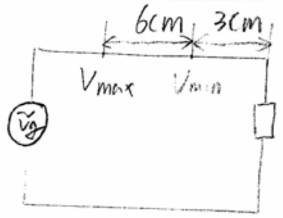
1. 1. (10 points)On a 100-Ω lossless line, the following observation was noted: distance of the first voltage minimum from the load is 3cm, distance from the first voltage maximum from the load is 9cm, S=3, find ZL.



$Z_0 = 100\Omega \quad S = 3$

First step: find $\lambda = ?$

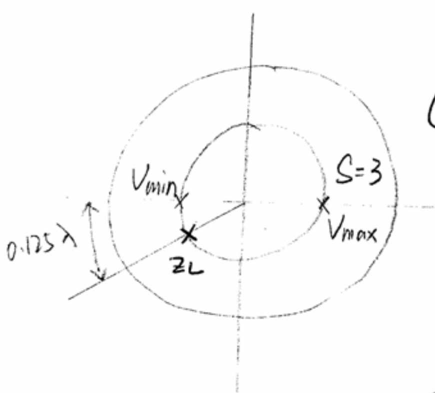
$\frac{\lambda}{4} = 9 - 3$ (Distance from Load for V_{max} − Distance from Load for V_{min})

$\lambda = 24\,cm$

Second step: Translate distance in term of wavelength

$L_{min} = \frac{3}{24} = \frac{1}{8}\lambda = 0.125\lambda \qquad L_{max} = \frac{9}{24} = \frac{3}{8}\lambda = 0.375\lambda$

Third step. Smith chart



① Location of V_{max} . V_{min}

② rotate load Z_L 0.125λ to rich V_{min} towoard generator (clockwise)

also you can rotate Z_L 0.375λ to rich V_{max} towoard generator. get same result.

③ $z_L = 0.6 - 0.8j$

$Z_L = Z_0 \cdot z_L$

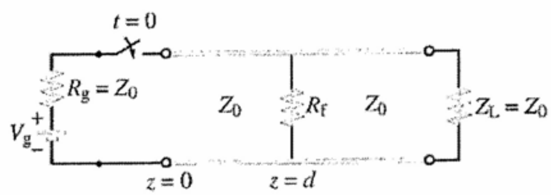
$= 100(0.6 - 0.8j) = 60 - 80j$

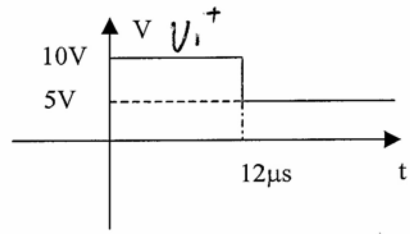
2. (10 points) A time domain reflector is an instrument to locate faults on a transmission line. A voltage meter is connected to a 50-Ω lossless matched transmission line, and the measured voltage waveform is shown in following figure. The line insulating material is Teflon with $\varepsilon_r = 2.0$. If we know that the resistance of the generator is also 50-Ω, determine

(a) the generator voltage,

(b) the location of the fault

(c) the fault resistance





(a). $V_1^+ = \dfrac{V_g Z_0}{R_g + Z_0} = \dfrac{V_g Z_0}{2Z_0} = \dfrac{V_g}{2}$

$V_g = 2V_1^+ = 2 \times 10 = 20V$.

(b) $2T = 12\,\mu s$

$T = 6\,\mu s$

1.3 km

$\dfrac{L_{Rf}}{C/\sqrt{2}} = T \qquad L_{Rf} = T \cdot C = 6 \times 10^{-6} \times \dfrac{3 \times 10^{8}}{\sqrt{2}} =$

(c) $V = V_{trans} + V_{ref}$

$5 = 10 + V_{ref}$

$V_{ref} = -5V$

$\Gamma_f = \dfrac{V_0^-}{V_0^+} = \dfrac{V_{ref}}{V_{tran}} = \dfrac{-5}{10} = -\dfrac{1}{2}$

$\Gamma_f = \dfrac{Z_{Lf} - Z_0}{Z_{Lf} + Z_0} \qquad \dfrac{1}{Z_{Lf}} = \dfrac{1}{Z_0} + \dfrac{1}{R_f}$

$= -\dfrac{1}{2}$

$Z_{Lf} = \dfrac{50}{3}\,\Omega \qquad R_f = 25\,\Omega.$

3. (10 points) Determine the closed loop line integral of $\vec{F} = \hat{x} + 2z\hat{y} + 2y\hat{z}$ from P1(-1,3,-2) to P2(2,4,1), from P2 to P3(2, 5, 1) and from P3 back to P1, where points are specified in rectangular coordinate system

$$\oint_c \vec{F} \cdot dl = \int \nabla \times \vec{F} \cdot ds$$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 1 & 2z & 2y \end{vmatrix}$$

$$= 0$$

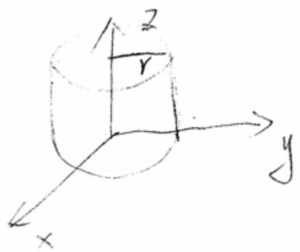
$$\oint_c \vec{F} \cdot dl = \int 0 \cdot ds = 0 .$$

4. (10 points) If an electric field is given $\vec{E} = 3r\hat{r} + 3r\phi\hat{\phi} - 2\hat{z}$ in the region

$0 \le r \le 2,\ -\pi \le \phi \le \pi, \quad 0 \le z \le 4$, determine:

(1) The volume charge density in this region

(2) Total charge in the volume $0 \le r \le 2,\ -\pi \le \phi \le \pi, \quad 0 \le z \le 4$.



(1) $\rho_v = \nabla \cdot D$

$\vec{D} = \varepsilon \cdot \vec{E}$

$\rho_v = \varepsilon (\nabla \cdot \vec{E})$

$= \varepsilon \left\{ \frac{1}{r} \frac{\partial}{\partial r}(rA_r) + \frac{1}{r} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z} \right]$

$= \varepsilon \left\{ \frac{1}{r} \frac{\partial}{\partial r}(3r^2) + \frac{1}{r}(3r) + 0 \right]$

$= \varepsilon \{ 6 + 3 + 0] = 9\varepsilon$

(2) $Q = \iiint \rho_v \cdot dV$

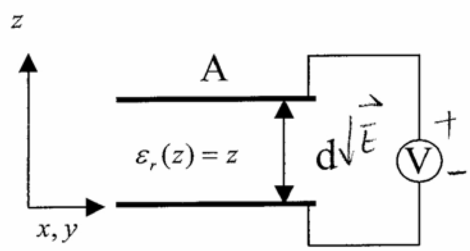
$= \iiint 9\varepsilon \cdot dV$

$= \int_0^2 \int_{-\pi}^{\pi} \int_0^4 9\varepsilon \cdot r \, dr \, d\phi \, dz$

$= 9\varepsilon \left| \frac{r^2}{2} \cdot \phi \cdot z \right) \Big|_{r=2,\ \phi=2\pi,\ z=4}$

$= 9\varepsilon \cdot \frac{4}{2} \cdot 2\pi \cdot 4 = 144\pi\varepsilon$

5. (10 points) A parallel-plate capacitor has the dielectric filling region $\varepsilon_r(z) = \cancel{z+d}$ between the plate, A1 = 1cm^2, d = 2cm, V=10V, ignore Fringing field effect. $2+\frac{z}{d}$



Determine:

(a) The Electric field density E of the region.

(b) The capacitance of the capacitor

a $\quad D = E\cdot\varepsilon \qquad$ D is constant

$$E = D/\varepsilon$$

$$\int_0^d \frac{D}{2+z/d}\cdot dz = +V$$

$$D\cdot\int_0^d \frac{d}{2d+z}\, d(2d+z) = +10$$

$$D\cdot d\ \ln(2d+z)\Big|_0^d = +10$$

$$D\cdot d\left(\ln\frac{3d}{2d}\right) = +10$$

$$\vec{D} = \frac{10}{d}\ln 1.5\ (+\hat{z})$$

$$\vec{E}(z) = \frac{\frac{10}{d}\ln 1.5}{2+\frac{z}{d}}(+\hat{z}) = \frac{10\ln 1.5}{2d+z}\ (+\hat{z})$$

b $\quad C = \frac{Q}{V}$

$$Q = \oint_s D\cdot ds = \frac{10}{+d}\ln 1.5\cdot A$$

$$C = \frac{+10\ln 1.5\cdot A}{+d\cdot 10} = \frac{\ln 1.5\, A}{d}$$

6. (10 points) Determine the electric field intensity vector from the following potential distributions: (a) $V(x,y,z)=\frac{1}{(x^2+y^2+z^2)^{1/2}}$, (b) $V(r,\phi,z)=re^{-z}\cos(\phi)$.

$$V=-\int_{\infty}^{P} E\cdot dL \qquad E=-\nabla V$$

(a). $V(x\cdot y\cdot z)=\frac{1}{(x^2+y^2+z^2)^{1/2}}$

$$-\nabla V=-\left(\frac{\partial V}{\partial x}\hat{x}+\frac{\partial V}{\partial y}\hat{y}+\frac{\partial V}{\partial z}\hat{z}\right)$$

$$=-\left[\frac{2x\cdot(-\frac{1}{2})}{(x^2+y^2+z^2)^{3/2}}\hat{x}+\frac{2y\cdot(-\frac{1}{2})}{(x^2+y^2+z^2)^{3/2}}\hat{y}+\frac{2z(-\frac{1}{2})}{(x^2+y^2+z^2)^{3/2}}\hat{z}\right]$$

$$\vec{E}=(x^2+y^2+z^2)^{-\frac{3}{2}}\{x\cdot\hat{x}+y\cdot\hat{y}+z\hat{z}]$$

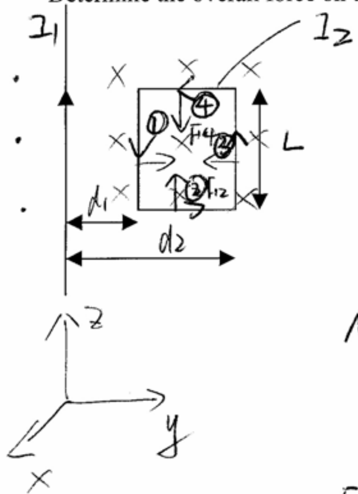
(b). $V(r\cdot\phi\cdot z)=r\cdot e^{-z}\cos(\phi)$

$$-\nabla V=-\{\hat{r}\left(\frac{\partial V}{\partial r}\right)+\hat{\phi}\frac{1}{r}\left(\frac{\partial V}{\partial\phi}\right)+\hat{z}\left(\frac{\partial V}{\partial z}\right)]$$

$$=-\{\hat{r}\cdot e^{-z}\cos\phi+\hat{\phi}e^{-z}(-\sin\phi)+\hat{z}(-z)\cdot r\,e^{-z}\cos\phi]$$

$$\vec{E}=-\hat{r}\cdot e^{-z}\cos\phi+\hat{\phi}e^{-z}\sin\phi+\hat{z}\,z\cdot r\cdot e^{-z}\cdot\cos\phi.$$

7. (10 points) An infinitely long wire carrying current I1=100mA is adjacent to a rectangular loop carries a current I2=200mA. d1 = 5cm, d2=10cm and L=5cm. Determine the overall force on the loop.



assuming I_2 direction: anticlockwise

F_4's (segmen 4) direction is $-\hat{z}$

F_3's direction is $\hat{z}$

Magniude of F_4 and F_3 is the same.

$$F_4 + F_3 = 0.$$

$$F_1 = \hat{y} \cdot |I_2| \times |B_2|$$

$$F_2 = -\hat{y}\ |I_2| \times |B_3|$$

$$\vec{H} = \frac{I_1}{2\pi r}(-\hat{x})$$

$$\vec{B} = u\vec{H} = \frac{u \cdot I_1}{2\pi r}(-\hat{x})$$

$$F_1 = \hat{y}\ \frac{uI_1I_2L}{2\pi d_1} \qquad F_2 = -\hat{y}\ \frac{uI_1I_2L}{2\pi d_2}$$

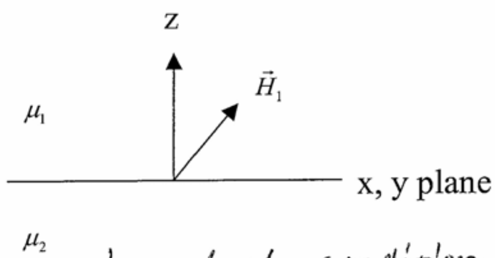
$$F_{all} = \hat{y}\left(\frac{uI_1I_2L}{2\pi d_1} - \frac{uI_1I_2}{2\pi d_2} \right)$$

8. (10 points) The x-y plane separates two magnetic media with magnetic permeability:

$\mu_1 = \mu_0, \mu_2 = 2\mu_0$, if we know $\vec{H}_1 = 2\hat{x} + 3\hat{y} + 4\hat{z}$,

(a) if no surface current, determine: $\vec{H}_2$

(b) if surface current $\vec{J}_s = 1\hat{x} + 1\hat{y}$, determine $\vec{H}_2$.



boundary condition

$B_{1n} = B_{2n} \qquad \mu_1 H_{1n} = \mu_2 H_{2n}$

$J_s = 0 \qquad H_{1t} = H_{2t}$

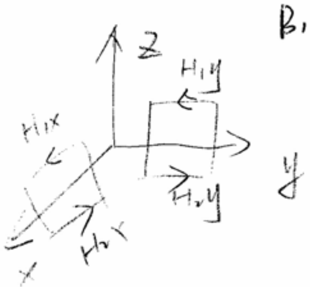
(a) $H_{2n} = \dfrac{\mu_1 H_{1n}}{\mu_2} = \dfrac{\mu_0}{2\mu_0} \cdot 4\hat{z} = 2\hat{z}$

$H_{1t} = H_{2t}$

$\vec{H}_2 = 2\hat{x} + 3\hat{y} + 2\hat{z}$

(b) $J_s \neq 0$

$B_{1n} = B_{2n} \qquad H_{2n} = 2\hat{z}$



$-H_{1y} + H_{2y} = J \cdot \hat{x} \qquad J_s = 1\hat{x} + 1\hat{y}$

$H_{2y} = 3 + 1 = 4$

$H_{1x} - H_{2x} = J \cdot \hat{y}$

$H_{2x} = 2 - 1 = 1$

$\vec{H}_2 = \hat{x} + 4\hat{y} + 2\hat{z}$

9. (10 points)Write down the Maxwell equations and their integral forms.

Ditterential form		Intergral form
Gauss's law	$\nabla \cdot D = \rho_v$	$\oint_s D \cdot ds = Q$
Faraday's law	$\nabla \times E = -\frac{\partial B}{\partial t}$	$\oint_c E \cdot dL = -\int_s \frac{\partial B}{\partial t} \cdot ds$
No magnetic charge	$\nabla \cdot B = 0$	$\oint_s B \cdot ds = 0$
Amperes law	$\nabla \times H = J + \frac{\partial D}{\partial t}$	$\oint_c H \cdot dl = \int_s (J + \frac{\partial D}{\partial t}) \cdot ds$

10. (10 points) For an electromagnetic wave of a single frequency f = 200MHz, the phasor of magnetic filed is: $\tilde{\vec{H}}(z,) = \hat{x}e^{-j2z}$, find (a) E, (b) displacement current Jd.

(a) $E = \frac{1}{j\omega\varepsilon} \nabla \times \tilde{H}$

$\tilde{H} = \hat{x} e^{-j2z}$

$$\nabla \times \tilde{H} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ e^{-j2z} & 0 & 0 \end{vmatrix} = (-\hat{y}) \cdot (2j \cdot e^{-j2z})$$

$$\tilde{E} = \frac{1}{j\omega\varepsilon} \cdot 2j \cdot e^{-j2z} (-\hat{y}) = (-\hat{y}) \frac{2}{\omega\varepsilon} \cdot e^{-j2z}$$

$$E = Re\{\frac{2}{\omega\varepsilon} \cdot e^{-j2z} \cdot e^{j\omega t}\} (-\hat{y})$$

$$= -\hat{y} \frac{2}{\omega\varepsilon} \cos(\omega t - 2z)$$

(b). $J_D = \partial D / \partial t = \frac{\partial E \cdot \varepsilon}{\partial t}$

$$= \frac{\partial\{(-\hat{y}) \frac{2}{\omega\varepsilon} \cdot \varepsilon \cos(\omega t - 2z)\}}{\partial t}$$

$$= \frac{2}{\omega} \cdot \omega \cdot \sin(\omega t - 2z) \hat{y}$$

$$= 2 \sin(\omega t - 2z) \hat{y}$$

$\omega = 2\pi f = 2\pi \times 2 \times 10^8 = 4\pi \times 10^8$ rad/s