbf{R} & \boldsymbol{\theta} R & \boldsymbol{\phi} R\sin\theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & RA_\theta & (R\sin\theta)A_\phi \end{vmatrix}$$

$$= \mathbf{R}\frac{1}{R\sin\theta}\left[\frac{\partial}{\partial \theta}(A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial \phi}\right] + \boldsymbol{\theta}\frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial \phi} - \frac{\partial}{\partial R}(RA_\phi)\right] + \boldsymbol{\phi}\frac{1}{R}\left[\frac{\partial}{\partial R}(RA_\theta) - \frac{\partial A_R}{\partial \theta}\right]$$

$$\nabla^2 V = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

Table 3-1: Summary of vector relations.

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Coordinate variables	x, y, z	r, ϕ, z	R, θ, ϕ
Vector representation, A =	$\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{r}}A_r + \hat{\boldsymbol{\phi}}A_\phi + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{R}}A_R + \hat{\boldsymbol{\theta}}A_\theta + \hat{\boldsymbol{\phi}}A_\phi$
Magnitude of A, $\lvert A\rvert$ =	$\sqrt[+]{A_x^2 + A_y^2 + A_z^2}$	$\sqrt[+]{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt[+]{A_R^2 + A_\theta^2 + A_\phi^2}$
Position vector $\overrightarrow{OP_1}$ =	$\hat{\mathbf{x}}x_1 + \hat{\mathbf{y}}y_1 + \hat{\mathbf{z}}z_1$, for $P(x_1, y_1, z_1)$	$\hat{\mathbf{r}}r_1 + \hat{\mathbf{z}}z_1$, for $P(r_1, \phi_1, z_1)$	$\hat{\mathbf{R}}R_1$, for $P(R_1, \theta_1, \phi_1)$
Base vectors properties	$\hat{\mathbf{x}}\cdot\hat{\mathbf{x}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}}\times\hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}}\times\hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}}\cdot\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}$	$\hat{\mathbf{R}}\cdot\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = 1$ $\hat{\mathbf{R}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}}\times\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}$ $\hat{\boldsymbol{\theta}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product, A · B =	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product, A × B =	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\phi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length, $d\mathbf{l}$ =	$\hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}\,dy + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{r}}\,dr + \hat{\boldsymbol{\phi}}r\,d\phi + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{R}}\,dR + \hat{\boldsymbol{\theta}}R\,d\theta + \hat{\boldsymbol{\phi}}R\sin\theta\,d\phi$
Differential surface areas	$d\mathbf{s}_x = \hat{\mathbf{x}}\,dy\,dz$ $d\mathbf{s}_y = \hat{\mathbf{y}}\,dx\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}\,dx\,dy$	$d\mathbf{s}_r = \hat{\mathbf{r}}r\,d\phi\,dz$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}\,dr\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}r\,dr\,d\phi$	$d\mathbf{s}_R = \hat{\mathbf{R}}R^2\sin\theta\,d\theta\,d\phi$ $d\mathbf{s}_\theta = \hat{\boldsymbol{\theta}}R\sin\theta\,dR\,d\phi$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}R\,dR\,d\theta$
Differential volume, $d\mathcal{V}$ =	$dx\,dy\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

Boundary conditions:

$$D_{1n} - D_{2n} = \rho_s$$

$$H_{1x} - H_{2x} = J_y$$

$$H_{1y} - H_{2y} = -J_x$$