Problem 3.39 For the vector field $\mathbf{E} = \hat{\mathbf{x}}xz - \hat{\mathbf{y}}yz^2 - \hat{\mathbf{z}}xy$, verify the divergence theorem by computing:

(a) the total outward flux flowing through the surface of a cube centered at the origin and with sides equal to 2 units each and parallel to the Cartesian axes, and

(b) the integral of $\nabla \cdot \mathbf{E}$ over the cube's volume.

Problem 3.40 For the vector field $\mathbf{E} = \hat{\mathbf{r}}10e^{-r} - \hat{\mathbf{z}}3z$, verify the divergence theorem for the cylindrical region enclosed by $r = 2$, $z = 0$, and $z = 4$.

Problem 3.41 A vector field $\mathbf{D} = \hat{\mathbf{r}}r^3$ exists in the region between two concentric cylindrical surfaces defined by $r = 1$ and $r = 2$, with both cylinders extending between $z = 0$ and $z = 5$. Verify the divergence theorem by evaluating:

(a) $\oint_S \mathbf{D} \cdot d\mathbf{s}$,

(b) $\int_{\mathcal{V}} \nabla \cdot \mathbf{D} \, d\mathcal{V}$.