

Electric Field

| | | | |
|---------------------|---|--------------------------|---|
| Current density | $\mathbf{J} = \rho_v \mathbf{u}$ | Point charge | $\mathbf{E} = \hat{\mathbf{R}} \frac{q}{4\pi \epsilon R^2}$ |
| Poisson's equation | $\nabla^2 V = -\frac{\rho_v}{\epsilon}$ | Many point charges | $\mathbf{E} = \frac{1}{4\pi \epsilon} \sum_{i=1}^N \frac{q_i (\mathbf{R} - \mathbf{R}_i)}{ \mathbf{R} - \mathbf{R}_i ^3}$ |
| Laplace's equation | $\nabla^2 V = 0$ | Volume distribution | $\mathbf{E} = \frac{1}{4\pi \epsilon} \int_{V'} \frac{\hat{\mathbf{R}}' \rho_v dV'}{R'^2}$ |
| Resistance | $R = \frac{-\int_l \mathbf{E} \cdot d\mathbf{l}}{\int_s \sigma \mathbf{E} \cdot d\mathbf{s}}$ | Surface distribution | $\mathbf{E} = \frac{1}{4\pi \epsilon} \int_{S'} \frac{\hat{\mathbf{R}}' \rho_s ds'}{R'^2}$ |
| Boundary conditions | Table 4-3 | Line distribution | $\mathbf{E} = \frac{1}{4\pi \epsilon} \int_{l'} \frac{\hat{\mathbf{R}}' \rho_\ell dl'}{R'^2}$ |
| Capacitance | $C = \frac{\int_s \epsilon \mathbf{E} \cdot d\mathbf{s}}{-\int_l \mathbf{E} \cdot d\mathbf{l}}$ | Infinite sheet of charge | $\mathbf{E} = \hat{\mathbf{z}} \frac{\rho_s}{2\epsilon_0}$ |
| RC relation | $RC = \frac{\epsilon}{\sigma}$ | Infinite line of charge | $\mathbf{E} = \frac{\mathbf{D}}{\epsilon_0} = \hat{\mathbf{r}} \frac{D_r}{\epsilon_0} = \hat{\mathbf{r}} \frac{\rho_\ell}{2\pi \epsilon_0 r}$ |
| Energy density | $w_e = \frac{1}{2} \epsilon E^2$ | Dipole | $\mathbf{E} = \frac{qd}{4\pi \epsilon_0 R^3} (\hat{\mathbf{R}} 2 \cos \theta + \hat{\boldsymbol{\theta}} \sin \theta)$ |
| | | Relation to V | $\mathbf{E} = -\nabla V$ |

Chapter 5 Relationships

Maxwell's Magnetostatics Equations

Gauss's Law for Magnetism

$$\nabla \cdot \mathbf{B} = 0 \quad \longleftrightarrow \quad \oint_s \mathbf{B} \cdot d\mathbf{s} = 0$$

Ampère's Law

$$\nabla \times \mathbf{H} = \mathbf{J} \quad \longleftrightarrow \quad \oint_c \mathbf{H} \cdot d\mathbf{l} = I$$

Lorentz Force on Charge q

$$\mathbf{F} = q(\mathbf{E} + \mathbf{u} \times \mathbf{B})$$

Magnetic Force on Wire

$$\mathbf{F}_m = I \oint_c d\mathbf{l} \times \mathbf{B} \quad (\text{N})$$

Magnetic Torque on Loop

$$\mathbf{T} = \mathbf{m} \times \mathbf{B} \quad (\text{N}\cdot\text{m})$$

$$\mathbf{m} = \hat{\mathbf{n}} NIA \quad (\text{A}\cdot\text{m}^2)$$

Biot-Savart Law

$$\mathbf{H} = \frac{I}{4\pi} \int_l \frac{d\mathbf{l} \times \hat{\mathbf{R}}}{R^2} \quad (\text{A/m})$$

Magnetic Field

Infinitely Long Wire $\mathbf{B} = \hat{\boldsymbol{\phi}} \frac{\mu_0 I}{2\pi r} \quad (\text{Wb/m}^2)$

Circular Loop $\mathbf{H} = \hat{\mathbf{z}} \frac{Ia^2}{2(a^2 + z^2)^{3/2}} \quad (\text{A/m})$

Solenoid $\mathbf{B} \simeq \hat{\mathbf{z}} \mu_n I = \frac{\hat{\mathbf{z}} \mu_n N I}{l} \quad (\text{Wb/m}^2)$

Vector Magnetic Potential

$$\mathbf{B} = \nabla \times \mathbf{A} \quad (\text{Wb/m}^2)$$

Vector Poisson's Equation

$$\nabla^2 \mathbf{A} = -\mu \mathbf{J}$$

Inductance

$$L = \frac{\Lambda}{I} = \frac{\Phi}{I} = \frac{1}{I} \int_s \mathbf{B} \cdot d\mathbf{s} \quad (\text{H})$$

Magnetic Energy Density

$$w_m = \frac{1}{2} \mu H^2 \quad (\text{J/m}^3)$$

Chapter 6 Relationships

Faraday's Law

$$V_{\text{emf}} = -\frac{d\Phi}{dt} = -\frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{s} = V_{\text{emf}}^{\text{tr}} + V_{\text{emf}}^{\text{m}}$$

Transformer

$$V_{\text{emf}}^{\text{tr}} = -N \int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s} \quad (N \text{ loops})$$

Motional

$$V_{\text{emf}}^{\text{m}} = \oint_C (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l}$$

Charge-Current Continuity

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho_V}{\partial t}$$

EM Potentials

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

Current Density

$$\text{Conduction} \quad \mathbf{J}_c = \sigma \mathbf{E}$$

$$\text{Displacement} \quad \mathbf{J}_d = \frac{\partial \mathbf{D}}{\partial t}$$

Conductor Charge Dissipation

$$\rho_V(t) = \rho_{V0} e^{-(\sigma/\epsilon)t} = \rho_{V0} e^{-t/\tau_c}$$