2.1 A transmission line of length l connects a load to a sinusoidal voltage source with an oscillation frequency f. Assuming the velocity of wave propagation on the line is c, for which of the following situations is it reasonable to ignore the presence of the transmission line in the solution of the circuit:

(a) $l = 20$ cm, $f = 20$ kHz,
(b) $l = 50$ km, $f = 60$ Hz,
(c) $l = 20$ cm, $f = 600$ MHz,
(d) $l = 1$ mm, $f = 100$ GHz.

Solution: A transmission line is negligible when $l/\lambda \leq 0.01$.

(a) $$\frac{l}{\lambda} = \frac{lf}{u_p} = \frac{(20 \times 10^{-2} \text{ m}) \times (20 \times 10^{3} \text{ Hz})}{3 \times 10^{8} \text{ m/s}} = 1.33 \times 10^{-5} \text{ (negligible)}.$$

(b) $$\frac{l}{\lambda} = \frac{lf}{u_p} = \frac{(50 \times 10^{3} \text{ m}) \times (60 \times 10^{0} \text{ Hz})}{3 \times 10^{8} \text{ m/s}} = 0.01 \text{ (borderline)}.$$

(c) $$\frac{l}{\lambda} = \frac{lf}{u_p} = \frac{(20 \times 10^{-2} \text{ m}) \times (600 \times 10^{6} \text{ Hz})}{3 \times 10^{8} \text{ m/s}} = 0.40 \text{ (nonnegligible)}.$$

(d) $$\frac{l}{\lambda} = \frac{lf}{u_p} = \frac{(1 \times 10^{-3} \text{ m}) \times (100 \times 10^{9} \text{ Hz})}{3 \times 10^{8} \text{ m/s}} = 0.33 \text{ (nonnegligible)}.$$

2.19 A 50-Ω lossless transmission line is terminated in a load with impedance $Z_L = (30 - j50)$ Ω. The wavelength is 8 cm. Find:

(a) the reflection coefficient at the load,
(b) the standing-wave ratio on the line,
(c) the position of the voltage maximum nearest the load,
(d) the position of the current maximum nearest the load.

Solution:

(a) From Eq. (2.59),

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(30 - j50) - 50}{(30 - j50) + 50} = 0.57 \exp - j79.8^\circ.$$

(b) From Eq. (2.73),

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.57}{1 - 0.57} = 3.65.$$

(c) From Eq. (2.70)

$$d_{max} = \frac{\theta_r \lambda}{4\pi} + \frac{n\lambda}{2} = \frac{-79.8^\circ \times 8 \text{ cm}}{4\pi} \frac{\pi \text{ rad}}{180^\circ} + \frac{n \times 8 \text{ cm}}{2}$$
$$= -0.89 \text{ cm} + 4.0 \text{ cm} = 3.11 \text{ cm}.$$

(d) A current maximum occurs at a voltage minimum, and from Eq. (2.72),

$$d_{min} = d_{max} - \lambda/4 = 3.11 \text{ cm} - 8 \text{ cm}/4 = 1.11 \text{ cm}.$$

2.20 A 300-Ω lossless air transmission line is connected to a complex load composed of a resistor in series with an inductor, as shown in Fig. P2.20. At 5 MHz, determine: (a) Γ, (b) S, (c) location of voltage maximum nearest to the load, and (d) location of current maximum nearest to the load.

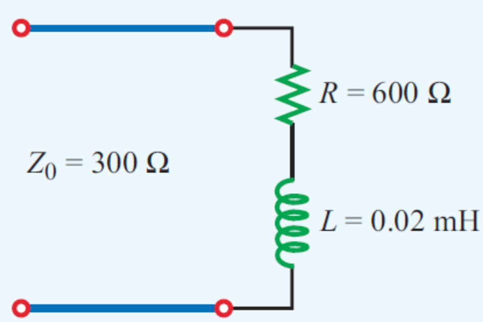

Figure P2.20 Circuit for Problem 2.20.

Solution:

(a)

$$Z_L = R + j\omega L$$
$$= 600 + j2\pi \times 5 \times 10^6 \times 2 \times 10^{-5} = (600 + j628)\ \Omega.$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$
$$= \frac{600 + j628 - 300}{600 + j628 + 300}$$
$$= \frac{300 + j628}{900 + j628} = 0.63e^{j29.6^\circ}.$$

(b)

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.63}{1 - 0.63} = 1.67.$$

(c)

$$l_{max} = \frac{\theta_r \lambda}{4\pi} \qquad \text{for } \theta_r > 0.$$
$$= \left(\frac{29.6^\circ \pi}{180^\circ}\right) \frac{60}{4\pi}, \qquad \left(\lambda = \frac{3 \times 10^8}{5 \times 10^6} = 60 \text{ m}\right)$$
$$= 2.46 \text{ m}$$

(d) The locations of current maxima correspond to voltage minima and vice versa. Hence, the location of current maximum nearest the load is the same as location of voltage minimum nearest the load. Thus

$$l_{min} = l_{max} + \frac{\lambda}{4}, \qquad \left(l_{max} < \frac{\lambda}{4} = 15 \text{ m}\right)$$
$$= 2.46 + 15 = 17.46 \text{ m}.$$

2.24 A 50-Ω lossless line terminated in a purely resistive load has a voltage standing-wave ratio of 3. Find all possible values of Z_L.

Solution:

$$|\Gamma| = \frac{S-1}{S+1} = \frac{3-1}{3+1} = 0.5.$$

For a purely resistive load, $\theta_r = 0$ or π. For $\theta_r = 0$,

$$Z_L = Z_0 \left[\frac{1+\Gamma}{1-\Gamma}\right] = 50 \left[\frac{1+0.5}{1-0.5}\right] = 150\ \Omega.$$

For $\theta_r = \pi,\ \Gamma = -0.5$ and

$$Z_L = 50 \left[\frac{1-0.5}{1+0.5}\right] = 15\ \Omega.$$