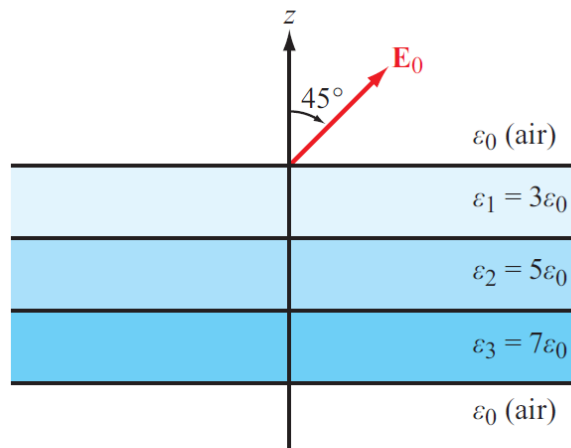


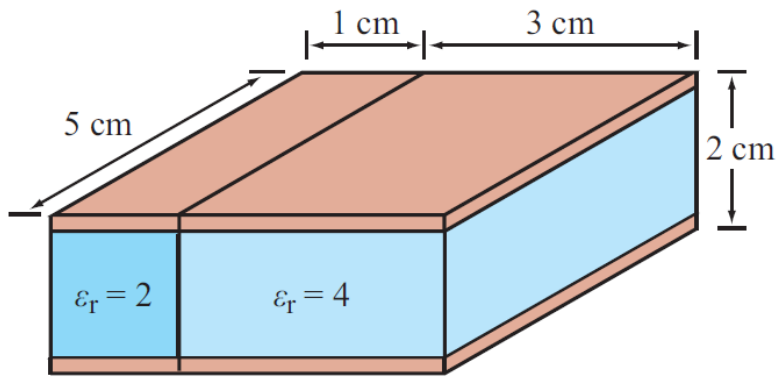
**Problem 4.51** Figure P4.51 shows three planar dielectric slabs of equal thickness but with different dielectric constants. If  $\mathbf{E}_0$  in air makes an angle of  $45^\circ$  with respect to the  $z$ -axis, find the angle of  $\mathbf{E}$  in each of the other layers.



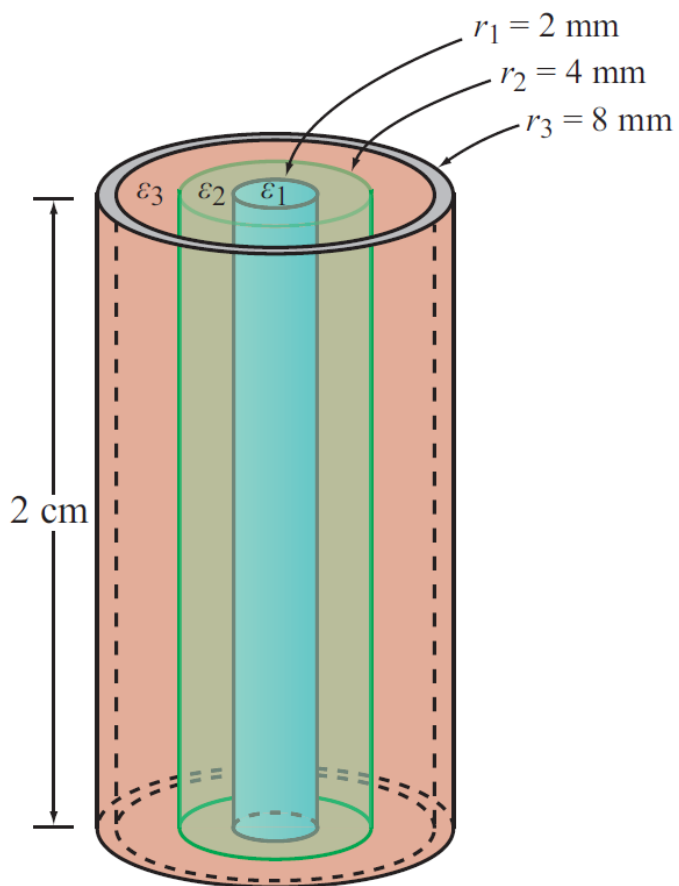
**Figure P4.51:** Dielectric slabs in Problem 4.51.

**Problem 4.57** Use the result of Problem 4.56 to determine the capacitance for each of the following configurations:

- (a) Conducting plates are on top and bottom faces of the rectangular structure in Fig. P4.57(a).
- (b) Conducting plates are on front and back faces of the structure in Fig. P4.57(a).
- (c) Conducting plates are on top and bottom faces of the cylindrical structure in Fig. P4.57(b).



(a)



$$\epsilon_1 = 8\epsilon_0; \epsilon_2 = 4\epsilon_0; \epsilon_3 = 2\epsilon_0$$

(b)

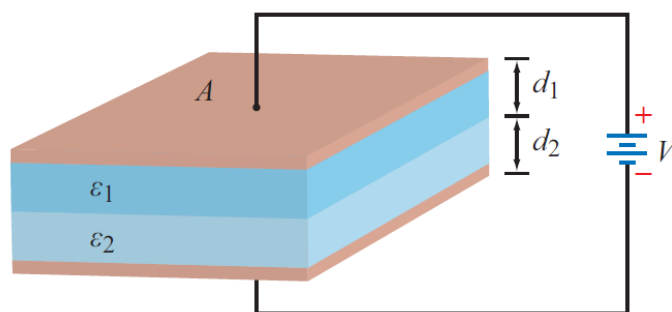
**Problem 4.58** The capacitor shown in Fig. P4.58 consists of two parallel dielectric layers. Use energy considerations to show that the equivalent capacitance of the overall capacitor,  $C$ , is equal to the series combination of the capacitances of the individual layers,  $C_1$  and  $C_2$ , namely

$$C = \frac{C_1 C_2}{C_1 + C_2} \quad (22)$$

where

$$C_1 = \epsilon_1 \frac{A}{d_1}, \quad C_2 = \epsilon_2 \frac{A}{d_2}$$

- (a) Let  $V_1$  and  $V_2$  be the electric potentials across the upper and lower dielectrics, respectively. What are the corresponding electric fields  $E_1$  and  $E_2$ ? By applying the appropriate boundary condition at the interface between the two dielectrics, obtain explicit expressions for  $E_1$  and  $E_2$  in terms of  $\epsilon_1$ ,  $\epsilon_2$ ,  $V$ , and the indicated dimensions of the capacitor.
- (b) Calculate the energy stored in each of the dielectric layers and then use the sum to obtain an expression for  $C$ .
- (c) Show that  $C$  is given by Eq. (22).



(a)



(b)

**Figure P4.58:** (a) Capacitor with parallel dielectric layers, and (b) equivalent circuit (Problem 4.58).