1.1 A 2-kHz sound wave traveling in the x-direction in air was observed to have a differential pressure $p(x,t) = 10$ N/m^2 at $x = 0$ and $t = 50$ μs. If the reference phase of $p(x,t)$ is 36°, find a complete expression for $p(x,t)$. The velocity of sound in air is 330 m/s.

Solution: The general form is given by Eq. (1.17),

$$p(x,t) = A\cos\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \phi_0\right),$$

where it is given that $\phi_0 = 36°$. From Eq. (1.26), $T = 1/f = 1/\left(2\times 10^3\right) = 0.5$ ms. From Eq. (1.27),

$$\lambda = \frac{u_\text{p}}{f} = \frac{330}{2\times 10^3} = 0.165 \text{ m}.$$

Also, since

$$\begin{aligned} p(x=0,\ t=50\ \mu\text{s}) = 10\ (\text{N/m}^2) &= A\cos\left(\frac{2\pi\times 50\times 10^{-6}}{5\times 10^{-4}} + 36°\,\frac{\pi \text{ rad}}{180°}\right) \\ &= A\cos(1.26 \text{ rad}) = 0.31A, \end{aligned}$$

it follows that $A = 10/0.31 = 32.36$ N/m^2. So, with t in (s) and x in (m),

$$\begin{aligned} p(x,t) &= 32.36\cos\left(2\pi\times 10^6\,\frac{t}{500} - 2\pi\times 10^3\,\frac{x}{165} + 36°\right) \qquad (\text{N/m}^2) \\ &= 32.36\cos(4\pi\times 10^3 t - 12.12\pi x + 36°) \qquad (\text{N/m}^2). \end{aligned}$$

1.4 A wave traveling along a string is given by

$$y(x,t) = 2\sin(4\pi t + 10\pi x) \qquad \text{(cm)},$$

where x is the distance along the string in meters and y is the vertical displacement. Determine: (a) the direction of wave travel, (b) the reference phase ϕ_0, (c) the frequency, (d) the wavelength, and (e) the phase velocity.

Solution:

(a) We start by converting the given expression into a cosine function of the form given by (1.17):

$$y(x,t) = 2\cos\left(4\pi t + 10\pi x - \frac{\pi}{2}\right) \qquad \text{(cm)}.$$

Since the coefficients of t and x both have the same sign, the wave is traveling in the negative x-direction.

(b) From the cosine expression, $\phi_0 = -\pi/2$.

(c) $\omega = 2\pi f = 4\pi$,

$$f = 4\pi/2\pi = 2 \text{ Hz}.$$

(d) $2\pi/\lambda = 10\pi$,

$$\lambda = 2\pi/10\pi = 0.2 \text{ m}.$$

(e) $u_p = f\lambda = 2 \times 0.2 = 0.4$ (m/s).

1.7 A wave traveling along a string in the $+x$ direction is given by

$$y_1(x,t) = A\cos(\omega t - \beta x),$$

where $x = 0$ is the end of the string, which is tied rigidly to a wall, as shown in Fig. P1.7. When wave $y_1(x,t)$ arrives at the wall, a reflected wave $y_2(x,t)$ is generated. Hence, at any location on the string, the vertical displacement y_s is the sum of the incident and reflected waves:

$$y_s(x,t) = y_1(x,t) + y_2(x,t).$$

(a) Write an expression for $y_2(x,t)$, keeping in mind its direction of travel and the fact that the end of the string cannot move.

(b) Generate plots of $y_1(x,t)$, $y_2(x,t)$ and $y_s(x,t)$ versus x over the range $-2\lambda \le x \le 0$ at $\omega t = \pi/4$ and at $\omega t = \pi/2$.

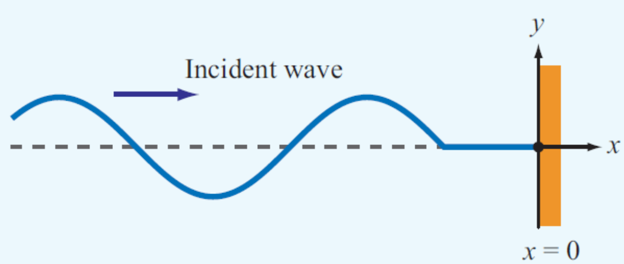


Figure P1.7 Wave on a string tied to a wall at $x = 0$ (Problem 1.7).

Solution:

(a) Since wave $y_2(x,t)$ was caused by wave $y_1(x,t)$, the two waves must have the same angular frequency ω, and since $y_2(x,t)$ is traveling on the same string as $y_1(x,t)$, the two waves must have the same phase constant β. Hence, with its direction being in the negative x-direction, $y_2(x,t)$ is given by the general form

$$y_2(x,t) = B\cos(\omega t + \beta x + \phi_0), \qquad (1)$$

where B and ϕ_0 are yet-to-be-determined constants. The total displacement is

$$y_s(x,t) = y_1(x,t) + y_2(x,t) = A\cos(\omega t - \beta x) + B\cos(\omega t + \beta x + \phi_0).$$

Since the string cannot move at $x = 0$, the point at which it is attached to the wall, $y_s(0,t) = 0$ for all t. Thus,

$$y_s(0,t) = A\cos\omega t + B\cos(\omega t + \phi_0) = 0. \qquad (2)$$

(i) Easy Solution: The physics of the problem suggests that a possible solution for (2) is $B = -A$ and $\phi_0 = 0$, in which case we have

$$y_2(x,t) = -A\cos(\omega t + \beta x). \tag{3}$$

(ii) Rigorous Solution: By expanding the second term in (2), we have

$$A\cos\omega t + B(\cos\omega t\cos\phi_0 - \sin\omega t\sin\phi_0) = 0,$$

or

$$(A + B\cos\phi_0)\cos\omega t - (B\sin\phi_0)\sin\omega t = 0. \tag{4}$$

This equation has to be satisfied for all values of t. At $t = 0$, it gives

$$A + B\cos\phi_0 = 0, \tag{5}$$

and at $\omega t = \pi/2$, (4) gives

$$B\sin\phi_0 = 0. \tag{6}$$

Equations (5) and (6) can be satisfied simultaneously only if

$$A = B = 0 \tag{7}$$

or

$$A = -B \quad \text{and} \quad \phi_0 = 0. \tag{8}$$

Clearly (7) is not an acceptable solution because it means that $y_1(x,t) = 0$, which is contrary to the statement of the problem. The solution given by (8) leads to (3).

(b) At $\omega t = \pi/4$,

$$y_1(x,t) = A\cos(\pi/4 - \beta x) = A\cos\left(\frac{\pi}{4} - \frac{2\pi x}{\lambda}\right),$$

$$y_2(x,t) = -A\cos(\omega t + \beta x) = -A\cos\left(\frac{\pi}{4} + \frac{2\pi x}{\lambda}\right).$$

Plots of y_1, y_2, and y_3 are shown in Fig. P1.7(b).

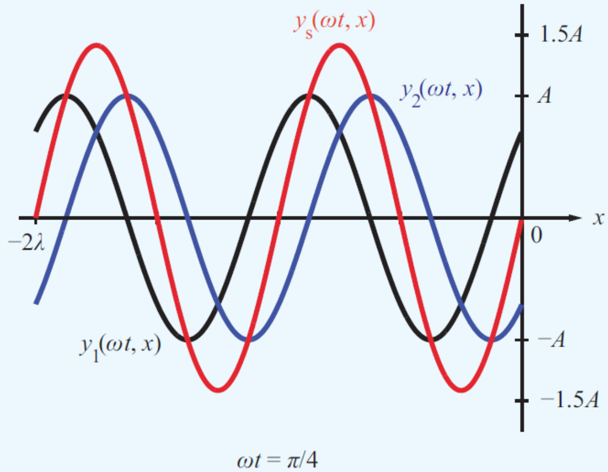


Figure P1.7 (b) Plots of y_1, y_2, and y_s versus x at $\omega t = \pi/4$.

At $\omega t = \pi/2$,

$$y_1(x,t) = A\cos(\pi/2 - \beta x) = A\sin\beta x = A\sin\frac{2\pi x}{\lambda},$$

$$y_2(x,t) = -A\cos(\pi/2 + \beta x) = A\sin\beta x = A\sin\frac{2\pi x}{\lambda}.$$

Plots of y_1, y_2, and y_3 are shown in Fig. P1.7(c).

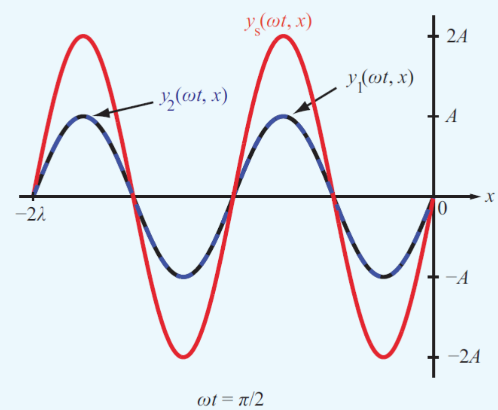


Figure P1.7 (c) Plots of y_1, y_2, and y_s versus x at $\omega t = \pi/2$.

1.9 Give expressions for $y(x,t)$ for a sinusoidal wave traveling along a string in the negative x-direction, given that $y_{\max} = 40$ cm, $\lambda = 30$ cm, $f = 10$ Hz, and

(a) $y(x,0) = 0$ at $x = 0$,

(b) $y(x,0) = 0$ at $x = 3.75$ cm.

Solution: For a wave traveling in the negative x-direction, we use Eq. (1.17) with $\omega = 2\pi f = 20\pi$ (rad/s), $\beta = 2\pi/\lambda = 2\pi/0.3 = 20\pi/3$ (rad/s), $A = 40$ cm, and x assigned a positive sign:

$$y(x,t) = 40\cos\left(20\pi t + \frac{20\pi}{3}x + \phi_0\right) \qquad \text{(cm)},$$

with x in meters.

(a) $y(0,0) = 0 = 40\cos\phi_0$. Hence, $\phi_0 = \pm\pi/2$, and

$$\begin{aligned} y(x,t) &= 40\cos\left(20\pi t + \frac{20\pi}{3}x \pm \frac{\pi}{2}\right) \\ &= \begin{cases} -40\sin\left(20\pi t + \frac{20\pi}{3}x\right) \text{ (cm), if } \phi_0 = \pi/2, \\ 40\sin\left(20\pi t + \frac{20\pi}{3}x\right) \text{ (cm), } \quad \text{if } \phi_0 = -\pi/2. \end{cases} \end{aligned}$$

(b) At $x = 3.75$ cm $= 3.75 \times 10^{-2}$ m, $y = 0 = 40\cos(\pi/4 + \phi_0)$. Hence, $\phi_0 = \pi/4$ or $5\pi/4$, and

$$y(x,t) = \begin{cases} 40\cos\left(20\pi t + \frac{20\pi}{3}x + \frac{\pi}{4}\right) \text{ (cm), } \text{ if } \phi_0 = \pi/4, \\ 40\cos\left(20\pi t + \frac{20\pi}{3}x + \frac{5\pi}{4}\right) \text{ (cm), if } \phi_0 = 5\pi/4. \end{cases}$$

1.25 A voltage source given by

$$\upsilon_s(t) = 25\cos(2\pi \times 10^3 t - 30^\circ) \qquad \text{(V)}$$

is connected to a series RC load as shown in Fig. 1-20. If $R = 1$ MΩ and $C = 200$ pF, obtain an expression for $\upsilon_c(t)$, the voltage across the capacitor.

Solution: In the phasor domain, the circuit is a voltage divider, and

$$\widetilde{V}_c = \widetilde{V}_s \frac{1/j\omega C}{R + 1/j\omega C} = \frac{\widetilde{V}_s}{(1 + j\omega RC)}.$$

Now $\widetilde{V}_s = 25\exp -j30^\circ$ V with $\omega = 2\pi \times 10^3$ rad/s, so

$$\begin{aligned}\widetilde{V}_c &= \frac{25\exp -j30^\circ \text{ V}}{1 + j((2\pi \times 10^3 \text{ rad/s}) \times (10^6\ \Omega) \times (200 \times 10^{-12}\text{ F}))} \\ &= \frac{25\exp -j30^\circ \text{ V}}{1 + j2\pi/5} = 15.57\exp -j81.5^\circ \text{ V}.\end{aligned}$$

Converting back to an instantaneous value,

$$v_c(t) = \mathfrak{Re}\widetilde{V}_c \exp j\omega t = \mathfrak{Re}\, 15.57\exp j(\omega t - 81.5^\circ) \text{ V} = 15.57\cos\left(2\pi \times 10^3 t - 81.5^\circ\right) \text{ V},$$

where t is expressed in seconds.

1.26 Find the phasors of the following time functions:

(a) $\upsilon(t) = 9\cos(\omega t - \pi/3)$ (V)

(b) $\upsilon(t) = 12\sin(\omega t + \pi/4)$ (V)

(c) $i(x,t) = 5e^{-3x}\sin(\omega t + \pi/6)$ (A)

(d) $i(t) = -2\cos(\omega t + 3\pi/4)$ (A)

(e) $i(t) = 4\sin(\omega t + \pi/3) + 3\cos(\omega t - \pi/6)$ (A)

Solution:

(a) $\widetilde{V} = 9\exp - j\pi/3$ V.

(b) $v(t) = 12\sin(\omega t + \pi/4) = 12\cos(\pi/2 - (\omega t + \pi/4)) = 12\cos(\omega t - \pi/4)$ V, $\widetilde{V} = 12\exp - j\pi/4$ V.

(c)

$$\begin{aligned} i(t) &= 5\exp - 3x\sin(\omega t + \pi/6) \text{ A} = 5\exp - 3x\cos[\pi/2 - (\omega t + \pi/6)] \text{ A} \\ &= 5\exp - 3x\cos(\omega t - \pi/3) \text{ A}, \\ \widetilde{I} &= 5\exp - 3x\exp - j\pi/3 \text{ A}. \end{aligned}$$

(d)

$$\begin{aligned} i(t) &= -2\cos(\omega t + 3\pi/4), \\ \tilde{I} &= -2e^{j3\pi/4} = 2e^{-j\pi}e^{j3\pi/4} = 2e^{-j\pi/4} \text{ A}. \end{aligned}$$

(e)

$$\begin{aligned} i(t) &= 4\sin(\omega t + \pi/3) + 3\cos(\omega t - \pi/6) \\ &= 4\cos[\pi/2 - (\omega t + \pi/3)] + 3\cos(\omega t - \pi/6) \\ &= 4\cos(-\omega t + \pi/6) + 3\cos(\omega t - \pi/6) \\ &= 4\cos(\omega t - \pi/6) + 3\cos(\omega t - \pi/6) = 7\cos(\omega t - \pi/6), \\ \tilde{I} &= 7e^{-j\pi/6} \text{ A}. \end{aligned}$$

1.27 Find the instantaneous time sinusoidal functions corresponding to the following phasors:

(a) $\widetilde{V} = -5e^{j\pi/3}$ (V)
(b) $\widetilde{V} = j6e^{-j\pi/4}$ (V)
(c) $\tilde{I} = (6 + j8)$ (A)
(d) $\tilde{I} = -3 + j2$ (A)
(e) $\tilde{I} = j$ (A)
(f) $\tilde{I} = 2e^{j\pi/6}$ (A)

Solution:

(a)

$$\widetilde{V} = -5\exp j\pi/3 \text{ V} = 5\exp j(\pi/3 - \pi) \text{ V} = 5\exp -j2\pi/3 \text{ V},$$
$$v(t) = 5\cos(\omega t - 2\pi/3) \text{ V}.$$

(b)

$$\widetilde{V} = j6\exp -j\pi/4 \text{ V} = 6\exp j(-\pi/4 + \pi/2) \text{ V} = 6\exp j\pi/4 \text{ V},$$
$$v(t) = 6\cos(\omega t + \pi/4) \text{ V}.$$

(c)

$$\widetilde{I} = (6 + j8) \text{ A} = 10\exp j53.1^\circ \text{ A},$$
$$i(t) = 10\cos(\omega t + 53.1^\circ) \text{ A}.$$

(d)

$$\tilde{I} = -3 + j2 = 3.61\, e^{j146.31^\circ},$$
$$i(t) = \mathfrak{Re}\{3.61\, e^{j146.31^\circ} e^{j\omega t}\} = 3.61\cos(\omega t + 146.31^\circ) \text{ A}.$$

(e)

$$\tilde{I} = j = e^{j\pi/2},$$
$$i(t) = \mathfrak{Re}\{e^{j\pi/2} e^{j\omega t}\} = \cos(\omega t + \pi/2) = -\sin\omega t \text{ A}.$$

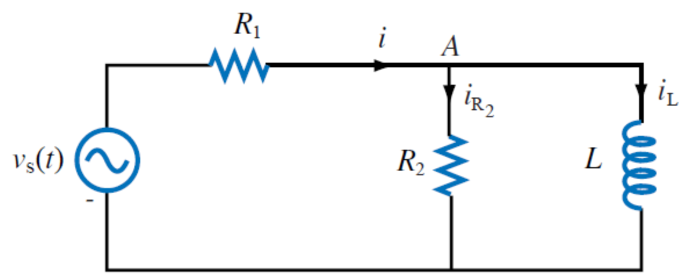
(f)

$$\tilde{I} = 2e^{j\pi/6},$$
$$i(t) = \mathfrak{Re}\{2e^{j\pi/6} e^{j\omega t}\} = 2\cos(\omega t + \pi/6) \text{ A}.$$

1.29 The voltage source of the circuit shown in Fig. P1.29 is given by

$$v_s(t) = 25\cos(4\times 10^4 t - 45^\circ) \qquad \text{(V)}.$$

Obtain an expression for $i_L(t)$, the current flowing through the inductor.



$R_1 20\Omega, R_2 30\Omega, L$ 0.4 mH

Figure P1.29

Solution: Based on the given voltage expression, the phasor source voltage is

$$\widetilde{V}_s = 25e^{-j45^\circ} \qquad \text{(V)}. \tag{9}$$

The voltage equation for the left-hand side loop is

$$R_1 i + R_2 i_{R_2} = v_s \tag{10}$$

For the right-hand loop,

$$R_2 i_{R_2} = L\frac{di_L}{dt}\,, \tag{11}$$

and at node A,

$$i = i_{R_2} + i_L. \tag{12}$$

Next, we convert Eqs. (2)–(4) into phasor form:

$$R_1\widetilde{I} + R_2\widetilde{I}_{R_2} = \widetilde{V}_s \tag{13}$$

$$R_2\widetilde{I}_{R_2} = j\omega L\widetilde{I}_L \tag{14}$$

$$\widetilde{I} = \widetilde{I}_{R_2} + \widetilde{I}_L \tag{15}$$

Upon combining (6) and (7) to solve for $\widetilde{I}_{R_2}$ in terms of $\widetilde{I}$, we have:

$$\widetilde{I}_{R_2} = \frac{j\omega L}{R_2 + j\omega L} I\,. \tag{16}$$

Substituting (8) in (5) and then solving for $\tilde{I}$ leads to:

$$R_1\tilde{I} + \frac{jR_2\omega L}{R_2 + j\omega L}\tilde{I} = \widetilde{V}_s$$
$$\tilde{I}\left(R_1 + \frac{jR_2\omega L}{R_2 + j\omega L}\right) = \widetilde{V}_s$$
$$\tilde{I}\left(\frac{R_1R_2 + jR_1\omega L + jR_2\omega L}{R_2 + j\omega L}\right) = \widetilde{V}_s$$
$$\tilde{I} = \left(\frac{R_2 + j\omega L}{R_1R_2 + j\omega L(R_1 + R_2)}\right)\widetilde{V}_s. \qquad (17$$

Combining (6) and (7) to solve for $\tilde{I}_L$ in terms of $\tilde{I}$ gives

$$\tilde{I}_L = \frac{R_2}{R_2 + j\omega L}\tilde{I}. \qquad (18$$

Combining (9) and (10) leads to

$$\tilde{I}_L = \left(\frac{R_2}{R_2 + j\omega L}\right)\left(\frac{R_2 + j\omega L}{R_1R_2 + j\omega L(R_1 + R_2)}\right)\widetilde{V}_s$$
$$= \frac{R_2}{R_1R_2 + +j\omega L(R_1 + R_2)}\widetilde{V}_s.$$

Using (1) for $\widetilde{V}_s$ and replacing R_1, R_2, L and ω with their numerical values, we have

$$\tilde{I}_L = \frac{30}{20 \times 30 + j4 \times 10^4 \times 0.4 \times 10^{-3}(20 + 30)}\, 25e^{-j45^\circ}$$
$$= \frac{30 \times 25}{600 + j800}\, e^{-j45^\circ}$$
$$= \frac{7.5}{6 + j8}\, e^{-j45^\circ} = \frac{7.5e^{-j45^\circ}}{10e^{j53.1^\circ}} = 0.75e^{-j98.1^\circ} \qquad \text{(A)}.$$

Finally,

$$i_L(t) = \mathfrak{Re}[\tilde{I}_L e^{j\omega t}]$$
$$= 0.75\cos(4 \times 10^4 t - 98.1^\circ) \qquad \text{(A)}.$$